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Application of a Modified Gauss Elimination Technique for Separable Fuzzy Nonlinear Programming Problems

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ABSTRACT

In this study, a novel approach to resolving separable fuzzy nonlinear programming problems is presented. Utilizing a parametric form, the issues associated with separable fuzzy nonlinear programming, particularly those arising from uncertainty, ambiguity, and vagueness, are addressed. To resolve these issues, each separable function within the Separable Fuzzy Nonlinear Programming Problem (SFNPP) is approximated via a piecewise linear function. This approximation is then subjected to the standard graphical and simplex techniques to obtain a solution. Significantly, a novel variant of the Gauss elimination method for inequalities, specifically designed for separable fuzzy nonlinear programming problems, has been developed and implemented. Compared to previous methods, our approach offers notable advantages in terms of reduced computational time and enhanced precision, due to the simplicity of the calculations involved.

1. INTRODUCTION

Separable programming problems are a specific class of challenges within the domain of nonlinear programming. These problems share the characteristic that both their objective functions and constraints can be represented as summations of functions involving a single variable. Nonlinear programming, which serves as a broader model for optimization, is by no means a narrow framework. Instead, it encompasses a wide range of scenarios characterized by nonlinearity, a prevalent feature in our natural surroundings.

To illustrate, consider the case of doubling the dosage of a medicinal drug. The relationship between dosage and its efficacy is rarely a straightforward linear correlation due to the intricate nonlinear dynamics that underlie biological processes. Similarly, increasing the manpower on a project by twofold doesn't inherently lead to a proportional reduction in completion time. These practical instances highlight how nonlinearity is a pervasive aspect of diverse real-world situations.

Nonlinear programming techniques play a pivotal role in optimizing objective functions involving non-negative variables while adhering to both linear and nonlinear constraints. The core objective within this context is to streamline the computational process required to optimize the given problem. This involves treating the objective function as an inherent constraint. Such constraints are prevalent in various domains, including but not limited to statistical data fitting, logistics, the design and management of water distribution systems, and the analysis of electrical networks. Within the scope of separable fuzzy nonlinear programming, a distinctive situation arises. Here, the objective function and constraints, some of which might contain nonlinear elements, can be effectively defined and articulated.

Given the intricacies surrounding the elimination of inequalities, Gauss' initial method has undergone significant refinements within specialized fields. The computational framework of variable elimination presents a clear and straightforward approach, making it particularly appealing for integration into interpreters designed to tackle the complexities of next-generation constraint programming problems. Exploring the possibilities of variable elimination within broader linear constraints, utilizing techniques like Gaussian elimination and its extended methodologies, emerges as a topic of substantial importance and scholarly curiosity.

Adabitabar et al. [1] applied the Gauss elimination technique and the investigation into variable elimination within linear constraints over real numbers has been explored by Allahviranloo et al. [2]. Chakraborty and Singh [3] applied some note that successfully addressing separable fuzzy nonlinear programming problems demands a solid grasp of optimization principles and fuzzy logic. Tailoring the approach to the problem's unique characteristics and available resources is essential for effective implementation studied by Chandru [4]. Darby-Dowman et al. [5] present a comprehensive strategy for dealing with a wide spectrum of optimization problems. This approach aims to establish a common framework that can be applied cohesively to address diverse problem types, including linear, integer, separable, and fuzzy programming problems. These studies have specifically concentrated on constraints that exist within discrete domains. Hedayatfar et al. [6] introduce a novel approach to address optimization challenges characterized by separable programming problems and constraints expressed through max-product fuzzy relation equations.

Jain and Mangal [7-10] undertook a comprehensive exploration of various elimination techniques, especially for fractional programming problems. Their later work [10] extended this inquiry to the realm of problems involving multiple linear objectives. Other researchers, such as Kanniappan and Thangavel [11], Karmarker [12], Kohler [13], Whereas Mayer [14] significantly contributed to enhancing the applicability of the interval Gaussian method. Sharma and Bhargava [15] proposed They have introduced unique elimination methodologies customized to effectively tackle the difficulties posed by linear programming problems, Umamaheswari and Ganesan [16] present a methodological approach for tackling optimization problems that involve both fuzziness and nonlinearity studied by Williams [17], have presented distinct elimination methodologies tailored to address challenges in linear programming problems.

This research focuses on a novel approach that leverages piecewise fuzzy linear approximation to effectively tackle separable fuzzy nonlinear programming problems. The concept of separable fuzzy is elucidated within this context, and a modified Gauss elimination method is employed to provide a robust solution for these nonlinear programming challenges.

The organization of the remaining content in this paper is as follows: In Section 2, the foundational aspects are introduced, followed by an exploration of the application of the modified Gauss elimination technique to address inequalities in Section 3. Section 4 delves into the integration of the SFNPP's objective function within the discussed constraints. The subsequent section provides insights into the resolution of the SFNPP's systems of inequalities using the adapted Gauss elimination method. Finally, the paper concludes by presenting a numerical example.

2. PRELIMINARIES

2.1 Fuzzy set

If X is a universal set and $x \in X$, then a fuzzy set $\widetilde{A}$ defined as a collection of ordered pairs:

$$\widetilde{A} = \{x, \mu_{\widetilde{A}}(x); x \in X\} \tag{1}$$

where, $\mu_{\widetilde{A}}$ is called the membership function that maps X to the membership space M.

2.2 Triangular fuzzy number

A fuzzy number $\widetilde{A}$ on R is said to be a triangular fuzzy number if its membership function $\widetilde{A}: R \rightarrow [0,1]$ has the following characteristics:

$$\widetilde{A}(x) \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 \le x \le a_3 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

We denote this triangular fuzzy numbers $\widetilde{A} = (a_1, a_2, a_3)$. We use F(R) to denote the set of all triangular fuzzy numbers.

2.3 Separable programming

A Function $f(\mathbf{x_1}, \mathbf{x_2}, \dots . \mathbf{x_n})$ is said to be separable if it can be expressed as the sum of n single valued functions $\mathbf{f_1}(\mathbf{x_1}), \mathbf{f_2}(\mathbf{x_2}), \dots . \mathbf{f_n}(\mathbf{x_n})$.

$$\text{i.e. } f(\mathbf{x_1}, \mathbf{x_2}, \dots . \mathbf{x_n}) = \mathbf{f_1}(\mathbf{x_1}), \mathbf{f_2}(\mathbf{x_2}), \dots . \mathbf{f_n}(\mathbf{x_n}). \tag{3}$$

2.4 Parametric form

Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number then the parametric form of the TFN is defined as $\tilde{A} = (a_0, a_*, a^*)$. Where $a_* = a_0 - \underline{a}$, $a^* = \overline{a} - a_0$, $a_0 = a_2.r \in [0,1]$.

2.5 Theorem: (Separation)

Let $\tilde{A}$ and $\tilde{B}$ represent two nonempty disjoint convex subsets of R_n. Then a hyper plane exists that separates them, i.e., there is a nonzero vector *c* in R_n and a scalar α such that:

$$\left.\begin{array}{ll} cx \le \alpha, & for\ all\ x \in \tilde{A} \\ cx \ge \alpha, & for\ all\ x \in \tilde{B} \end{array}\right\} \tag{4}$$

3. FORMULATION OF THE MODIFIED GAUSS ELIMINATION PROBLEM [14]

Here, we take the LPP as:

$$\begin{gathered} \text{Max } Z = mx + \alpha \\ Ax \le b \\ x \ge 0 \end{gathered} \tag{5}$$

In order to apply the modified Gauss elimination approach to the LPP, by rephrasing it with the objective function as constraints and all constraints having the same sign of inequality. Hence, the reduced form of LPP using the improved Gauss elimination approach is as:

$$\begin{gathered} Max\ Z \\ Z - (mx + \alpha) \le 0 \\ Ax \le b \\ -x \le 0 \end{gathered} \tag{6}$$

We write $l_j \le x_j \le u_j$, a minimum of one ordered pair (l_j, u_j) gives a practical solution.

4. SEPARABLE NON-LINEAR PROGRAMMING PROBLEM

Consider the nonlinear programming problem (NLPP)

$$\begin{gathered} \text{Max (or Min) } f_j(x_1, x_2, \dots . x_n) \\ g_{ij}(x_1, x_2, \dots . x_n) \le b_i \\ x_j \ge 0 \end{gathered} \tag{7}$$

If the restrictions and objective function can be expressed separately, it can be written as:

$$\begin{gathered} f_j(x_1, x_2, \dots . x_n) = \sum_{j=1}^{n} f_j(x_j) \\ g_{ij}(x_1, x_2, \dots . x_n) = \sum_{j=1}^{n} g_{ij}(x_j) \end{gathered} \tag{8}$$

Hence, the separable nonlinear programming issue is expressed as:

$$\text{Max (or Min) } \sum_{j=1}^{n} f_j(x_j)$$
$$\sum_{j=1}^{n} g_{ij}(x_j) \leq b_i \quad (9)$$
$$x_j \geq 0$$

where, some or all g_{ij}, x_{ij} and $f(x_j)$ are non linear.

$$\text{Max.(or Min.) } \sum_{j=1}^{n} \sum_{k=1}^{K_j} f_j(a_{jk})\, w_{jk}$$
$$\sum_{j=1}^{n} \sum_{k=1}^{K_j} g_{ij}(a_{jk})\, w_{jk} \leq b_i$$
$$0 \leq w_{j1} \leq y_{j1}$$
$$0 \leq w_{jk} \leq y_{j,k-1} + y_{jk} \quad (10)$$
$$0 \leq w_j K_j \leq y_{j,} K_{j-1}$$
$$\sum_{K=1}^{K_j} w_{jk} = 1, \sum_{k=1}^{K_{j-1}} y_{jk} = 1$$
$$y_{jk} = 0 \text{ or } 1.$$

5. NUMERICAL EXAMPLES

Here we take our proposed method an example of SFNPP as:

$$\text{Max } Z = \tilde{x}_1 + \tilde{x}_2^4 \quad (11)$$

Subject to:

$$3\tilde{x}_1 + 2\tilde{x}_2^2 \leq \tilde{9}$$
$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

Let us assume that,

$$\tilde{1} = (0,1,2) \qquad \tilde{1} = (0,1,3) \qquad \tilde{3} = (2,3,4)$$
$$\tilde{2} = (1,2,3) \qquad \tilde{9} = (8,9,10)$$

Hence, the problem may be expressed as,

$$\text{Max } Z = (0,1,2)\tilde{x}_1 + (0,1,3)\tilde{x}_2^4$$

Subject to:

$$(2,3,4)\tilde{x}_1 + (1,2,3)\tilde{x}_2^2 \leq (8,9,10)$$
$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

Step-1: Creating the provided problem's standard form requires taking the objective function and putting it in maximising form, however in this case the problem is already in maximising form, therefore move on to the next step.

Step-2: Gather the separable components of the given problem.

$$f_1(x_1) = (0,1,2)\tilde{x}_1,\ f_2(x_2) = (0,1,3)\tilde{x}_2^4$$
$$g_{11}(x_1) = (2,3,4)\tilde{x}_1,\ g_{12}(x_2) = (1,2,3)\tilde{x}_2^2$$

Used the parametric form,

$$(a_0, a_*, a^*) = (a_1, a_2, a_3)$$
$$(a_0, a_*, a^*) = (1, 1-\kappa, 1-\kappa)$$
$$f_1(x_1) = (1, 1-\kappa, 1-\kappa)\tilde{x}_1,$$
$$f_2(x_2) = (1, 1-1\kappa r, 2-2\kappa)\tilde{x}_2^4$$
$$g_{11}(x_1) = (3, 1-\kappa, 1-\kappa)\tilde{x}_1$$
$$g_{12}(x_2) = (2, 1-\kappa, 1-\kappa)\tilde{x}_2^2$$

Step-3: When limitations are applied, it is shown that:

$$\text{Max } Z = (1, 1-\kappa, 1-\kappa)\tilde{x}_1 + (1, 1-\kappa, 2-2\kappa)\tilde{x}_2^4$$

Subject to:

$$(3, 1-\kappa, 1-\kappa)\tilde{x}_1 + (2, 1-\kappa, 1-\kappa)\tilde{x}_2^2$$
$$\leq (9, 1-\kappa, 1-\kappa)$$
$$\tilde{x}_1, \tilde{x}_2 \geq 0 \quad (12)$$
$$x_1 \leq (3, 1-\kappa, 1-\kappa) \qquad x_2 \leq \left(\sqrt{\tfrac{9}{2}}, 1-r, 1-r\right)$$
$$x_1 \leq (0, 1-\kappa, 1-\kappa) \qquad x_2 \leq (2.1, 1-\kappa, 1-\kappa)$$

The upper limit for the variables $x_1 \& x_2$ is $(3, 1-\kappa, 1-\kappa)$ and the lower limit for the variables $x_1 \& x_2$ are $(0, 1-\kappa, 1-\kappa)$.

Step-4: Now considering non-linear $f_2(x_2)$ and $g_{12}(x_2)$ by converting breaking points at (*k*=4) into linear form:

Table 1. Breaking point

$\boldsymbol{\kappa}$	$\mathbf{a_{2k}}$	$\mathbf{f_2(a_{2k}) = (1, 1-\kappa, 2-2\kappa)\tilde{x}_2^4}$	$\mathbf{g_{12}(a_{2k}) = (2, 1-\kappa, 1-\kappa)\tilde{x}_2^2}$
1	0	0	0
2	1	$(1, 1-\kappa, -2\kappa)$	$(2, 1-\kappa, 1-\kappa)$
3	2	$(16, 1-\kappa, 2-2\kappa)$	$(8, 1-\kappa, 1-\kappa)$
4	3	$(81, 1-\kappa, 2-2\kappa)$	$(18, 1-\kappa, 1-\kappa)$

From the Table 1 breaking point given as,

$$f_2(x_2) \cong w_{21}f_2(a_{21}) + w_{22}f_2(a_{22}) + w_{23}f_2(a_{23}) + w_{24}f_2(a_{24})$$
$$\cong w_{21}\,0 + w_{22}(1, 1-\kappa, 2-2\kappa) + w_{23}(16, 1-\kappa, 2-2\kappa) + w_{24}(81, 1-\kappa, 2-2\kappa)$$
$$= w_{22}(1, 1-\kappa, 2-2\kappa) + w_{23}(16, 1-\kappa, 2-2\kappa) + w_{24}(81, 1-r, 2-2r)$$

$$g_{12}(x_2) \cong w_{21}g_{12}(a_{21}) + w_{22}g_{12}(a_{22}) + w_{23}g_{12}(a_{23}) + w_{24}g_{12}(a_{24})$$
$$\cong w_{21}0 + w_{22}(2, 1-\kappa, 1-\kappa) + w_{23}(8, 1-\kappa, 1-\kappa) + w_{24}(18, 1-\kappa, 1-\kappa)$$
$$= w_{22}(2, 1-\kappa, 1-\kappa) + w_{23}(8, 1-\kappa, 1-\kappa) + w_{24}(18, 1-\kappa, 1-\kappa)$$

Step-5: Thus, the reduced LPP is:

$$\text{Max } Z = (1, 1-\kappa, 1-\kappa)\tilde{x}_1 + w_{22}(1, 1-\kappa, 2-2\kappa) + w_{23}(16, 1-\kappa, 2-2\kappa) + w_{24}(81, 1-\kappa, 2-2\kappa)(3, 1-\kappa, 1-\kappa)\tilde{x}_1 + w_{22}(2, 1-\kappa, 1-\kappa) + w_{23}(8, 1-\kappa, 1-\kappa) + w_{24}(18, 1-\kappa, 1-\kappa) \leq (9, 1-\kappa, 1-\kappa)(1, 1-\kappa, 1-\kappa)w_{21} + (1, 1-\kappa, 1-\kappa)w_{22} + (1, 1-\kappa, 1-\kappa)w_{23} + (1, 1-\kappa, 1-\kappa)w_{24} = (1, 1-\kappa, 1-\kappa) \quad (13)$$

$$w_{21}, w_{22}, w_{23}, w_{24} \geq 0$$

With the additional restrictions that:

(i) There are more than two positive w_{jk} for each j=1, 2 and

(ii) A pair of positive w_{jk} must be neighbouring points if there are two of them.

Modified Gauss elimination techniques will be applied to solve the given approximate LPP.

Making use of the modified Gauss elimination approach and using all inequalities of the same sign as constraints, we have:

$$\begin{gathered}\text{Max } \tilde{Z} \\ Z - (1,1-\kappa,1-\kappa)\tilde{x}_1 - w_{22}(1,1-\kappa,2-2\kappa) \\ - w_{23}(16,1-\kappa,2-2\kappa) \\ - w_{24}(81,1-\kappa,2-2\kappa) \leq 0 \\ (3,1-\kappa,1-\kappa)\tilde{x}_1 + w_{22}(2,1-\kappa,1-\kappa) + \\ w_{23}(8,1-\kappa,1-\kappa) + w_{24}(18,1-\kappa,1-\kappa) \leq \\ (9,1-\kappa,1-\kappa) \\ (1,1-\kappa,1-\kappa)w_{21} + (1,1-\kappa,1-\kappa)w_{22} \\ + (1,1-\kappa,1-\kappa)w_{23} \\ + (1,1-\kappa,1-\kappa)w_{24} \\ \leq (1,1-\kappa,1-\kappa) \\ -(1,1-\kappa,1-\kappa)w_{21} \leq 0 \\ -(1,1-\kappa,1-\kappa)w_{22} \leq 0 \\ -(1,1-\kappa,1-\kappa)w_{23} \leq 0 \\ -(1,1-\kappa,1-\kappa)w_{24} \leq 0 \\ -(1,1-\kappa,1-\kappa)\tilde{x}_1 \leq 0\end{gathered} \tag{14}$$

Using modified Gauss elimination in the first step to remove w_{21}, we have:

$$\begin{gathered}\text{Max } \tilde{Z} \\ Z - (1,1-\kappa,1-\kappa)\tilde{x}_1 - w_{22}(1,1-\kappa,2-2\kappa) \\ - w_{23}(16,1-\kappa,2-2\kappa) \\ -w_{24}(81,1-\kappa,2-2\kappa) \leq 0 \\ (3,1-\kappa,1-\kappa)\tilde{x}_1 + w_{22}(2,1-\kappa,1-\kappa) \\ + w_{23}(8,1-\kappa,1-\kappa) \\ +w_{24}(18,1-\kappa,1-\kappa) \leq (9,1-\kappa,1-\kappa) \\ (1,1-\kappa,1-\kappa)w_{22} + (1,1-\kappa,1-\kappa)w_{23} \\ + (1,1-\kappa,1-\kappa)w_{24} \\ \leq (1,1-\kappa,1-\kappa) \\ -(1,1-\kappa,1-\kappa)w_{22} \leq 0 \\ -(1,1-\kappa,1-\kappa)w_{23} \leq 0 \\ -(1,1-\kappa,1-\kappa)w_{24} \leq 0 \\ -(1,1-\kappa,1-\kappa)\tilde{x}_1 \leq 0\end{gathered} \tag{15}$$

Using modified Gauss elimination in the first step to remove w_{22}, we have:

$$\begin{gathered}\text{Max } \tilde{Z} \\ Z - (1,1-\kappa,1-\kappa)\tilde{x}_1 - w_{23}(16,1-\kappa,2-2\kappa) \\ -w_{24}(81,1-\kappa,2-2\kappa) \leq 0 \\ (3,1-\kappa,1-\kappa)\tilde{x}_1 + w_{23}(8,1-\kappa,1-\kappa) \\ +w_{24}(18,1-\kappa,1-\kappa) \leq (9,1-\kappa,1-\kappa) \\ (1,1-\kappa,1-\kappa)w_{23} + (1,1-\kappa,1-\kappa)w_{24} \\ \leq (1,1-\kappa,1-\kappa) \\ -(1,1-\kappa,1-\kappa)w_{23} \leq 0 \\ -(1,1-\kappa,1-\kappa)w_{24} \leq 0 \\ -(1,1-\kappa,1-\kappa)\tilde{x}_1 \leq 0\end{gathered} \tag{16}$$

Using modified Gauss elimination in the first step to remove $\tilde{x}_1$, we have:

$$\begin{gathered}\text{Max } \tilde{Z} \\ Z - w_{23}(16,1-\kappa,2-2\kappa) - w_{24}(81,1-\kappa,2 \\ - 2\kappa) \leq 0 \\ w_{23}(8,1-\kappa,1-\kappa) + w_{24}(18,1-\kappa,1-\kappa) \leq \\ (9,1-\kappa,1-\kappa) \\ (1,1-\kappa,1-\kappa)w_{23} + (1,1-\kappa,1-\kappa)w_{24} \\ \leq (1,1-\kappa,1-\kappa) \\ -(1,1-\kappa,1-\kappa)w_{23} \leq 0 \\ -(1,1-\kappa,1-\kappa)w_{24} \leq 0\end{gathered} \tag{17}$$

Using modified Gauss elimination in the first step to remove w_{23}, we have:

$$\begin{gathered}\text{Max } \tilde{Z} \\ Z - w_{24}(65,1-\kappa,2-2\kappa) \leq (16,1-\kappa,1-\kappa) \\ w_{24}(10,1-\kappa,1-\kappa) \leq (1,1-\kappa,1-\kappa) \\ -(1,1-\kappa,1-\kappa)w_{24} \leq (1,1-\kappa,1-\kappa) \\ -(1,1-\kappa,1-\kappa)w_{24} \leq 0\end{gathered} \tag{18}$$

Rewritten as:

$$\begin{gathered}\text{Max } \tilde{Z} \\ Z - w_{24}(65,1-\kappa,2-2\kappa) \leq (16,1-\kappa,1-\kappa) \\ w_{24} \leq (1/10,1-\kappa,1-\kappa) \\ -(1,1-\kappa,1-\kappa)w_{24} \leq 0\end{gathered} \tag{19}$$

Using modified Gauss elimination in the first step to remove w_{24}, we have:

$$\begin{gathered}\text{Max } \tilde{Z} \\ Z \leq (\tfrac{45}{2},1-\kappa,2-2\kappa) \\ Z = (\tfrac{45}{2},1-\kappa,2-2\kappa)\end{gathered} \tag{20}$$

Now, Max $\tilde{Z} = \frac{45}{2}$ and using back substituting by putting $\tilde{Z} = \frac{45}{2}$ in the above Inequalities. We get $w_{24} = (1/10,1-\kappa,1-\kappa)$. Now putting $\tilde{Z} = \frac{45}{2}$ and $w_{24} = (1/10,1-\kappa,1-\kappa)$,we get $w_{23} = (9/10,1-\kappa,1-\kappa)$.

Using back substituting by putting $\tilde{Z} = \frac{45}{2}$, $w_{24} = (1/10,1-r,1-r)$ and $w_{23} = (9/10,1-\kappa,1-\kappa)$ in the inequalities, we have $w_{21} = \tilde{x}_1 = w_{22} = 0$.

Now, the solution of original SFNPP in terms of original variables $\tilde{x}_1$ and $\tilde{x}_2$, we consider $w_{24} = (1/10,1-\kappa,1-\kappa)$, $w_{23} = (9/10,1-\kappa,1-\kappa$, and $w_{21} = \tilde{x}_1 = w_{22} = 0$. Therefore, $\tilde{x}_2 = 2w_{23} + 3w_{24} = 2(9/10,1-\kappa,1-\kappa) + 3\,(1/10,1-\kappa,1-\kappa)$ = 2.1, $\tilde{x}_1 = 0$.

Hence the optimal solution of SFNPP is:

$$\begin{gathered}\tilde{x}_1 = 0, \tilde{x}_2 = (2.1,1-\kappa,1-\kappa) \\ \text{Max } \tilde{Z}=\tilde{x}_1 + \tilde{x}_2^4 \\ =0 + (2.1,1-\kappa,1-\kappa)^4 \\ \text{Max } \tilde{Z} = (19.45,1\text{-}\kappa,1\text{-}\kappa).\end{gathered}$$

5.1 Comparison chart

The proposed parametric form in the fuzzy separable programming issue yields equivalent results without converting into a crisp form, assisting in obtaining the fuzzy optimum solution.

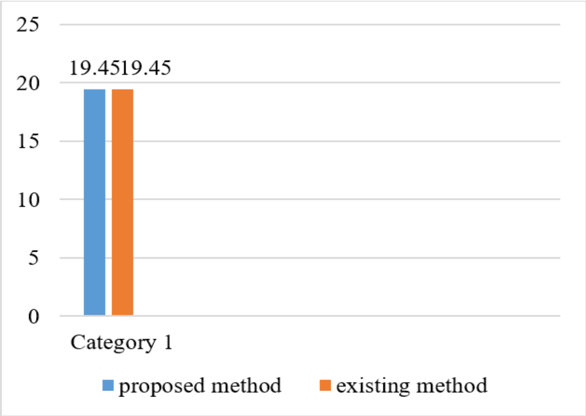

Figure 1. Graphical representation for comparison of proposed method

6. CONCLUSION

The current research introduces an innovative methodology for tackling the challenges posed by fuzzy non-linear programming problems (FNLPP). This approach is rooted in a combination of separable fuzzy programming problem (SFPP) and fuzzy set (FS) theory. The traditional technique for addressing separable fuzzy programming problems has been enhanced through the application of the Modified Gauss Elimination Technique. This modification involves scalarization and has proven to yield outcomes that are satisfactory to decision-makers. In situations where interconnected decisions come into play, the manipulation of these variables can become intricate. To overcome such complexities, this study proposes an integrated strategy built upon the foundations of separable fuzzy programming (SFP).

The strategy involves the creation of parameterized models for non-linear programming. This is achieved through the adaptation of modified Gauss elimination, utilizing trigonal fuzzy numbers. These models are specifically designed to address the challenges associated with fuzzy separable programming. The overarching aim of the Separable Fuzzy Programming approach is to provide an effective means of resolving the complexities associated with fuzzy nonlinear programming problems. To provide a tangible illustration of the approach, a practical example is presented towards the conclusion of the paper. The optimal solution obtained from this example grants decision-makers the flexibility to select a value for $\kappa \in [0,1]$ .This selection process is adaptable to specific circumstances and personal preferences, and can be executed by employing the proposed methodology outlined in Table 2.

Table 2. For various values of *r* we get the solution table

κ	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{Z}$
0	0	(2.1,1,1)	(19.45,1,1)
0.25	0	(2.1,0.75,0.75)	(19.45,0.75,0.75)
0.5	0	(2.1,0.5,0.5)	(19.45,0.5,0.5)
1	0	(2.1)	(19.45)

In the prospective trajectory, the horizon of fuzzy sets may witness the integration of diverse extensions, such as picture fuzzy sets and their subsequent variations, into the landscape of fuzzy non-linear programming. This harmonization has the potential to significantly augment the proficiency of resolving intricate challenges within this domain. Additionally, numerical methodologies, exemplified by the bisection technique and similar approaches, hold promise for effectively tackling the intricacies presented by fuzzy non-linear programming problems. Incorporating such techniques has the capacity to yield refined solutions and further elevate the advancements in this field.

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NOMENCLATURE

R	Real number
F(R)	Set of all real number
r	Parametric