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Convective Heat Transfer in a Three-Dimensional Tubular Exchanger Filled with Pure/Hybrid Water-Based Nanofluid and Exposed to the Magnetic Field Effects

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ABSTRACT

The aim of this study is to evaluate, numerically, the effect of different nanoparticle volume fractions, on a heat transfer in a tubular heat exchanger. The main objective is to control this process under the effect of a magnetic field. The nanoparticles used for this analysis include water-based pure (alumina: Al_2O_3 and copper Cu) and hybrid (Al_2O_3-Cu) nanoparticles. This work is considered for laminar and stationary conditions in co-current mode flow. The computational analysis is performed under the CFD/Fluent code. The magnetic induction used is around [0.1 to 0.6] Tesla, and is applied in conjunction with the exchanger axis. The comparative study shows that copper nanofluids have a significant effect on heat transfer compared with alumina because: in co-current mode and for B=0T, Re=50 and φ=1% volume fraction of nanoparticles, the efficiency of copper reached 88.25%, while alumina was 87.12%. In addition, the heat transfer coefficient and the friction factor can be controlled by the magnetic field because curves h(B) and C_f(B) show, under the growth of a magnetic field B, the heat transfer coefficient increases autonomously, while the friction coefficient decreases.

1. INTRODUCTION

Heat transfer and its improvement is the subject that continues to attract the scientific community attention. The interest into this process is due to its various industrial applications. Transport, renewable energies, food processing and petrochemicals are among the most popular areas of heat exchangers application [1-10]. The intensive use of these tools has extended to the adaptation of heat-sensitive components (such as batteries, oils, solar cells, etc.) to the most difficult thermal operating conditions. In addition, to meet industrial requirements in terms of reliability and durability, heat exchangers have undergone several innovations in the geometric structure. Moreover, to assess the performance of nanofluids in heat exchange, several tests have been carried out on suspension of pure [1, 2, 9-12] and hybrid water-based nanoparticle [6, 13-16]. To highlight the performance of nanofluids in this area, it is important, to look at the previous works;

Choi and Eastman [7] and Eastman et al. [8] have shown that a suspension of metal nanoparticles placed in the heat transfer fluid improves its thermal conductivity. They also observed that, for a volume concentration of alumina (Al_2O_3) between 1% and 5%, the effective thermal conductivity of the water- Al_2O_3 mixture increased by 20%.

The study reported by Das et al. [10] discusses in detail the growth of thermal conductivity as a function of temperature for nanofluids composed of water-based Al_2O_3/Cu nanoparticles.

In their paper, Nakhchi and Esfahani [12] presented a numerical analysis of a Cu-water nanofluid flow within a circular conduit embedded in a twisted ribbon with an alternating axis. Their results show that the heat transfer coefficient rises to 23.20% when the nanoparticle volume fraction increases from 0% to 1.5%. Furthermore, thermal properties improve with increasing nanoparticle volume fraction within the duct.

A study performed by Moghadassi et al. [15] has highlighted the effect of a 0.1% volume fraction of pure water/Al_2O_3 and hybrid water/ Al_2O_3-Cu nanofluid on forced convection heat transfer under laminar flow conditions. Results revealed that the convective heat transfer coefficient was more significant for the hybrid nanofluid.

Xuan and Li [11] carried out an experimental study of convective heat transfer and flow characteristics of a nanofluid inside a tubular heat exchanger. Their results showed the nanofluid exhibited a higher heat transfer coefficient than the base liquid for a given Reynolds number. The friction coefficient obtained for the nanofluid (water/Cu) was approximately equal to that of the basic liquid.

In their experimental study, Suresh et al. [13] developed a hybrid nanofluid (Al_2O_3-Cu/water) with a volume density of 0.1% to 2% nanoparticles. Their results showed that the thermal conductivity and viscosity of hybrid nanofluids increased proportionally to the nanoparticle density. Considering both parameters, the increase in viscosity was found to be significantly greater than that of thermal conductivity. When thermal conductivity was measured experimentally, a maximum increase of 12.11% was observed at a volume concentration of 2%.

The experimental study carried out by Suresh et al. [14] showed that the Nusselt number improved by a maximum of 13.56% at a Reynolds number of 1,730 compared to the water Nusselt number. It was also shown that Al_2O_3-Cu/water hybrid nanofluids at 0.1% have a slightly higher friction factor than Al_2O_3/water nanofluids at 0.1%.

In parallel, a large number of researchers have been interested in hybrid nanofluids submitted to the magnetic field; among them:

Das et al. [17] conducted a research on the magnetohydrodynamic (MHD) flow of Al_2O_3/water nanofluid and Cu- Al_2O_3/water hybrid nanofluid through a porous channel in the presence of a transverse magnetic field.

Sheikholeslami [18] used a novel computational method, the MHD flow of a nanofluid through a porous enclosure where Al_2O_3 of different shapes have been dispersed in water. Results illustrated that convection decreased with increasing magnetic forces.

In this light, the goal of this work is to investigate, at first, the performances of a tubular heat exchanger using a volume fraction of pure (Al_2O_3, Cu,) and hybrids (Al_2O_3/Cu or Cu/ Al_2O_3) water based nanoparticles. The choice of tubular heat exchangers as a topic for our study is due, on one hand, to their robustness under extreme temperature and pressure conditions. On the other hand, these devices are used in refrigeration and heating systems, as well as in car radiators, the main focus of our study. The vehicle heat exchanger is the device that increases engine power by acting on the inlet air temperature within the engine block. So, to ensure the reliability of traction motors, automotive heat exchanger need to transfer heat more efficiently. This survey may be completed by subjecting the nanofluids to the effects of magnetic fields [4-19]. This test is dedicated to control the heat transfer stability and to master flow viscosity and friction. Concerning the computational analysis of the nanofluid performances, it will be carried out for different volume fractions of pure and hybrid water-based nanoparticles [1-23]. The co-current mode flow will be studied for a laminar Reynolds number. The mathematical formulation [24-26] will be established and defined by the flow mode and boundary conditions, as well as by the configuration and dimensioning of the studied sample. In this context, the study will be divided into six sections organized as follows: The first section is devoted to the presentation of generalities on nanofluids and the interest of the magnetic field, as well as a bibliographical synthesis of the various works published in this field. The second one describes the study model. The third deals specifically with the governing equations and boundary conditions, in 3D. The fourth presents the thermo-physical properties of the nanofluid. The fifth section presents the main numerical results of the study, together with interpretations and analyses of the various results. Finally, a general conclusion summarizes the main obtained results.

2. DESCRIPTION OF THE STUDY MODEL

Heat exchangers have a direct impact on a system's energy efficiency. Consequently, the existing heat exchanger architecture is based on exchangers with specific characteristics and almost exclusively using metallic materials such as copper or aluminum. Regarding exchanger sizing, it depends on operating conditions. In vehicles, priority is given to reducing size and weight, and increasing performance [27].

The configuration of the study model is given by Figure 1. The tubular exchanger is composed by a simple coaxial tube with a circular section of 1m long with an internal diameter Di = 0.01 m and an external one D_{ex}=0.02 m, the thickness of the internal wall is considered negligible. The hot water flows through the inner tube while the nanofluid (water/% nanoparticles) passes through the annular section. It should be remembered that nanoparticles (Cu or Al_2O_3) are nanotubes. Also, they are considered to be homogeneously suspended in water, the basic fluid [28].

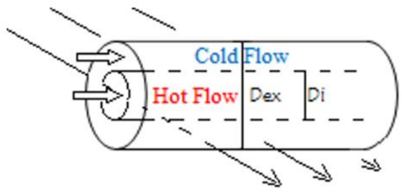


Figure 1. Study model

Both hot and cold fluids are animated by velocity calculated from the chosen Reynolds number (50, 100 and 200). The thermo physical properties of the exchanger fluids are given in Table 1:

Table 1. Thermo-physical properties of the exchanger fluids

	K (w/m k)	C_p (J/kg k)	ρ (kg/m³)	μ (kg/m s)	P_r
Pure Water	0.5984	4174.84	997.21	0.001001	6.6
(Al_2O_3)	36.96	785.02	3950	/	/
Cu	400	385	8933	/	/

3. MATHEMATIC FORMULATION

Governed by the incompressible and coupled Navier-Stokes and energy equations, the mathematical formulation is in line with the following model, with laminar and steady flow as a function of the Reynolds number.

3.1 Continuity equation

Deduced from the principle of mass conservation, continuity equation is described by the expression below:

$$\frac{d\rho}{dt} + \rho\nabla \cdot \vartheta = 0 \quad (1)$$

where, ρ denotes Density, (kg.m^{-3}), V represents velocity vector (m.s^{-1}).

3.2 Momentum equation

This equation is defined, according to the second principle of dynamics, as follows:

$$\rho\left(\frac{\partial\vartheta}{\partial t} + (\vartheta \cdot \nabla)\vartheta\right) = -\nabla P + \nabla \cdot (\mu(\nabla\vartheta + \acute{\nabla}\vartheta)) + f \quad (2)$$

where, $\frac{\partial \vartheta}{\partial t}$ represents the change of velocity with time, $\vartheta \cdot \nabla\vartheta$ is convective term, $-\nabla P$ is pressure gradient, $\nabla \cdot (\mu(\nabla\vartheta + \acute{\nabla}\vartheta))$ is the viscous force, f is external forces.

3.2.1 Momentum equations incomprehensible fluid

Considering the viscous and incomprehensible fluid, the momentum equations can be written in their compact form as in equation:

$$\rho\left(\frac{\partial \vartheta}{\partial t} + (\vartheta.\nabla)\vartheta\right) = -\nabla P + \mu\nabla^2\vartheta + f$$

3.2.2 MHD effects

If this fluid is in mobility in a magnetic field B_0, it will create, through Ohm's law, the electric current represented by:

$$j_{induit} = \sigma\vartheta \times B_0$$

These currents will, in return, generate an induced field:

$$rotB_{induit} = \mu_0 j_{induit}$$

This last modifies the total magnetic field given by:

$$B_{total} = B_0 + B_{induit}$$

We see here how the hydrodynamic force modifies the electromagnetic field.

Moreover, Laplace force is given by:

$$f_{Laplace} = j_{induit} \wedge B_0$$

It will influence the fluid to oppose the initial movement and we see here how the electromagnetic fields will modify the hydrodynamic force.

If placed in the limit where the flow is incompressible (divϑ = 0); Then we get:

$$\frac{\partial B}{\partial t} + (\vartheta.\nabla)B = (B.\nabla)\vartheta + \eta\Delta B$$

In this equation the term $(\vartheta.\nabla)B$ corresponds to the convective transport of the magnetic field, the term $(B.\nabla)\vartheta$ to the stretching of the velocity field by the magnetic field B and $\eta\Delta B$ to the transport term by diffusion of the magnetic field where $\eta = 1/(\mu_0\sigma)$ is the magnetic diffusibility.

Acknowledging that Lorentz's force is only interacting with the flux through its magnetic part, the Navier-Stokes equation takes the following form:

$$\rho\left(\frac{\partial \vartheta}{\partial t} + (\vartheta.\nabla)\vartheta\right) = -\nabla p + \mu\Delta\vartheta + j \wedge B \qquad (3)$$

3.3 Energy equation

Given by the first principle of thermodynamics, the energy equation is expressed in the following form:

$$\rho C_p\left(\frac{\partial T}{\partial t} + \vartheta \cdot \nabla T\right) = \nabla \cdot (k\,\nabla T) + q + \emptyset \qquad (4)$$

where, C_p is the specific heat at constant pressure (J.Kg^{-1}.K^{-1}). k is the thermal conductivity (W.m^{-1}.K^{-1}). T is the temperature (K). q is the heat flux lost by thermal conduction (Wm^{-2}); ϕ denotes the heat flux lost through radiation (Wm^{-2}).

4. THE NANOFLUID THERMOPHYSICAL PROPERTIES

Equations used to estimate the effective properties of the nanofluids derived, typically, from Einstein's (1906) work. Using the volume fraction φ given by Eq. (5):

$$\varphi = \frac{v_p}{v_T} \qquad (5)$$

where, v_p: volume of solid particles m^3; v_T: Total volume m^3.

The thermo-physical properties of the nanofluid, assumed constant and independent of temperature, are calculated from the following relations:

4.1 Thermal conductivity (K_{nf})

Thermal conductivity (K_{nf}) is a very important property for improving the thermal performance of the heat transfer fluid. The synthesis of nanoparticles allows increasing the thermal conductivity of the basic fluid. The model used to estimate the value of the suspension thermal conductivity is the Hamilton-Crosser model:

$$k_{nf} = \frac{k_p + (n-1)k_f - (n-1)(k_f - k_p)\varphi}{k_p + (n-1)k_f + \varphi(k_f - k_p)}$$

where, n is the empirical form factor which is equal to six (06) in the case of a cylindrical nanoparticle.

4.2 Dynamic viscosity (μ_{nf})

According to the Einstein Model, it is expressed by:

$$\mu_{nf} = \mu_f(2.5\varphi + 1)$$

where, μ_{nf}: The dynamic viscosity of the nano fluid. μ_f: The dynamic viscosity of the base fluid. φ: The volume fraction of the nanoparticles.

4.3 Density (ρ_{nf})

Pack and Cho model is given by,

$$\rho_{nf} = \varphi\rho_p + (1-\varphi)\rho_f$$

where, ρ_{nf}: Density of the nano fluid. ρ_p: Density of the nanoparticle. ρ_f: Density of the fluid.

4.4 The mass heat (C_p)

The term used is Pak and Cho:

$$(c_p)_{nf} = (1-\varphi)(c_p)_f + \varphi(c_p)_p$$

where,

Cp_{nf}, Cp_f and Cp_p: are the specific heat of the nanofluid, the fluid and the nanoparticle respectively. The inlet and outlet conditions are given by Table 2.

Table 2. Thermo-physical properties of the base fluid (water) and the nanoparticles

Limits	Hydrodynamic Conditions	Thermal Conditions
Z=0 (the inlet)	u=0; v=0; w=w_0	T_c=350k T_f=293.15k
Z=L (the outlet)	$\frac{\partial w}{\partial z}=0$ (pressure outlet)	T=T_s
Adiabatic sections	u=0; v=0; w=0	q=0 or $\frac{\partial T}{\partial r}=0$ for: r=R_e, $0\leq Z\leq L$ $0\leq \theta \leq 2\pi$

To improve their contribution, the researchers opted to use hybrid nanofluids [13, 14]. Based on the governing equations of mixture, the density and volumetric capacity of the hybrid nanofluid are calculated as follows:

$$\rho_{nf} = \varphi_{Cu}\rho_{Cu} + \varphi_{Al2O3}\rho_{Al2O3} + (1-\varphi)\rho_f$$

With: $\varphi = \varphi_{Cu} + \varphi_{Al2O3}$ the volume concentration of the nanoparticle mixture is the sum for copper and alumina concentrations.

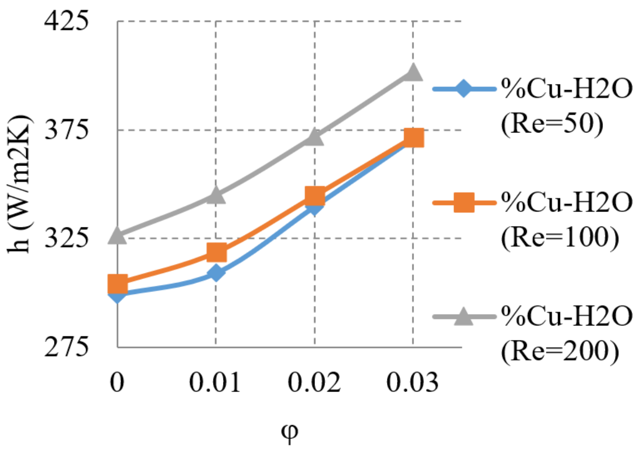
$$(\rho c_p)_{nf} = \varphi_{Cu}(\rho c_p)_{Cu} + \varphi_{Al2O3}(\rho c_p)_{Al2O3} + (1-\varphi)(c_p)_f$$

5. COMPUTATIONAL SIMULATION AND DISCUSSION

To investigate the performance of this kind of nanofluid, a 3D model is built in Gambit. The Navier-Stocks and energy equations are solved using the finite volume method of the Computational Fluid Dynamics (Fluent) code. Concerning the survey input parameter; the study is carried out for a given volume fraction φ (0≤φ≤0.03) of both pure and hybrid nanoparticles. To solve this set of equations, we adopted the following assumptions:

- At first, flow is unidirectional, steady and laminar.
- The nanofluid is considered Newtonian and incompressible.
- The outer wall is assumed to be adiabatic and the thickness of the inner wall negligible.

In light of the nanoparticles' thermophysical properties, the following Figures 2-7 illustrate the results:



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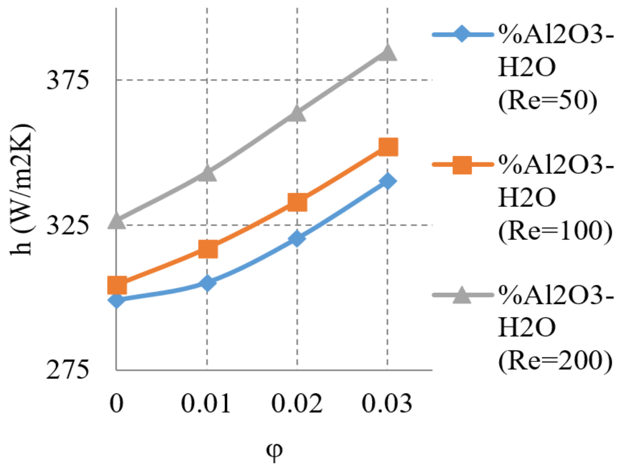

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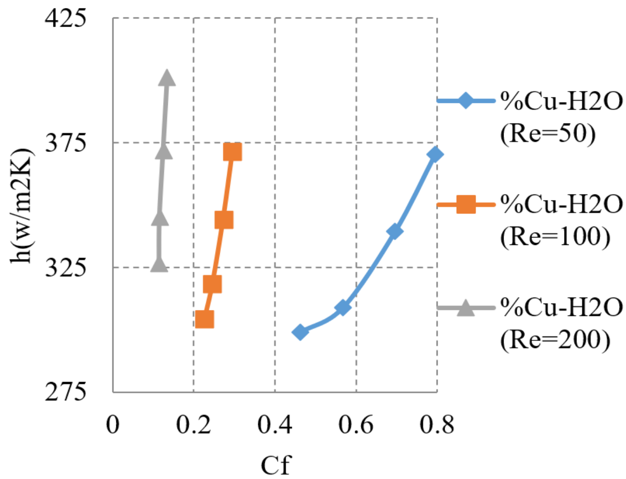
Figure 2. (a) Evolution of the heat exchange coefficient as a function of volume fraction of suspended Cu nanoparticles at different Reynolds numbers; (b) Evolution of the heat exchange coefficient as a function of volume fraction of suspended Al_2O_3 nanoparticles at different Reynolds numbers



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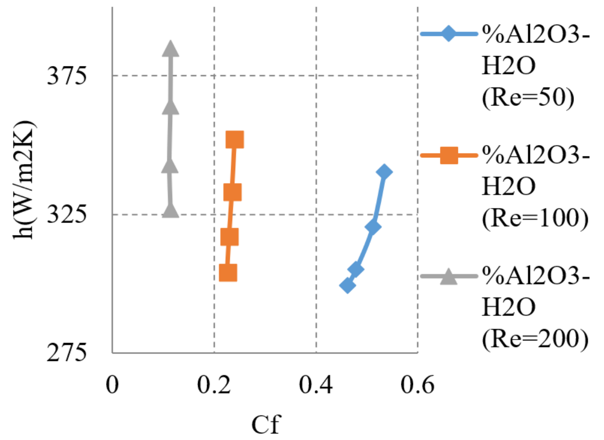


(b)

Figure 3. (a) Evolution of the coefficient of heat exchange as a function of the coefficient of friction of the nanofluid (H_2O/%Cu) h_{Cu} (C_f) for different Reynolds numbers; (b) Evolution of the coefficient of heat exchange as a function of the friction coefficient of the nanofluid (H_2O/% Al_2O_3) h_{Al2O3} (C_f) for different Reynolds number

Figure 2 shows, for different Reynolds numbers, the evolution of the heat exchange coefficient as a function of the volume fraction of Cu nanoparticles (i.e. Al_2O_3) water-based. The results showed that: increasing the volume fraction of nanoparticles φ improves the heat transfer. To validate this result, we computed the heat transfer efficiency for water alone and in presence of 1% of the tested nanoparticles. We found that the transfer efficiency increased from 72.92% for water alone to 88.25% in Cu and 87.12% in alumina for the case of B=0T and Re=50. The same remarks can be made when increasing the Reynolds number. These results are validated experimentally in references [11, 13].

Figure 3 presents the results of the study show a weak coefficient of friction (C_f) obtained for a high Reynolds number (Re=200) for the two pure nanofluids (H_2O/%Al_2O_3) and (H_2O/%Cu).

Figure 4 presents the comparative study carried out between the two pure nanofluids (H_2O/%Al_2O_3) and (H_2O/%Cu) in the same conditions for B=0T and Re=50 shows that the exchange coefficient is better in the case of a strong suspension of Cu nanoparticles. Moreover, the nanoparticle density led to a considerable increase in the coefficient of friction (C_f).

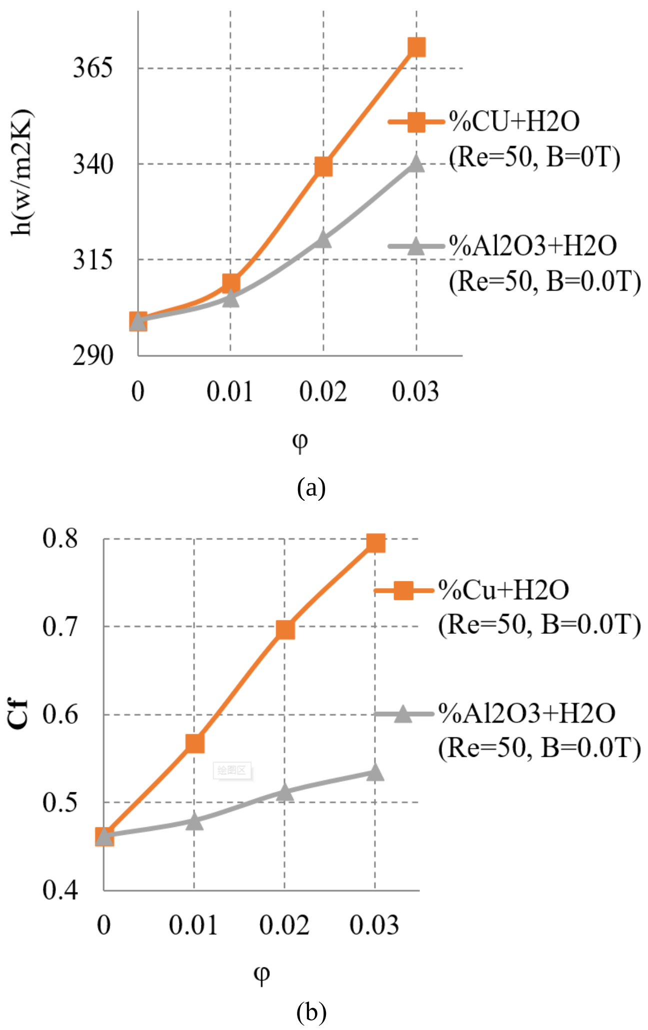


Figure 4. (a) Comparative study of the heat exchange coefficient of nanofluids (H_2O/% Al_2O_3) and (H_2O/%Cu) for different φ, with the Reynolds number Re=50 and no magnetic field B; (b) Comparative study of the friction coefficient of nanofluids (H_2O/% Al_2O_3) and (H_2O/%Cu) for different φ, with Reynolds number Re=50 and no B

Figure 5 defines two study zones for the two hybrid nanofluids (H_2O/1% Al_2O_3/%Cu) and (H_2O/1%Cu/% Al_2O_3). The first zone corresponds to φ<1%: In this zone, the hybrid nanofluid with the highest exchange coefficient corresponds to (H_2O/1%Cu/% Al_2O_3). However, to this highest heat exchange coefficient correspond the higher friction coefficient (C_f). In the second zone with φ>1%, the hybrid nanofluids corresponding to the highest h and Cf is (H_2O/1%Al_2O_3/%Cu).

Regarding the effect of the magnetic field on nanofluid heat transfer, the results are presented in both Figures 6 and 7.

Figure 6 illustrates, for Re=50 and Cu density of 0% to 3% range, the growth of the hybrid nanofluid (H_2O+10%Al_2O_3/%Cu) heat exchange coefficient. Using different values of B: B=0.1T, B=0.3T and B=0.6T, the heat exchange coefficient increases steadily. Concurrently, the evolution of the friction coefficient C_f can be divided into two phases: at low copper density (φ<1%), the coefficient of friction increases, but above this value, the growth of the friction coefficient decreases.

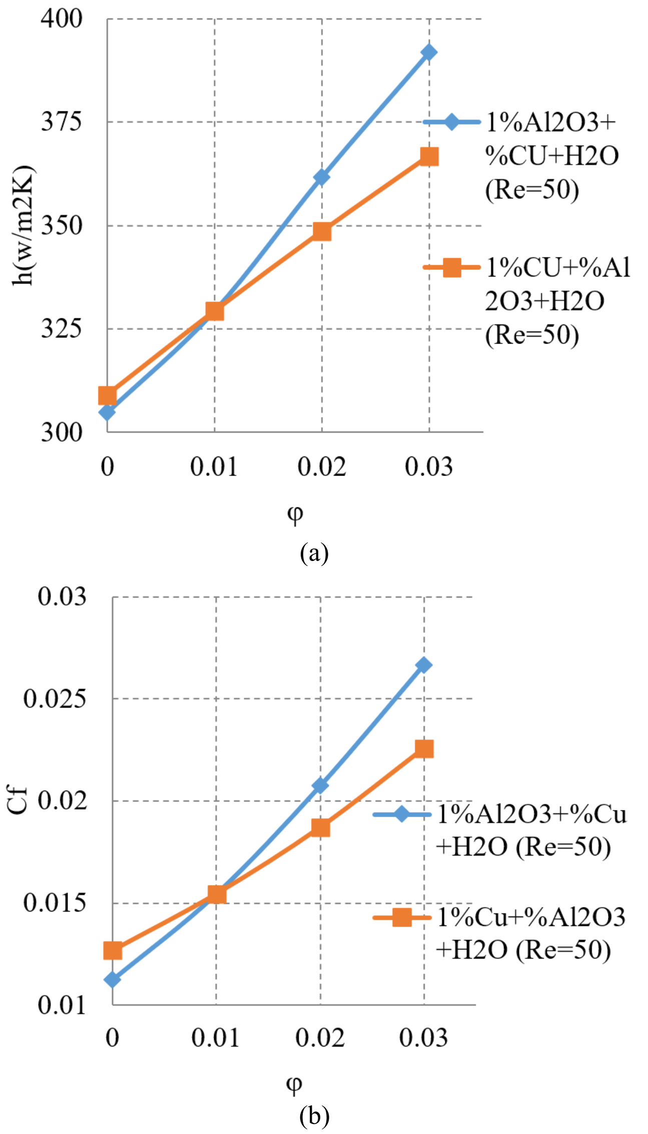
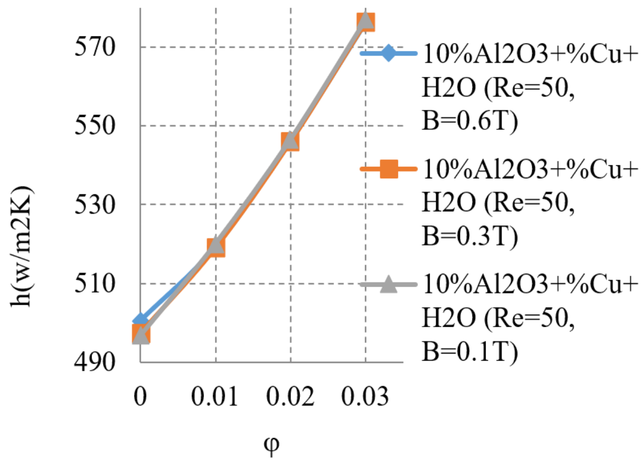


Figure 5. (a) Comparative study of the heat exchange coefficient of hybrid nanofluids (H_2O+1% Al_2O_3/%Cu) and (H_2O+1%Cu/% Al_2O_3) for different φ, at Re=50 and no magnetic field B; (b) Comparative study of the friction coefficient of hybrid nanofluids (H_2O+1% Al_2O_3/%Cu) and (H_2O+1%Cu/% Al_2O_3) for different φ, at Re=50 and no B



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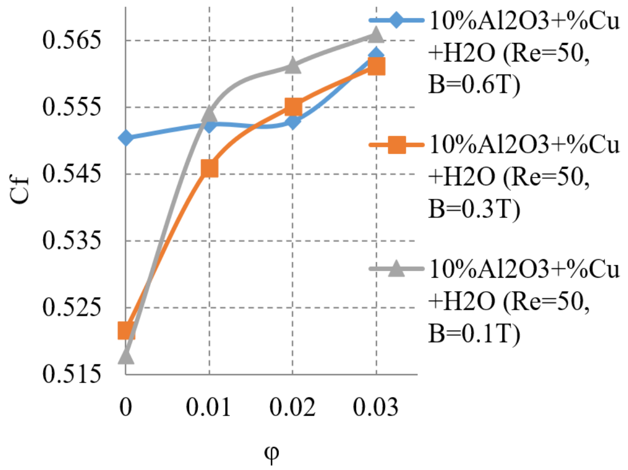

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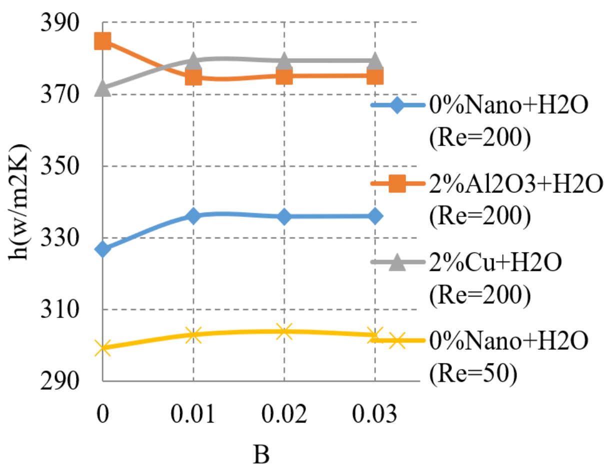
Figure 6. (a) Evolution of the heat exchange coefficient of the hybrid nanofluid (H_2O+10% Al_2O_3/%Cu) at Re=50 and for: 0%< φ < 3%, for different B: B=0.1T, B=0.3T and B=0.6T; (b) Evolution of the friction coefficient of the hybrid nanofluid (H_2O+10% Al_2O_3/%Cu) at Re=50 for: 0%<φ <3%, and B=0.1T, B=0.3T and B=0.6T

Figure 7 shows the evolution of the heat exchange coefficient as a function of the intensities of B. As B increases, h ceases to increase. The same observation can be made for variations in the coefficient of friction depending on B.



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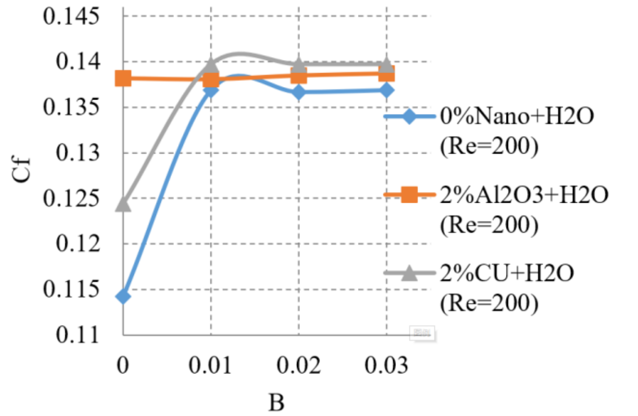

(b)

Figure 7. (a) Comparative study of the heat exchange coefficient of pure nanofluids (H_2O/2%Al_2O_3) and (H_2O/2%Cu) at Re=200 submitted to the magnetic field B intensities; (b) Evolution of the friction coefficient of nanofluids (H_2O/2%Al_2O_3) and (H_2O/2%Cu) at Re=200 submitted to the magnetic field B intensities

6. CONCLUSIONS

This study has assessed, numerically, various characteristics of a water-based nanofluid in a 3D tubular heat exchanger, of 1 meter length. The survey concerns the convective heat transfer in a three-dimensional tubular exchanger filled with pure/hybrid water-based nanofluid and exposed to the magnetic field. The aim of this study aims to improve the heat exchange efficiency of a tubular exchanger. Results show:

1-The assessment of the heat exchange coefficient as a function of the volume fraction of Cu nanoparticles at different Reynolds numbers, indicate that as the Reynolds number increases, the exchange coefficient increases accordingly. The same observation can be made for alumina.

2-Concerning the evolution of the heat exchange coefficient as a function of the nanofluids (H_2O/%Cu) and (H_2O/% Al_2O_3) friction coefficients, for various Reynolds numbers, we can say that to reach a suitable compromise between the heat transfer coefficient and the friction coefficient (i.e.: a high h and a low C_f), a high Reynolds number (Re=200) must be used.

3-With regard to the comparative study of the nanofluids (H_2O/% Al_2O_3) and (H_2O/%Cu) heat exchange coefficient, for the Reynolds number Re=50 and using different concentrations of the nanoparticle, we conclude that the best heat exchange coefficient is obtained for copper.

4-With regard to the comparative study of the nanofluid (H_2O/% Al_2O_3) and the nanofluid (H_2O/%Cu), it is observed that the best heat exchange coefficient is obtained for copper with Reynolds number Re=50, but this is at the detriment of a high friction coefficient.

5-Regarding the hybrid nanofluid (H_2O+10% Al_2O_3/%Cu) at Re=50 and for various concentrations of Cu nanoparticle, the heat exchange coefficient increases steadily and the growth of the friction coefficient decreases when the hybrid nanofluid is subjected to an increasing magnetic field B.

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NOMENCLATURE

B	magnetic fields Tesla
C_p	specific heat, J. kg^{-1}. K^{-1}
g	gravitational acceleration, m.s^{-2}
k	thermal conductivity, W.m^{-1}. K^{-1}
P_r	Prandtl number
p	pressure of the fluid [Pa]
$\dot{q}$	the source term [W.m^{-3}]
T	temperature [k]
u_j	velocity vector in three directions (m.s^{-1}).
V_p	volume of solid particles [m^3]

Greek Symbols

∂	operator of the partial derivative
β	coefficient of thermal expansion
φ	volume fraction of the nanoparticles
ν	Kinematic viscosity, [m^{2}s^{-1}]
μ	nanofluid dynamic viscosity (the considered material) [kg. m^{-1}.s^{-1}]
ρ	density of the considered material (nano fluid, fluid or nanoparticle) [kg.m^{-3}]

Subscripts

p	nanoparticle
f	fluid (pure water)
nf	nanofluid