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Pell Labeling in Special Graph Classes: An Exploration of Cycles, Stars, and Related Structures

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ABSTRACT

In a graph G, define Pell labeling is a map $f: V(G) \rightarrow \{0,1,\cdots,p-1\}$ with an induced function $f^*: E(G) \rightarrow N$ defined by $f^*(uv) = f(u) + 2f(v)$ for every $uv \in E(G)$ are all distinct where $u, v \leq 0$. In addition to this a graph which admits Pell labeling concept is known as Pell graph. In this paper, the Pell labeling concepts applied for the following graphs such as splitting of Star, cycle with parallel chords, alternate double triangular Snake, ladder, Triangular ladder, diagonal ladder, shadow and splitting of path, bistar, subdivision of bistar, prism, $B_{n,n}^2$ and friendship graphs are studied. Our analysis contributes to the understanding of Pell labeling across a broad spectrum of graph configurations, highlighting its applicability and the unique characteristics of each considered graph family.

1. INTRODUCTION

In 1960, the graph labeling was introduced by Gallian [1]; an interesting problem that has an assignment of integers to a set of vertices or edges as well as for both in a graph G satisfies certain mathematical conditions. It has been classified into two types such as vertex-labeling and edge-labeling. In general, a labeled graph represents a vertex-labelled graph in which each label is distinct labeled by the consecutive integers of almost the maximum number of vertices in a graph.

Shiama et al. [2-11] discussed the types of Pell-labeling concepts for some graphs namely path graph, star and double star graphs, bistar graph, cycle graph, coconut tree including $B_{m,n,k}$. Jagadeeswari et al. [3-7] introduced an extended duplication of each vertex, duplication of a pendant vertex $C_n \odot K_1$ and switching of a pendant vertex in a path by the edge to describe the square difference labeling (SDL). Shanthi Maheswari [4-10] defined a d_2-splitting of graph concept with some properties. Sumathi et al. [5, 6] presented a family of the following ladder graphs using quotient labeling numbers such as open and closed ladder graphs, open and closed triangular ladder graphs, slanting and step ladder graphs including an open diagonal ladder graph. Moussa et al. [7, 8] described an existence of difference cordial labeling, Pell labeling for inflated triangular snake graph and its an alternative snake graph. An extended duplicate graph of an arrow graph and a splitting graph defined with respect to a path that provides a Pell labeling graph in Gallian et al. [1, 8]. Sriram [9-12] discussed a Square of Path graph $2P_n$ with results related to the Pell labeling graph. Sumathi et al. [6, 10] described an extension of a quadrilateral snake graph for Pell labeling and mean square sum labeling graphs. Vaidya [11, 15] et al. explained the concepts of product cordial labeling for an alternate triangular with quadrilateral snake graphs. Shiama et al. [2, 3, 12, 13] studied the cordial, bi-conditional, difference cordial and Pell labeling ideas applied to an inflation of the alternate triangular snake graph $I(A(T_n))$ for the even values of n followed by the alternate blocks will be counted from second vertices. Sriram et al. [9, 14] discussed the value of a proper lucky labeling number of ladder graphs, slanting and triangular ladder graphs, open-triangular and diagonal ladder graphs; and also include an open diagonal ladder graphs.

In this article, we discuss Pell labeling for graph labels for graphs used in transportation, emergency response planning and topology for digital relationships, where the population is the vertices and the interconnection is the edge. We have used number theory concept to describe the functions for the vertices and their induced edges.

2. PRELIMINARIES

The basic definitions on graphs [15]. Some graphs are Splitting graph $S(G)$, Ladder graph (L_n), Triangular ladder graph $T(L_n)$, Diagonal ladder graph $D(L_n)$, Alternate Triangular Snake are referred [2, 8, 10, 11] are respectively.

3. MAIN RESULTS

Theorem 3.1
The splitting graph of $K_{1,n}$ admits Pell labeling.
Proof: Consider the graph $G = (V, E)$ be a splitting graph of $K_{1,n}$ with the vertex set $V = \{x, x', x_i, x_i'/1 \leq i \leq n\}$ and

edge set $E = \{xx_i', xx_i, x_i'x_i / 1 \leq i \leq n\}$, then $|V(G)| = 2(n+1)$ and $|E(\mathrm{G})| = 3n$.

Let G be a bijective function from $\phi: V \to \{0,1,2,\cdots\}$ defined for $1 \leq i \leq n$ as follows.

$$\phi(x) = 1,$$
$$\phi(x') = 0,$$
$$\phi(x_i) = 2i,$$
$$\phi(x_i') = 2i + 1,$$

and the induced function $\phi^*: E(G) \to N$ defined by $\phi^*(xx') = \phi(x) + 2\phi(x'), xx' \in E(G)$, yields the edge labeling as follows:

For $1 \leq i \leq n$,

$$\phi^*(xx_i') = 2i + 1,$$
$$\phi^*(xx_i) = 2i + 1,$$
$$\phi^*(x'x_i) = 4i.$$

Thus, all the vertex and edge labeling satisfy the Pell labeling.

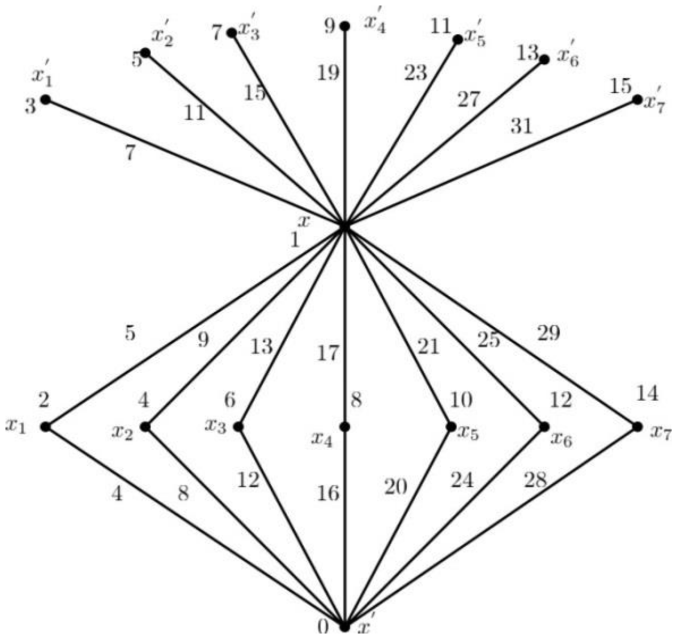

Figure 1. Pell labeling of splitting of $K_{1,7}$ graph

Thus, the entire edges are labeled and satisfies the required condition. Hence the splitting graph of $K(l, n)$ admits Pell labeling (Figure 1).

Theorem 3.2

The cycle $C_n (n \geq 6)$ with parallel chords admits Pell labeling.

Proof: Consider the graph H with vertex set $V = \{v_k / 0 \leq k \leq n-1\}$ and edge set $E = \{v_k v_{k+1}, v_k v_{n-k} / 0 \leq k \leq n-1\}$. The cardinality of vertices is n and for edges its $\frac{(3n-3)}{2}$ if odd and $\frac{(3n-2)}{2}$ if even. Let ϕ be a bijective function from the vertices $\{0,1,2,3,\cdots\}$ defined as follows:

$$\phi(v_k) = k, \quad 0 \leq k \leq n-1$$

and the induced function $\phi^*: E(H) \to N$ yields the edge labeling:

$$\phi^*(v_k v_{k+1}) = 3k + 2, \quad k = 1,3,5,\cdots$$

when n is odd $k = 4,6,8,\cdots$, when n is even, $\phi^*(v_k v_{k+2}) = 3k + 4, \quad k = 0,1,2,\cdots$, $\phi^*(v_0 v_1) = 2$.

Edge labeling is distinct hence the theorem acknowledges the labeling.

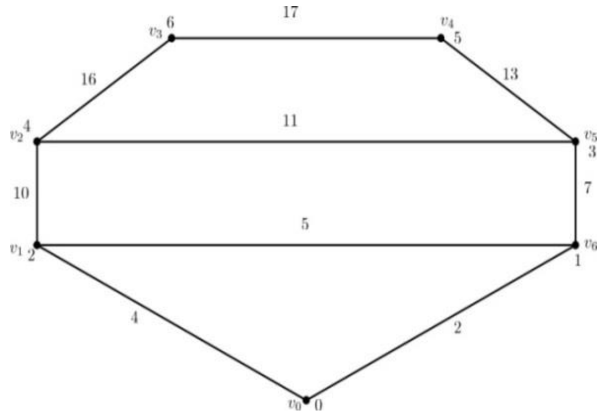

Figure 2. Pell labeling of odd cycle C_7 with parallel cords

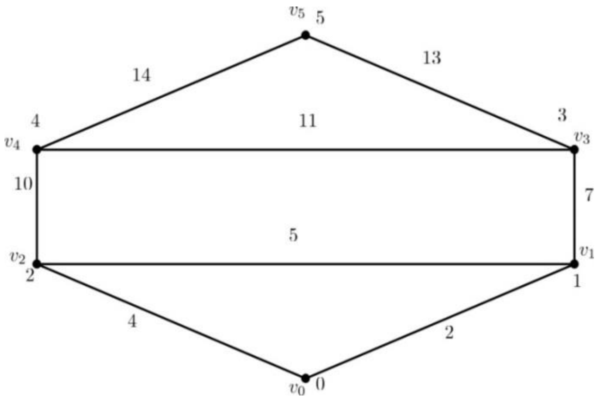

Figure 3. Pell labeling of even cycle C_6 with parallel cords

All the edge labeling are distinct. The cycle $C_n (n \geq 6)$ with parallel chords admits Pell labeling. Hence the result (Figures 2-3).

Theorem 3.3

Alternate double triangular snake admits Pell labeling (Figure 4).

Proof: Consider an alternate triangular snake graph.

Let $V = \{u_i, v_j / 0 \leq i \leq n-1, 0 \leq j \leq n\}$ be the vertices set and $E = \{u_i u_{i+1}, u_i v_j, v_j u_i / 0 \leq i \leq n-1, 0 \leq j \leq n\}$ be the edge set. The cardinality of vertices is $2n$ and edges is $3n - 1$. Let ϕ be a bijective function $\phi^*: E(G) \to N$ gives the vertices and edge labeling as follows.

$$\phi(u_i) = 2i, \quad 0 \leq i \leq n-1$$
$$\phi(v_j) = 2j - 1, \quad 0 \leq j \leq n$$
$$\phi^*(u_i u_{i+1}) = 6i + 4,$$
$$\phi^*(u_i v_j) = 5i + j + 1,$$
$$\phi^*(u_i v_j) = 6(i + 1),$$

when $i = 0,2,4,\cdots, n-1$, $j = 2,4,6,\cdots, n$; $\phi^*(v_j u_i) = 6j - 1, \quad i, j = 1,3,5,\cdots$

The entire edges are distinct. Hence the result.

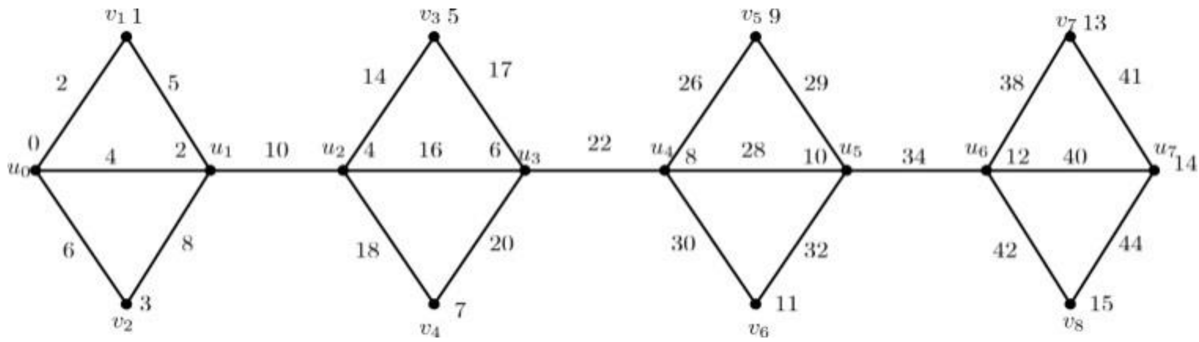

Figure 4. Pell labeling of alternate double triangular snake $A(T_8)$ graph

Theorem 3.4

The ladder graph satisfies Pell labeling (Figure 5).

Proof: Consider a ladder graph L_n. Let $V = \{u_i, v_i / 1 \leq i \leq n\}$ be the vertices set and $E = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_i / 1 \leq i \leq n\}$ be the edge set. The cardinality of vertices is $2n$ and edges is $3n - 2$. Let ϕ be a bijective function $\phi^*: E(G) \rightarrow N$ gives the vertices and edge labeling as follows.

For $0 \leq i \leq n$

$$\phi(u_i) = 2i - 1,$$
$$\phi(v_i) = 2i - 2,$$
$$\phi^*(u_i u_{i+1}) = 6i + 1,$$
$$\phi^*(v_i v_{i+1}) = 6i - 2,$$
$$\phi^*(u_i v_i) = 6i - 4.$$

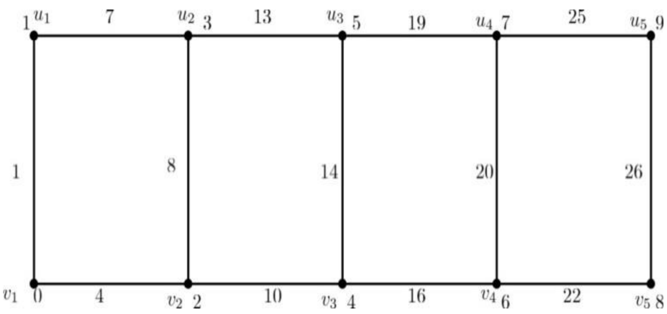

Figure 5. Pell labeling of ladder graph

The entire edges are distinct. The ladder graph admits a Pell labeling. Hence the result.

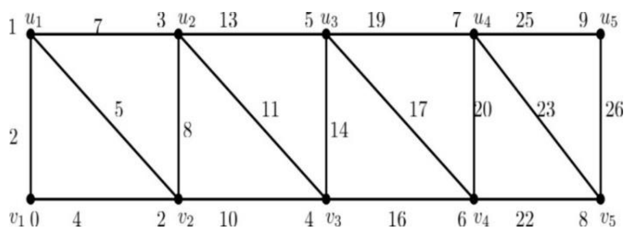

Figure 6. Pell labeling of triangular ladder graph

Theorem 3.5

The triangular ladder TL_n graph satisfies Pell labeling (Figure 6).

Proof: Consider triangular ladder TL_n. Let $V = \{u_i, v_i / 1 \leq i \leq n\}$ be the vertices set and $E = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_i, v_i u_{i+1} / 1 \leq i \leq n\}$ be the edge set. The cardinality of vertices is $2n$ and edges is $4n - 3$. Let ϕ be a bijective function $\phi^*: E(G) \rightarrow N$ gives the vertices and edge labeling as follows.

$$\phi(u_i) = 2i - 1, \quad 0 \leq i \leq n$$
$$\phi(v_j) = 2i - 2, \quad 0 \leq i \leq n$$
$$\phi^*(u_i u_{i+1}) = 6i + 1, \quad 0 \leq i \leq n$$
$$\phi^*(v_i v_{i+1}) = 6i - 2,$$
$$\phi^*(u_i v_i) = 6i - 4,$$
$$\phi^*(u_i v_{i+1}) = 6i - 1. \quad 1 \leq i \leq n$$

The entire edges are distinct. The Triangular ladder $T(L_n)$ graph admits a Pell labeling. Hence the result.

Theorem 3.6

The diagonal ladder DL_n graph satisfies Pell labeling (Figure 7).

Proof: Consider diagonal ladder DL_n. Let $V(G) = \{u_i, v_i / 1 \leq i \leq n\}$ be the vertices set and $E(G) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_i, v_i u_{i+1} / 1 \leq i \leq n\}$ be the edge set. The cardinality of vertices is $2n$ and edges is $5n - 4$. Let ϕ be a bijective function $\phi^*: E(G) \rightarrow N$ gives the vertices and edge labeling as follows.

For $1 \leq i \leq n$,

$$\phi(u_i) = 2i - 1,$$
$$\phi(v_j) = 2i - 2,$$
$$\phi^*(u_i u_{i+1}) = 6i + 1,$$
$$\phi^*(v_i v_{i+1}) = 6i - 2,$$
$$\phi^*(u_i v_i) = 6i - 4,$$
$$\phi^*(u_i v_{i+1}) = 6i - 1,$$
$$\phi^*(v_i u_{i+1}) = 6i.$$

The entire edges are distinct. Hence the result.

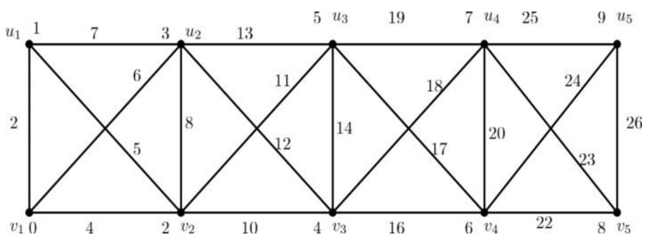

Figure 7. Pell labeling of diagonal ladder graph

Theorem 3.7

The shadow graph $D_2(p_n)$ satisfies Pell labeling (Figure 8).

Proof: Consider the shadow graph G. Let $V = \{u_i, v_i / 1 \leq i \leq n\}$ be the vertices set and $E = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1}, v_i u_{i+1} / 1 \leq i \leq n\}$ be the edge set. The cardinality of vertices is $2n$ and edges is $4n - 4$.

Let ϕ be a bijective function $\phi^*: E(G) \rightarrow N$ gives the vertices and edge labeling as follows.

For $1 \leq i \leq n$,

$$\phi(u_i) = 2i - 1,$$
$$\phi(v_j) = 2(i-1),$$
$$\phi^*(u_i u_{i+1}) = 6i + 1,$$
$$\phi^*(v_i v_{i+1}) = 6i - 2,$$
$$\phi^*(u_i v_{i+1}) = 6i - 1,$$
$$\phi^*(v_i u_{i+1}) = 6i.$$

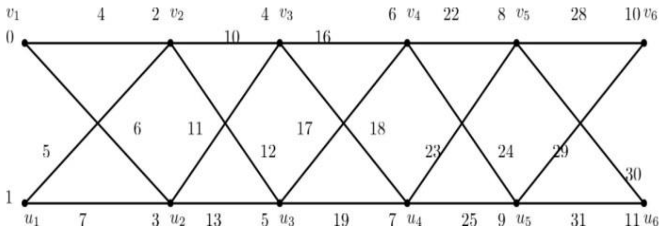

Figure 8. Pell labeling of shadow $D_2(p_n)$ graph

The entire edges are distinct. The shadow graph $D_2(P_n)$ admits a Pell labeling. Hence the result.

Theorem 3.8

The graph $S(P_n)$ satisfies Pell labeling (Figure 9).

Proof: Consider the graph G. Let $V = \{u_i, v_i / 1 \leq i \leq n\}$ be the vertices set and $E = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1}, v_i u_{i+1} / 1 \leq i \leq n\}$ be the edge set. The cardinality of vertices is $2n$ and edges is $3n - 3$.

Let ϕ be a bijective function $\phi^*: E(G) \rightarrow N$ gives the vertices and edge labeling as follows:

For $1 \leq i \leq n$,

$$\phi(u_i) = 2i - 1,$$
$$\phi(v_j) = 2(i-1),$$
$$\phi^*(u_i u_{i+1}) = 6i + 1,$$
$$\phi^*(u_i v_{i+1}) = 6i - 1,$$
$$\phi^*(v_i u_{i+1}) = 6i.$$

The entire edges are distinct. Hence the result.

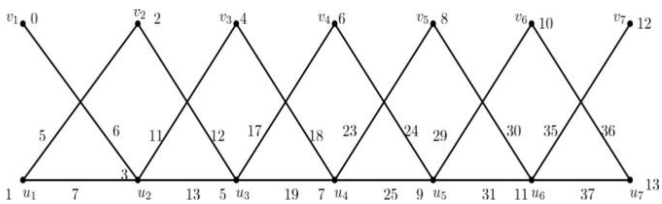

Figure 9. Pell labeling of $S(P_n)$ graph

Theorem 3.9

The bistar $B_{(n,n)}$ graph admits Pell labeling (Figure 10).

Proof: Consider the bistar graph G. Let $V = \{x, x', u_i, v_i / 1 \leq i \leq n\}$ be the vertices set and $E = \{x'u_i, xv_i / 1 \leq i \leq n\}$ be the edge set. The cardinality of vertices is $2n + 2$ and edges is $2n + 1$.

Let ϕ be a bijective function $\phi^*: E(G) \rightarrow N$ gives the vertices and edge labeling as follows:

For $1 \leq i \leq n$,

$$\phi(u_i) = 2i + 1,$$
$$\phi(v_i) = 2i,$$
$$\phi(x) = 1,$$
$$\phi(x') = 0,$$
$$\phi^*(x'u_i) = 4i + 3,$$
$$\phi^*(xv_i) = 4i.$$

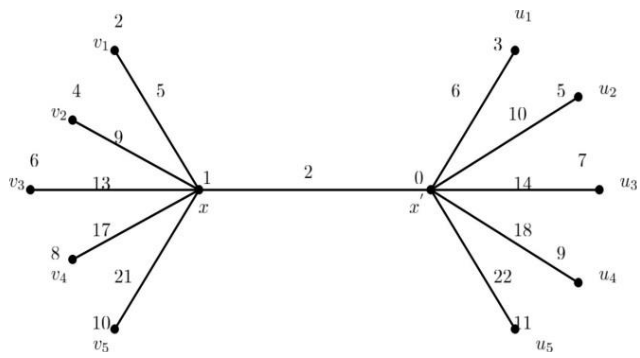

Figure 10. Pell labeling of bistar $B_{(4,4)}$ graph

The entire edges are distinct. The Bistar $B_{(n,n)}$ graph admits Pell labeling. Hence the result.

Theorem 3.10

The subdivision of bistar satisfies Pell labeling (Figure 11).

Proof: Consider the subdivision of bistar graph G.

Let $V = \{c, u, v, u_i, v_i, u_i', v_i' / 1 \leq i \leq n\}$ be the vertices set and $E = \{cu, cv, uu_i, vv_i, u_i u_i', v_i v_i' / 1 \leq i \leq n\}$ be the edge set. The cardinality of vertices is $4n + 3$ and edges is $4n + 2$. Let ϕ be a bijective function $\phi^*: E(G) \rightarrow N$ gives the vertices and edge labeling as follows:

For $1 \leq i \leq n$,

$$\phi(c) = 0,$$
$$\phi(u) = 2,$$
$$\phi(v) = 1,$$
$$\phi(u_i) = 2i + 2,$$
$$\phi(v_i) = 2i + 1,$$
$$\phi^*(cu) = 4,$$
$$\phi^*(cv) = 2,$$
$$\phi^*(uu_i) = 4i + 6,$$
$$\phi^*(u_i u_i') = 16i + 20,$$
$$\phi^*(vv_i) = 4i + 3,$$
$$\phi^*(v_i v_i') = 6i + 27.$$

The subdivision of bistar graph admits a Pell labeling.

The entire edges are distinct. Hence the result.

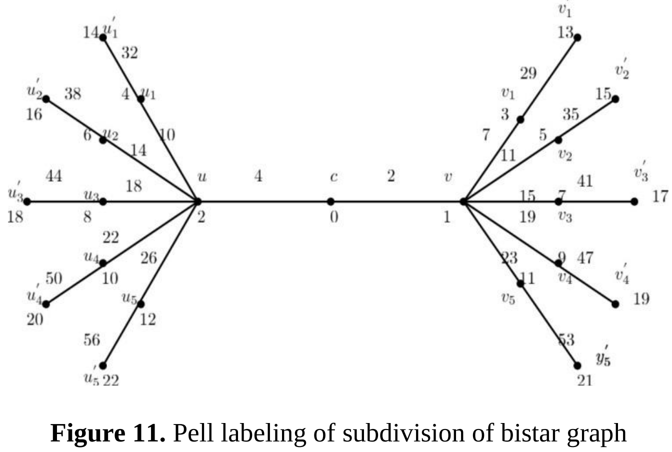

Figure 11. Pell labeling of subdivision of bistar graph

Theorem 3.11

The prism graph y_7 is a Pell labeling (Figure 12).

Proof: Consider the prism graph G.

Let $V = \{u_i, v_i / 1 \le i \le 7\}$ be the vertices set and $E = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_i, v_1 v_n, u_1 u_n / 1 \le i \le 7\}$ be the edge set. The cardinality of vertices is $2n$ and edges is $3n$. Let ϕ be a bijective function $\phi^*: E(G) \to N$ gives the vertices and edge labeling as follows:

For $1 \le i \le 7$,

$$\phi(u_i) = 2(i-1),$$
$$\phi(v_i) = 2i-1,$$
$$\phi^*(u_i u_{i+1}) = 6i-2,$$
$$\phi^*(v_i v_{i+1}) = 6i+1,$$
$$\phi^*(u_i v_i) = 6i-4,$$
$$\phi^*(u_1 u_n) = 20,$$
$$\phi^*(v_1 v_n) = 23.$$

The entire edges are distinct. Hence the prism graph.

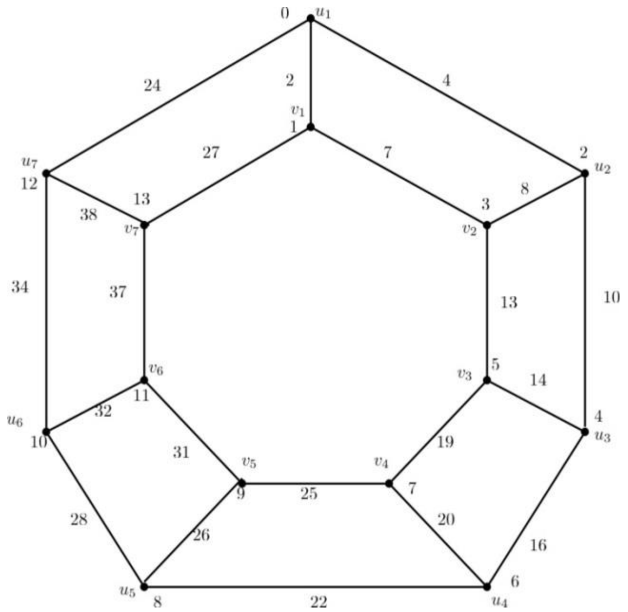

Figure 12. Pell labeling of prism graph y_7

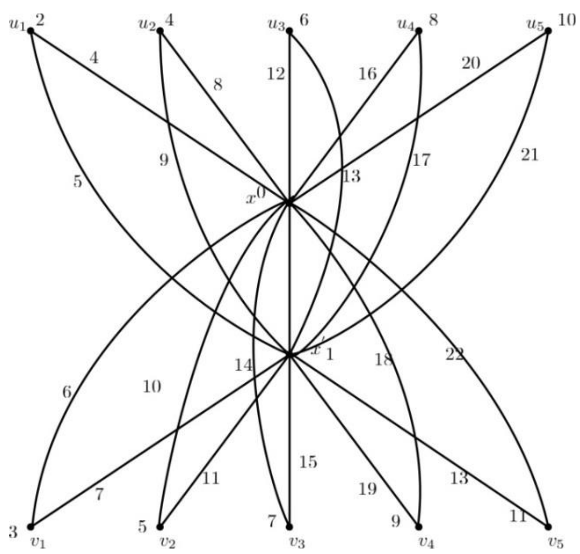

Figure 13. Pell labeling of $B^2_{5,5}$ graph

Theorem 3.12

The graph $B^2_{n,n}$ admits a Pell labeling (Figure 13).

Proof: Consider the $B^2_{n,n}$ graph G. Let $V = \{x, x', u_i, v_i / 1 \le i \le n\}$ be the vertex set and $E = \{xx', xu_i, xv_i, x'u_i, x'v_i / 1 \le i \le n\}$ be the edge set. The cardinality of vertices is $|v(G)| = 2n+2$ and edge $|E(G)| = 4n+1$ be the function $\phi^*: E(G) \to N$ gives the vertices and edge labeling as follows:

For $1 \le i \le n$,

$$\phi(x) = 0,$$
$$\phi(x') = 1,$$
$$\phi(u_i) = 2i,$$
$$\phi(v_i) = 2i+1,$$
$$\phi^*(xx') = 2,$$
$$\phi^*(xu_i) = 4i,$$
$$\phi^*(xv_i) = 4i+2,$$
$$\phi^*(x'u_i) = 4i+1,$$
$$\phi^*(x'v_i) = 4i+3.$$

The graph $B^2_{n,n}$ admits a Pell labeling. Hence the result.

Theorem 3.13

The friendship graph F_n admits a Pell labeling (Figure 14).

Proof: Consider the friendship graph G. Let $V = \{c, v_i / 1 \le i \le n\}$ be the vertices set and $E = \{v_i v_{i+1}, cv_i / 1 \le i \le n\}$ be the edge set. The cardinality of vertices is $2n+1$ and edges is $3n$. Let ϕ be a bijective function $\phi^*: E(G) \to N$ gives the vertices and edge labeling as follows:

For $1 \le i \le n$,

$$\phi(c) = 0,$$
$$\phi(v_i) = i,$$
$$\phi^*(cv_i) = 2i,$$
$$\phi^*(v_i v_{i+1}) = 6i-1,$$

Hence the result.

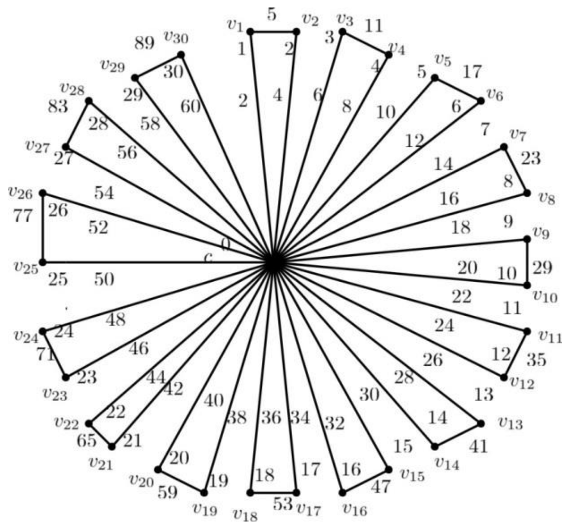

Figure 14. Pell labeling of friendship graph

Theorem 3.14

$Z - P_n$ is a Pell graph (Figure 15).

Proof: Let G be a graph with *2n* and *3n-3* number of vertices and edges respectively. Consider $V(G) = \{u_i, v_i \le i \le n\}$ and $E(G) = \{u_i u_{i+1}, v_i v_{i+1}, u_i v_{i+1} \le i \le n-1\}$.

Define a onto mapping $\phi: V(G) \to E(G)$ as:

$$\phi(u_i) = 2(i-1)$$
$$\phi(v_i) = 2i-1$$

From the above labeling, ϕ^* is defined as:

$$\phi^*(u_iu_{i+1}) = 6i - 2$$
$$\phi^*(v_iv_{i+1}) = 6i + 1$$
$$\phi^*(u_iv_{i+1}) = 6i$$

Thus $Z - P_n$ is a Pell graph.

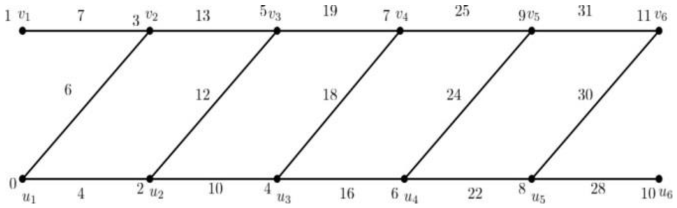

Figure 15. Pell labeling of $Z - P_6$ graph

Theorem 3.15

Moth graph is a Pell graph (Figure 16).

Proof: Let G be a Moth graph with 6 vertices and 7 edges. Let $V(G) = \{w_j \text{ for } j = 1,2,\dots,n\}$ and $E(G) = \{w_jw_{j+1}\ for\ j = 1,2\dots,n-1\}$. Then the bijective function ϕ is defined as $\phi(w_j) = j - 1$ and ϕ^* for the above labeling is defined as:

$$\phi^*(w_1w_3) = 4$$
$$\phi^*(w_2w_3) = 5$$
$$\phi^*(w_3w_4) = 8$$
$$\phi^*(w_4w_5) = 11$$
$$\phi^*(w_5w_6) = 14$$
$$\phi^*(w_3w_5) = 10$$
$$\phi^*(w_3w_6) = 12$$

Thus entire 7 edges are distinct. Hence Moth graph is Pell graph.

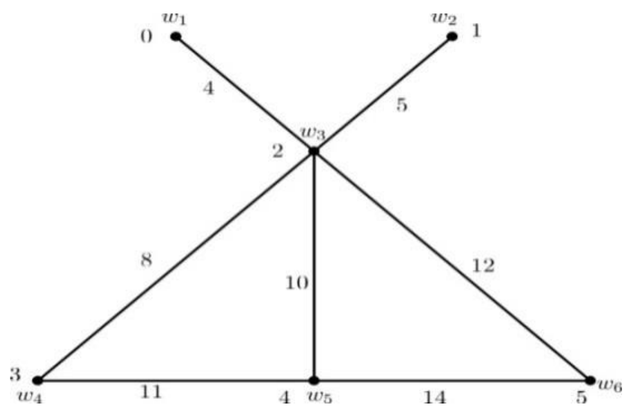

Figure 16. Pell labeling of Moth graph

Theorem 3.16

Degree splitting of Moth graph admits Pell labeling (Figure 17).

Proof: Let G be a degree splitting graph of Moth graph with 8 vertices and 11 edges. Consider a bijective function $\phi: V \to \{0,1,\dots,7\}$ defined as:

$$\phi(u_j) = j - 1\,, \quad 1 \le j \le 5$$
$$\phi(x) = 6$$
$$\phi(y) = 7$$

Thus, we receive 1-1 function as:

$$\phi^*(u_3u_4) = 8$$
$$\phi^*(u_4u_5) = 11$$
$$\phi^*(u_5u_6) = 14$$
$$\phi^*(u_3u_6) = 9$$
$$\phi^*(u_1u_3) = 4$$
$$\phi^*(u_2u_3) = 5$$
$$\phi^*(u_1x) = 12$$
$$\phi^*(u_2x) = 13$$
$$\phi^*(u_4y) = 17$$
$$\phi^*(u_6y) = 19$$
$$\phi^*(u_3u_5) = 10$$

Hence Degree splitting of Moth graph admits Pell labeling.

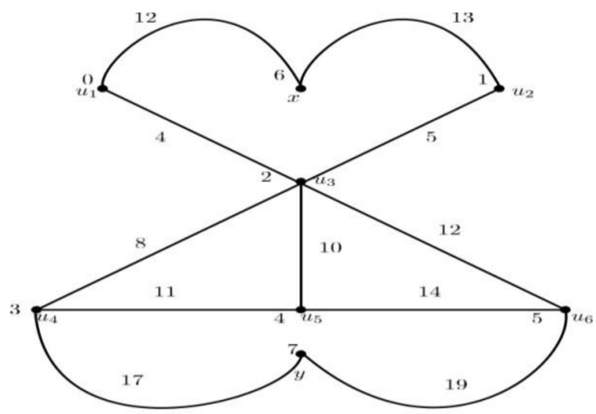

Figure 17. Pell labeling of degree splitting of Moth graph

Theorem 3.17

The graph K_1K_{n+1} is a Pell graph (Figure 18).

Proof: Let G be a graph with vertices $V(G) = \{u, v\,, u_k/1 \le k \le n\}$ and edges $E(G) = \{uu_k, uv, vu_k/1 \le k \le n-1\}$. $|V(G)| = n + 2$ and $|E(G)| = 2n + 1$. Define a mapping $\phi: V(K_1K_{n+1}) \to \{0,1,\dots,n-1\}$ as:

$$\phi(u) = 0$$
$$\phi(v) = 1$$
$$\phi(u_k) = k + 1$$

Thus, the entire $2n + 1$ edges have distinct labeling. Hence the graph K_1K_{n+1} is Pell graph.

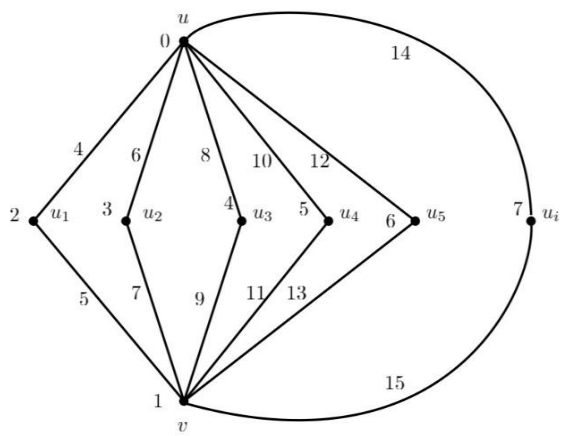

Figure 18. Pell labeling of K_1K_{5+1} graph

Theorem 3.18

Quadrilateral snake Q_n admits Pell labeling (Figure 19).

Proof: Consider the Quadrilateral snake Q_n. Let $\{u_i, v_i, w_i/$

$1 \leq i \leq n\}$ be the vertices and $\{u_i u_{i+1}, v_i w_i, u_i v_i, w_i u_{i+1} / 1 \leq i \leq n\}$ be the edges. The cardinality of the vertices is 3n-2, $n \geq 1$ and edges is $4(n-1)$. Let ϕ be a bijective function from $f^*: E(G) \to N$ gives the vertices and edge labeling as follows:

For $1 \leq i \leq n$,

$$\phi(u_i) = 3i\text{-}3$$
$$\phi(v_i) = 3i - 2$$
$$\phi(w_i) = 3i\text{-}1$$
$$\phi^*(u_i u_{i+1}) = 9i - 3$$
$$\phi^*(v_i w_i) = 9i - 4$$
$$\phi^*(u_i v_i) = 9i - 7$$
$$\phi^*(w_i u_{i+1}) = 9i - 1$$

Hence Quadrilateral snake Q_n admits Pell labeling.

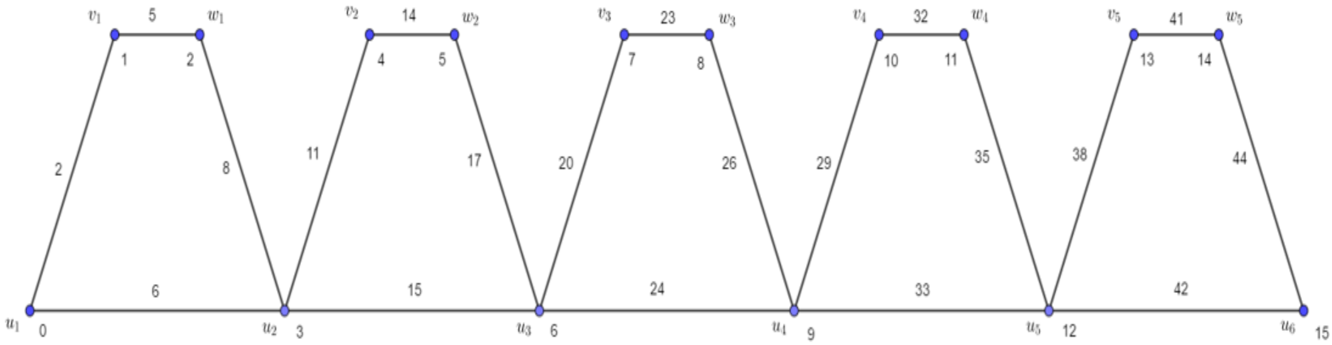

Figure 19. Quadrilateral snake Q_6

Theorem 3.19

Alternate Quadrilateral snake AQ_n admits Pell labeling (Figure 20).

Proof: Consider the Alternate Quadrilateral snake AQ_n. Let $\{u_i, v_i, x_i, y_i \,/ 1 \leq i \leq n\}$ be the vertices and $\{v_i u_{i+1}, v_i y_i, u_i v_i, u_i x_i, x_i y_i / 1 \leq i \leq n\}$ be the edges. Let f be a bijective function from $\phi^*: E(G) \to N$ gives the vertices and edge labeling as follows:

For $1 \leq i \leq n$,

$$\phi(u_i) = 4i\text{-}4$$
$$\phi(v_i) = 4i\text{-}1$$
$$\phi(x_i) = 4i - 3$$
$$\phi(y_i) = 4i - 2$$
$$\phi^*(u_i v_i) = 6(2i - 1)$$
$$\phi^*(v_i u_{i+1}) = 12i - 1$$
$$\phi^*(u_i x_i) = 2(6i - 5)$$
$$\phi^*(x_i y_i) = 12i - 7$$
$$\phi^*(v_i y_i) = 12i - 4$$

Hence Alternate Quadrilateral snake AQ_n admits Pell labeling.

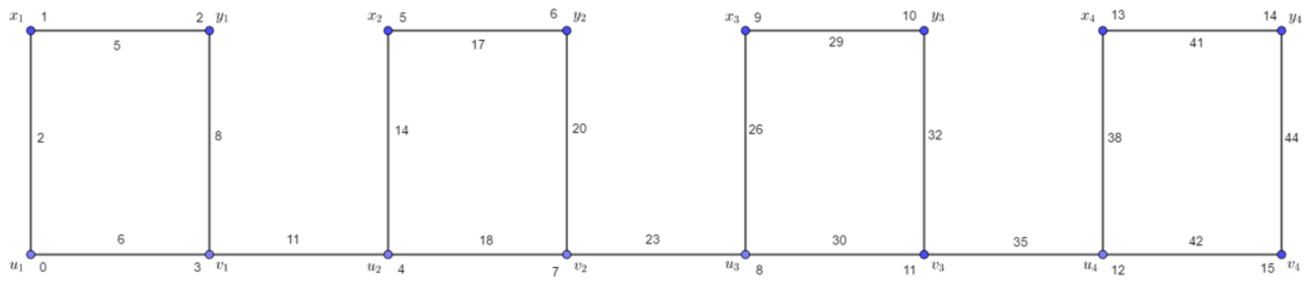

Figure 20. Alternate quadrilateral snake AQ_8

4. CONCLUSIONS

In this paper the Pell labeling for some graphs like splitting of Star graph, cycle graphs with parallel chords, Alternate double triangular Snake graph, ladder, Triangular ladder, diagonal ladder, shadow and splitting of Path, bistar, subdivision of bistar, prism, $B_{n,n}^2$ and friendship graphs are investigated. Future research work on Pell labeling for swastik, one step grid, double step grid graphs will be investigated.

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