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Normalization of Regular Scheduling Criteria with Dynamic Constraint

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ABSTRACT

The introduction of dynamic constraints to multi - criteria scheduling problems with regular objective function makes release dates a variable. Therefore, normalization equations for linear composite objective function are influenced by the release date. This work established the equations using the linear min-max method. Normalization equations for twelve (12) different objective functions, with both the cost and benefit orientation equations are established. The need for normalization was also established and the basis for deriving normalization equation for any multicriteria scheduling problems from the single criteria objective was established. The normalization equations for some multicriteria scheduling problems found in the literature were also established. This work will encourage researcher to explored composite objective functions for quantitative analysis of multicriteria problems.

1. INTRODUCTION

In a multi-criteria scheduling problem, two or more performance measures may be aggregated to form a composite function (simultaneous optimization) or a complex problem is divided into simpler sub-problems, and each level is optimized independently according to their importance [1]. This is called hierarchical optimization. A dominated and a non-dominated solution can also be obtained for a multi-criteria scheduling problem using a continuously updated algorithm. This is called pareto optimization.

In simultaneous optimization, each objective function is expressed as a component of a composite function [2]. Three different types of composite functions; linear, quadratic and an arbitrary composite function are extensively discussed by Józefowska [3]. For each of the classes, the objective function is expressed mathematically. The general expression for the three classes is given in the Eqs. (1)-(3) using a bicriteria problem with objective function, *A* and *B*.

Linear Composite Function

$$\sum_{i=1}^{n}(\alpha_i A_i + \beta_i B_i) \quad (1)$$

Quadratic Composite Function

$$\sum_{i=1}^{n}(\alpha_i A_i^2 + \beta_i B_i^2) \quad (2)$$

Arbitrary Composite Objective Function (ACOF)

$$\sum_{i=1}^{n} f_i^A(A_i) + f_i^B(B_i) \quad (3)$$

The solution approach of interest in this work is simultaneous approach using the linear composite objective function (LCOF). The challenges of skewness when the value of a performance measure dominates over other as well as dimensional imbalance for heterogeneous input data has been reported for simultaneous optimization [4]. Normalization of the input data before aggregation to form composite function is the proffered solution [5].

Five different normalization techniques; three of which are linear methods (sum, max, and max-min) and two non –linear (enhanced accuracy and vector method) was reported by Aytekin [6]. According to the study made by Vafaei et al. [7], linear max-min is the best techniques for a simple addition weighting system like linear composite function. The linear max–min method involves the determination of the maximum and minimum values of the criterion to be optimized. These parameters are called the extreme values. The normalization expression changes with the different objective functions as well as the job environments. In dynamic job environment, release dates is the parameter, and thus it influences the normalization equation. The objective of this work is to derive normalization equations for regular scheduling criteria under non-zero release date constraints using the linear min-max method.

Dynamic environment with distinct release dates is considered. In this regard, this work established the linear max-min normalization equations for regular performance measures with distinct release dates jobs.

The remainder of this paper is organized as follows.

Relevant literature on the application of different normalization techniques in multi criteria decision making are discussed in section 2. Section 3 defines the problem using some examples found in the literature as a case study. Section 4 discusses in details the use of linear; min-max techniques to determines the normalization equations for Fourteen (14) different regular scheduling criteria with imposed dynamic constraint on job availability. The derived equations were also implemented for some existing problems found in the literature. Conclusion and further research were discussed in section 5.

2. LITERATURE SURVEY

Normalization is a pre-processing stage for multi-criteria optimization problems used in several fields ranging from computing, material selection, medical and biological application, financing, industrial and project management among others [8]. Numerous researchers in various fields have explored the process within different context. For an instance, in the field of synthetic biology, Degasperi et al. [9] compared objective functions that use data-driven normalisation of the simulations with those that use scaling factors. According to Ersoy [10], normalization for multicriteria optimization problems can be classified into various ways; the need for normalization of objective functions, selection of suitable normalization techniques, studying the effects of different normalization techniques among others. Jain et al. [11] proposed a method for dynamic selection (DS) of optimal normalization technique using data complexity measures. The work evaluates 14 popular learning algorithms for designing dynamic selection model for the selection of optimal normalization technique. In order to design this dynamic selection model, 12 different data complexity measures are extracted for 48 different benchmark datasets. Akande [12] studied the need for normalization techniques in multicriteria scheduling problems. The studies of suitable normalization techniques for simple weighting method of solving multicriteria problem as well as hierarchy solution method was explored by Vafaei et al. [7, 13] respectively. The comparative analysis of linear and vector normalization methods in decision making for learning quota assistance was studied by Budiman and Hairah [14]. Ranking of solution based on normalization techniques was also discussed by Lakshmi and Venkatesan [15].

According to Vafaei et al. [7], it was stated that the linear min max method is the suitable normalization techniques for simple weighting method of solving multicriteria problem. The method is a subset of orientation dependent normalization techniques which is either benefit orientation or cost orientation. For this class, the normalization equation changes for different criteria as well as the imposed constraints. There are numerous multicriteria scheduling problems with release dates found in the literature that requires normalization in order to explore simultaneous optimization by computing the LCOF. These include Generating bicriteria schedules for correlated parallel machines involving tardy jobs and weighted completion time by Lin and Yin [16], Bi-criteria scheduling problems: Number of tardy jobs and maximum weighted tardiness by Huo et al. [17], minimization of total tardiness and total flowtime on single machine with non-zero release dates [4].

However, there are literature that established normalization equations for different objective function with the imposed constraints. Establishing the normalization equations for different regular criteria with unavailability constraints (non zero release date) is the purpose of this work. Though, Oyetunji and Oluleye [18] established the equations for the extreme parameters for completion time using only the benefit orientation. The corresponding equations for other regular performance measures are missing.

3. PROBLEM DEFINITION

Consider a bi-criteria scheduling problem with flexible maintenance and job release dates with the objective of minimizing the makespan and total tardiness simultaneously. The problem was solved by Chen et al. [19]. The LCOF of the problem is defined as:

$$\text{F}\ (C_{max}, T_{tot}) = \alpha\ C_{max}\ +\ \beta T_{tot} \tag{4}$$

where:
C_{max} is the makespan.
T_{tot} is the total tardiness.
α and β are the attached weight of the two criteria.
Assuming α=β=0.5.
The two performance measures are defined as follows:
The makespan is the completion time of the last scheduled job. It is the highest or maximum completion time.

$$C_{max} = \max\ (C_1\ ,\ C_2\ ,\ C_3 \ldots,\ C_n) \tag{5}$$

The tardiness of job i is defined as:

$$T_i = \max\ \{0\ ,(C_i - d_i)\} \tag{6}$$

The total tardiness is the sum of tardiness of all the jobs.

$$T_{tot}\text{: } \sum_{i=1}^{n} T_i = \sum_{i=1}^{n} \max\ \{0\ ,(C_i - d_i)\} \tag{7}$$

The two performance measures have the same unit and the challenge of dimensional imbalance does not exist. However, the domination of makespan value over the total tardiness is inevitable for small job sizes while the total tardiness values will also dominate over the makespan for large job sizes. Therefore, multicriteria decision for application of scheduling methodology by direct application of LCOF values without normalization will results in skewed decision towards certain criteria. This is not effective given that the two criteria are of equal importance.

Furthermore, Lin and Lin [20] considered the problem of generating bicriteria schedules for correlated parallel machines with the objective of minimizing the number of tardy jobs and weighted completion time simultaneously. The LCOF of the problem is defined as:

$$\text{F}\ (C_{tot}, N_t) = \alpha\ C_{tot}\ +\ \beta N_t \tag{8}$$

where:
C_{tot} is total completion time.
N_t is the number of tardy jobs.
α and β are the attached weight of the two criteria.
Assuming α=β=0.5.
The unit of total completion time is the time unit and that of total number of tardy jobs is job unit. Therefore, direct combination to obtain LCOF is impossible because the two inputs are heterogeneous data. Furthermore, the completion time values will likely dominate over the total number of tardy jobs. To solve these challenges, input data will be pre-processed to becomes a dimensionless data and to eliminate the dominating of one data over the other before computing LCOF. This pre-processing is called normalization. This work presents the normalization equations for regular performance

measures with jobs release date constraint.

4. LINEAR MIN-MAX NORMALIZATION TECHNIQUES

This normalization technique performs a linear transformation of the objective function value obtained by a given solution method to a dimensionless and scaled data (0, 1). Two different orientations of linear min-max techniques are usually employed; the benefit and the cost orientation.

4.1 Benefit and cost orientation

The benefit optimization orientation implies that the increase in the performance values of the alternatives evaluated in criterion $\boldsymbol{j}$ is preferred to the decrease while the cost optimization orientation implies that the reduction in the performance values of the alternatives in criterion $\boldsymbol{j}$ is preferred to the increase [8]. The general equations for the two orientations are given in Eqs. (9) and (10).

$$Y_B = \frac{Y_{SM} - Y_{min}}{Y_{max} - Y_{min}} \quad (9)$$

$$Y_c = \frac{Y_{max} - Y_{SM}}{Y_{max} - Y_{min}} \quad (10)$$

where:

Y_b is the benefit orientation normalized value of the objective function, X.

Y_c is the cost orientation normalized value of the objective function, X.

Y_{SM} is the objective function value obtained from a given solution method.

Y_{min} is the minimum value of the objective function.

Y_{max} is the maximum value of the objective function.

The Y_{min} and Y_{max} are called the extreme parameters. The equations for these parameters are functions of objective function of interest as well as the imposed constraints.

Determination of extreme values

Regular performance measures are functions of completion time. Therefore, the equation for the extreme parameters for completion time will be determine and use for other performance measures.

Given a set of N jobs to be scheduled on a single machine with distinct release dates, randomly generated from R_{min} to R_{max}.

The minimum values (best case scenario) occur when

i. The first schedule job (i=1) is schedule at minimum R_{min}

ii. the waiting time, (W_i) of all the jobs is zero. This is possible if $R_{i+1} = C_i$

The Gantt chart under this condition is represented in Figure 1.

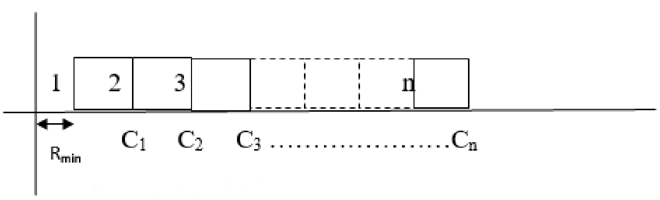

Figure 1. The Gantt chart for the best case scenario

$$C_{tot} = \sum_i^n C_t = C_1 + C_2 + C_3 + \dots + C_n$$
$$C_1 = P_1 + R_{min} = \sum_{i=1}^1 P_i + R_{min}$$
$$C_2 = C_1 + P_2 = \sum_{i=1}^2 P_i + R_{min}$$
$$C_3 = C_2 + P_3 = \sum_i^3 P_i + R_{min}$$
$$\dots$$
$$C_n = C_{n-1} + P_n = \sum_i^n P_i + R_{min} \quad (11)$$
$$C_{tot}^{min} = \sum_{i=1}^1 P_i + R_{min} + \sum_{i=1}^2 P_i + R_{min} + \sum_i^3 P_i + R_{min} + \dots + \sum_i^n P_i + R_{min}$$
$$C_{tot}^{min} = \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \dots \sum_{i=1}^n P_i + nR_{min}$$

The maximum values (worst case scenario) occurs when the waiting time, (w_i) of the first schedule job is maximum. The Gantt chart under this condition is represented in Figure 2.

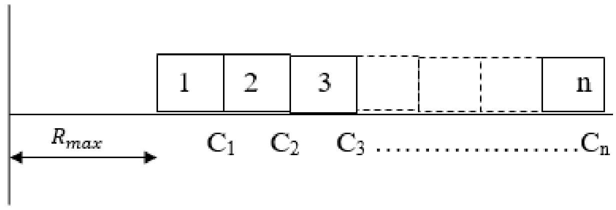

Figure 2. The Gantt chart for the worst case scenario

Therefore,

$$C_{tot} = \sum_i^n C_t = C_1 + C_2 + C_3 + \dots + C_n$$
$$C_1 = P_1 + R_{max} = \sum_i^1 P_i + R_{max}$$
$$C_2 = C_1 + P_2 = P_1 + R_{max} + P_2 = \sum_i^2 P_i + R_{max}$$
$$C_3 = C_2 + P_3 = \sum_i^2 P_i + R_{max} + P_3 = \sum_i^3 P_i + R_{max} \quad (12)$$
$$\dots$$
$$C_n = C_{n-1} + P_n = \sum_i^{n-1} P_i + R_{max} + P_n = \sum_i^n P_i + mR_{max}$$
$$C_{tot}^{max} = \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \dots \sum_{i=1}^n P_i + nR_{max}$$

The benefit and cost orientation equations can be determined from the extreme values.

Benefit orientation

$$C_N = \frac{C_{SM} - C_{min}}{X_{max} - X_{min}}$$
$$X_{max} - X_{min} = (nR_{max} + P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \dots + \sum_i^n P_i) - (P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \dots + \sum_i^n P_i + nR_{min}) = nR_{max} - nR_{min} \quad (13)$$
$$C_N = \frac{C_{SM} - (P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \dots + \sum_i^n P_i + R_{min})}{nR_{max}}$$
$$C_N = \frac{C_{SM} - (P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \dots + \sum_i^n P_i + nR_{min})}{nR_{max} - nR_{min}}$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}}$$
$$C_N = \frac{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \dots \sum_i^n P_i + nR_{max} - C_{SM}}{nR_{max} - nR_{min}} \quad (14)$$

Makespan (maximum completion time)

The maximum completion time, called the makespan is the completion time of the last job in a system.

$$C_{max} = \max (C_1, C_2, \ldots, C_3) \quad (15)$$

The minimum values of C_{max} is possible when all the jobs has zero waiting time and the first scheduled job has the minimum possible value of release date.

The Gantt chart under this condition is represented in Figure 3.

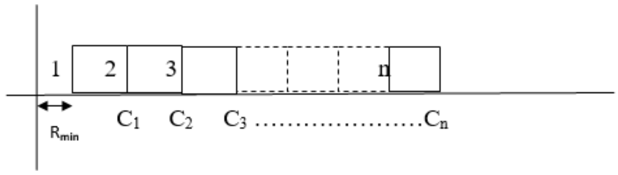

Figure 3. The Gantt chart for computing minimum C_{max}

Thus, the completion time of the last job is the summation of all the completion time.

$$C_{max}^{min} = max(P_1, \sum_i^2 P_i\ ,\ \sum_i^3 P_i\ ,\ \ldots\ ,\ \sum_i^n P_i + R_{min}) \quad (16)$$
$$C_{max}^{min} = \sum_i^n P_i + R_{min}$$

The maximum values (worst case scenario) occurs when the waiting time, (W_i) of the first schedule job is maximum. The Gantt chart is represented in Figure 4.

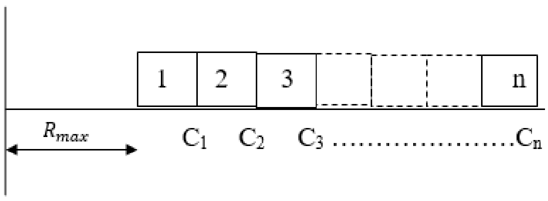

Figure 4. The Gantt chart for computing maximum C_{max}

Therefore,

$$C_{max}^{max} = \max (P_1 + R_{max}, \sum_i^2 P_i + R_{max}, \sum_i^3 P_i + R_{max}, + \ldots \sum_i^n P_i + R_{max})$$

$$C_{max}^{max} = \sum_i^n P_i + R_{max}$$

Benefit orientation

$$C_N = \frac{C_{SM} - C_{min}}{C_{max} - CX_{min}}$$
$$C_{max} - C_{min} = (R_{max} + \sum_i^n P_i) - (\sum_i^n P_i + R_{min}) = \quad (17)$$
$$C_N = \frac{C_{SM} - (\sum_i^n P_i + R_{min})}{R_{max} - R_{min}}$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}}$$
$$C_N = \frac{\sum_i^n P_i + R_{max} - C_{SM}}{nR_{max} - nR_{min}} \quad (18)$$

Total flow time

The flow time is the time a job spent in the shop after its availability. It is computed as the differences between the completions time of job *i* and its release date.

$$F_i = C_i - r_i \quad (19)$$

It is also the sum of job processing time and the waiting time.

$$F_i = P_i + W_i \quad (20)$$

$$F_{tot} = \sum_i^n F_{tot} = F_1 + F_2 + F_3 + \ldots + F_n \quad (21)$$

Just like the completion time, the extreme values for flowtime are also computed from the waiting time. The waiting time is computed as:

$$W_{i+1} = C_i - r_{i+1} \quad (22)$$

When $C_{i+1} > r_i$,
W_i is called the waiting time of job *i*.
When $C_{i+1} < r_i$,
W_i is the machine or processor waiting time, called the idle time.
When $C_i = r_{i+1}$,
$W_{i+1} = 0$, then F_{i+1} is minimum, thus $F_{i+1} = P_{i+1}$.

Given a set of N jobs to be scheduled on a single machine with distinct release dates, randomly generated from $R_{min} - R_{max}$.

The minimum values (best case scenario) occurs when the waiting time, (w_i) of all the jobs is zero. The Gantt chart is represented in Figure 5.

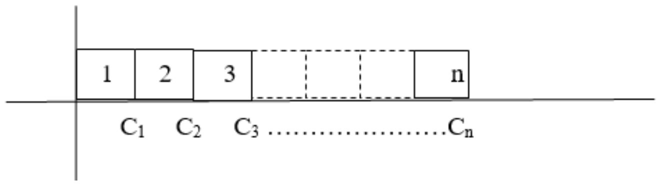

Figure 5. The Gantt chart for computing minimum F_{tot}

$$F_{tot}^{min} = \sum_i^n F_i^{min} = F_1^{min} + F_2^{min} + F_3^{min} + \ldots F_{tot}^{min}$$
$$F_{tot}^{min} = \sum_i^n F_i^{min} = P_1 + P_2 + P_3 + \ldots + P_n \quad (23)$$
$$F_{tot}^{min} = \sum_i^n P_i$$

The flow time is maximum, when

i. W_i is maximum.
ii. C_i (or C_{i+1}) is maximum and r_i is minimum.

$$F_i = C_i - r_i$$
$$F_i = P_i + W_i$$
$$F_{tot}^{max} = \sum_i^n F_{tot}^{max} = (C_1 - r_1 + C_2 - r_2 + C_3 - r_3 + \ldots + C_n - r_n) \quad (24)$$
$$F_{tot}^{max} = \sum_i^n F_{tot}^{max} = \sum_i^n (C_{tot}^{max} - R_i^{min})$$
$$F_{tot}^{max} = nR_{max} + P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \ldots \sum_i^n P_i - nR_{min}$$

Benefit orientation

$$F_N = \frac{X - X_{min}}{X_{max} - X_{min}}$$
$$X_{max} - X_{min} = (nR_{max} + P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \cdots \sum_i^n P_i - nR_{min}) - \sum_i^n P_i$$
$$(nR_{max} + P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \cdots \sum_i^{n-1} P_i - nR_{min}) \quad (25)$$
$$F_N = \frac{F_{SM} - \sum_i^n P_i}{P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \ldots + \sum_i^{n-1} P_i + nR_{max} - nR_{min}}$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}}$$
$$X_N = \frac{nR_{max} + P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \ldots + \sum_i^n P_i - nR_{min} - X}{P_1 + \sum_i^2 P_i + + \sum_i^3 P_i + \ldots + \sum_i^{n-1} P_i + nR_{max} - nR_{min}} \quad (26)$$

Maximum flow time

The maximum flow time is defined as:

$$F_{max} = max\,(F_1, F_2, F_3, \ldots, F_n) \quad (27)$$

From Eq. (23), $F_i = P_i$,

$$F_{max}^{min} = max\,(P_1, P_2, P_3, \ldots, P_n)$$
$$F_{max}^{min} = \max\,(P_i) \quad (28)$$

The flow time is maximum when,

i. W_i is maximum.

ii. $C_i\ (or\ C_{i+1})$ is maximum and r_i is minimum.

$$F_i = C_i - r_i$$

$$F_i = P_i + W_i$$

$$F_{max}^{max} = max(C_1 - r_1,\ C_2 - r_2,\ C_3 - r_3,\ \ldots\ C_n - r_n)$$

The maximum value of F_{max} is achieve when the completion time is maximum (makespan) and the release date is minimum.

$$F_{max}^{max} = C_{max} - r_{min} \quad (29)$$

Benefit orientation

$$NF_{max} = \frac{X - X_{min}}{X_{max} - X_{min}}$$
$$NF_{max} = \frac{F_{SM} - \max\,(P_i)}{(C_{max} - R_{min}) - \max\,(P_i)} \quad (30)$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}}$$
$$X_N = \frac{(C_{max} - R_{min}) - X}{(C_{max} - R_{min}) - \max\,(P_i)} \quad (31)$$

The total tardiness

This is the sum of tardiness of all the jobs. The tardiness of job *i* is defined as:

$$T_i = \max\{0, (C_i - d_i)\}$$
$$(T_{tot}): \sum_{i=1}^n T_i = \sum_{i=1}^n \max\{0, (C_i - d_i)\}$$

The minimum value of total tardiness s zero,

$$T_{tot}^{min} = 0 \quad (32)$$

The due date of job i, can never be 0 or negative, it implies that the total tardiness is maximum when the completion time is maximum.

The maximum value of total tardiness is given by:

$$T_{tot}^{max} = \left(C_{tot}^{max} - \sum_{i=1}^n d_i\right)$$
$$T_{tot}^{max} = \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d \quad (33)$$

Benefit orientation

$$T_N = \frac{T_{SM} - T_{min}}{T_{max} - T_{min}} = \frac{T_{SM}}{T_{max}}$$
$$T_N = \frac{T_{SM}}{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d} \quad (34)$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}}$$
$$X_N = \frac{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d - T_{SM}}{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d} \quad (35)$$

The maximum tardiness

Maximum tardiness (T_{max}) is given by:

$$T_{max} = \max\,(\max\{0, (C_i - d_i)\}$$

The minimum possible value of maximum tardiness is zero

$$T_{mxn}^{mix} = 0 \quad (36)$$

The maximum value of maximum total tardiness

$$T_{tot}^{max} = (C_{max} - d_{min})$$

Benefit orientation

$$NT_{max} = \frac{T_{SM} - T_{min}}{T_{max} - T_{min}} = \frac{T_{SM}}{C_{max} - d_{min}}$$
$$NT_{max} = \frac{T_{SM}}{C_{max} - d_{min}} \quad (37)$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}},\ X_N = \frac{C_{max} - d_{min} - X_{SM}}{C_{max} - d_{min}} \quad (38)$$

The total lateness

Lateness is measure as the difference between the completion time and the due date.

The lateness is given by:

$$\mathrm{L_i} = (\mathrm{C_i} - \mathrm{d_i})$$
$$L_{tot} = \sum_{i=1}^n L_i = \sum_{i=1}^n (\mathrm{C_i} - \mathrm{d_i})$$

Minimum total lateness

$$(L_{tot}^{min}) = \sum_{i=1}^n \mathrm{L_i} = (C_{tot}^{min} - \sum_{i=1}^n \mathrm{d_i})$$
$$L_{tot}^{min} = \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ \ldots \sum_{i=1}^n P_i + nR_{min} - \sum_{i=1}^n \mathrm{d_i} \quad (39)$$

Maximum total lateness

$$(L_{tot}^{max}) = \sum_{i=1}^n \mathrm{L_i} = (C_{tot}^{max} - \sum_{i=1}^n \mathrm{d_i})$$
$$L_{tot}^{max} = \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ \ldots \sum_{i=1}^n P_i + nR_{max} - \sum_{i=1}^n \mathrm{d_i} \quad (40)$$

Benefit orientation

$$L_N = \frac{L_{SM} - L_{min}}{L_{max} - L_{min}},\ L_N = \frac{L_{SM} - L_{min}}{L_{max} - L_{min}}$$
$$L_N = \frac{L_{SM} - \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ \ldots \sum_{i=1}^n P_i + nR_{min} - \sum_{i=1}^n \mathrm{d_i}}{(nR_{max} - nR_{min})} \quad (41)$$

Cost orientation

$$X_N = \frac{X_{max} - X_{SM}}{X_{max} - X_{min}}$$

$$X_N = \frac{\sum_{i=1}^{1} P_i + \sum_{i=1}^{2} P_i + \sum_{i=1}^{3} P_i \ \dots \ \sum_{i=1}^{n} P_i + nR_{min} - \sum_{i=1}^{n} \mathrm{d_i} - X_{SM}}{(nR_{max} - nR_{min})} \quad (42)$$

The maximum lateness

The maximum lateness is given by,

$$\mathrm{L_{max}} = max(\mathrm{C_i} – \mathrm{d_i})$$

$$\text{The } L_{max}^{min} = 0 \quad (43)$$

This occurs when $d_i \geq C_i$ for all i.

The L_{max}^{max} occurs when the completion time is maximum and the due date is minimum.

$$L_{max}^{max} = (C_{max} – d_{min}) \quad (44)$$

Benefit orientation

$$NL_{max} = \frac{L_{SM} - L_{min}}{L_{max} - L_{min}} = \frac{L_{max}}{\mathrm{C_{max}} – \mathrm{d_{min}}} \quad (45)$$

Cost orientation

$$X_N = \frac{X_{max} - X_{SM}}{X_{max} - X_{min}}$$

$$X_N = \frac{C_{max} – d_{min} - X_{SM}}{C_{max} – d_{min}} \quad (46)$$

The total earliness

The job earliness has been expressed in two different ways in the literature; the classical definition and the opposite of lateness definition.

Earliness, defined as the opposite of lateness is express as $E_i = (d_i – C_i)$.

Total earliness $(E_{tot}) = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n} (d_i - C_i) = (-L_i)$

With this definition, the earliness based performance measure is a maximization problem.

The extreme values

The E_{tot} is minimum when the completion time is maximum.

Minimum total earliness

$$(E_{tot}^{min}) = \sum_{i=1}^{n} E_i = (\sum_{i=1}^{n} d_i - C_{tot}^{max})$$

$$E_{tot}^{min} = \sum_{i=1}^{n} d_i - \sum_{i=1}^{1} P_i + \sum_{i=1}^{2} P_i + \sum_{i=1}^{3} P_i \ \dots \ \sum_{i=1}^{n} P_i + nR_{max} \quad (47)$$

Maximum total earliness

$$(E_{tot}^{max}) = \sum_{i=1}^{n} E_i = (\sum_{i=1}^{n} d_i - C_{tot}^{min})$$

$$E_{tot}^{max} = \sum_{i=1}^{n} d_i - \sum_{i=1}^{1} P_i + \sum_{i=1}^{2} P_i + \sum_{i=1}^{3} P_i \dots \sum_{i=1}^{n} P_i + nR_{min} \quad (48)$$

Benefit orientation

$$NE_{tot} = \frac{E_{SM} - E_{min}}{E_{max} - E_{min}}$$

$$NE_{tot} = \frac{E_{SM} - \sum_{i=1}^{n} \mathrm{d_i} - \sum_{i=1}^{1} P_i + \sum_{i=1}^{2} P_i + \sum_{i=1}^{3} P_i \ \dots \ \sum_{i=1}^{n} P_i + nR_{max}}{nR_{max} - nR_{min}} \quad (49)$$

Cost orientation

$$X_N = \frac{X_{max} - X_{SM}}{X_{max} - X_{min}}$$

$$NE_{max} = \frac{\sum_{i=1}^{n} \mathrm{d_i} - \sum_{i=1}^{1} P_i + \sum_{i=1}^{2} P_i + \sum_{i=1}^{3} P_i \dots \sum_{i=1}^{n} P_i + nR_{min} - E_{SM}}{nR_{max} - nR_{min}} \quad (50)$$

Maximum earliness

$$(\mathrm{E_{max}}) = max(\mathrm{d_i} – \mathrm{C_i}) = max(–\mathrm{L_i}), E_{max}^{min} = 0 \quad (51)$$

This occurs when $d_i \leq C_i$.

The E_{max}^{max} is when the completion time is minimum and the due date is maximum.

$$E_{max}^{max} = (d_{max} – C_{min}) \quad (52)$$

Benefit orientation

$$X_N = \frac{X - X_{min}}{X_{max} - X_{min}}, \quad X_N = \frac{X}{d_{max} - C_{min}} \quad (53)$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}} = \frac{d_{max} - C_{min} - X}{(d_{max} - C_{min})} \quad (54)$$

Classical definition of earliness

The earliness, E_i of job I is defined as:

$$\mathrm{E_i} = max\{-Li, 0\}.$$

Total earliness $(\mathrm{E_{tot}}) = \sum_{i=1}^{n} \mathrm{E_i} = \sum_{i=1}^{n} \max(\mathrm{d_i} – \mathrm{C_i}, 0) = \sum_{i=1}^{n} max\{-Li, 0\}$.

Minimum total earliness

$$(E_{tot}^{min}) = 0 \quad (55)$$

Maximum total earliness

$$(E_{tot}^{max}) = \sum_{i=1}^{n} \mathrm{E_i} = (\sum_{i=1}^{n} \mathrm{d_i} - C_{tot}^{min})$$

$$E_{tot}^{max} = \sum_{i=1}^{n} \mathrm{d_i} - \sum_{i=1}^{1} P_i + \sum_{i=1}^{2} \mathrm{P_i} + \sum_{i=1}^{3} P_i \ \dots \ \sum_{i=1}^{n} P_i + nR_{min} \quad (56)$$

Benefit orientation

$$X_N = \frac{X - X_{min}}{X_{max} - X_{min}} \quad (57)$$

$$X_N = \frac{X}{\sum_{i=1}^{n} \mathrm{d_i} - \sum_{i=1}^{1} P_i + \sum_{i=1}^{2} P_i + \sum_{i=1}^{3} P_i \ \dots \ \sum_{i=1}^{n} P_i + nR_{min}} \quad (58)$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}}$$

$$X_N = \frac{\sum_{i=1}^{n} \mathrm{d_i} - \sum_{i=1}^{1} P_i + \sum_{i=1}^{2} P_i + \sum_{i=1}^{3} P_i \ \dots \ \sum_{i=1}^{n} P_i + nR_{min} - X}{\sum_{i=1}^{n} \mathrm{d_i} - \sum_{i=1}^{1} P_i + \sum_{i=1}^{2} P_i + \sum_{i=1}^{3} P_i \ \dots \ \sum_{i=1}^{n} P_i + nR_{min}} \quad (59)$$

Maximum earliness

$$\mathrm{E_i} = max\{-Li, 0\}$$

For i =1: n

$$E_{max} = max\{E_I, E_2, E_3, .., E_n\} \quad E_{max}^{min} = 0 \tag{60}$$

This occurs when $d_i \leq C_i$ for all *i*.

$$E_{max}^{min} = 0,\ E_{max}^{max} = (\mathrm{d}_{\max} - \mathrm{C}_{\min}) \tag{61}$$

Benefit orientation

$$X_N = \frac{X - X_{min}}{X_{max} - X_{min}} = \frac{X_{SM}}{d_{max} - C_{min}} \tag{62}$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}} = \frac{d_{max} - C_{min} - X_{SM}}{d_{max} - C_{min}} \tag{63}$$

Baker and Trietsch [21] and Akande and Ajisegiri [22] explored the classical definition of earliness.

The Total Number of Tardy/Late Jobs

The minimum number of tardy/late job is zero while the maximum number is the total number of jobs.

Benefit orientation

$$X_N = \frac{X - X_{min}}{X_{max} - X_{min}} = \frac{X}{X_{max}} \quad T_N = \frac{N_{t_{SM}}}{N} \tag{64}$$

Cost orientation

$$X_N = \frac{X_{max} - X}{X_{max} - X_{min}} = \frac{X}{N} \tag{65}$$

Table 1 shows the summary of the extreme values equations for the considered performance measures.

Table 1. The extreme values of some performance objectives

Criteria	Benefit Orientation	Cost Orientation
C_{tot}	$\frac{C_{SM} - (P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \ldots + \sum_i^n P_i + nR_{min})}{nR_{max} - nR_{min}}$	$\frac{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - C_{SM}}{nR_{max} - nR_{min}}$
C_{max}	$\frac{C_{SM} - (\sum_i^n P_i + R_{min})}{R_{max} - R_{min}}$	$\frac{\sum_i^n P_i + R_{max} - C_{SM}}{nR_{max} - nR_{min}}$
F_{tot}	$\frac{F_{SM} - \sum_i^n P_i}{P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \ldots + \sum_i^{n-1} P_i + nR_{max} - nR_{min}}$	$\frac{nR_{max} + P_1 + \sum_i^2 P_i + + \sum_i^3 P_i + \ldots + \sum_i^n P_i - nR_{min} - X}{P_1 + \sum_i^2 P_i + + \sum_i^3 P_i + \ldots + \sum_i^{n-1} P_i + nR_{max} - nR_{min}}$
F_{max}	$\frac{F_{SM} - \max(P_i)}{(C_{max} - R_{min}) - \max(P_i)}$	$\frac{(C_{max} - R_{min}) - X}{(C_{max} - R_{min}) - \max(P_i)}$
T_{tot}	$\frac{T_{SM}}{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d}$	$\frac{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d - T_{SM}}{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d}$
T_{max}	$\frac{T_{SM}}{C_{max} - d_{min}}$	$\frac{C_{max} - d_{min} - X_{SM}}{C_{max} - d_{min}}$
L_{tot}	$\frac{L_{SM} - \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ldots \sum_{i=1}^n P_i + nR_{min} - \sum_{i=1}^n d_i}{(nR_{max} - nR_{min})}$	$\frac{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ldots \sum_{i=1}^n P_i + nR_{min} - \sum_{i=1}^n d_i - L_{SM}}{(nR_{max} - nR_{min})}$
L_{max}	$\frac{X}{d_{max} - C_{min}}$	$\frac{d_{max} - C_{min} - X}{d_{max} - C_{min}}$
E_{tot}	$\frac{E_{SM} - \sum_{i=1}^n d_i - \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ldots \sum_{i=1}^n P_i + nR_{max}}{nR_{max} - nR_{min}}$	$\frac{\sum_{i=1}^n d_i - \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ldots \sum_{i=1}^n P_i + nR_{min} - X}{\sum_{i=1}^n d_i - \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ldots \sum_{i=1}^n P_i + nR_{min}}$
E_{max}	$\frac{X}{d_{max} - C_{min}}$	$\frac{d_{max} - C_{min} - X}{d_{max} - C_{min})}$
E_{tot} (CD)	$\frac{X}{\sum_{i=1}^n d_i - \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ldots \sum_{i=1}^n P_i + nR_{min}}$	$\frac{\sum_{i=1}^n d_i - \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ldots \sum_{i=1}^n P_i + nR_{min} - X}{\sum_{i=1}^n d_i - \sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_{i=1}^3 P_i \ldots \sum_{i=1}^n P_i + nR_{min}}$
E_{max}	$\frac{X_{SM}}{d_{max} - C_{min}}$	$\frac{d_{max} - C_{min} - X_{SM}}{d_{max} - C_{min}}$
N_t/N_L	$\frac{N_{t_{SM}}}{N}$	$\frac{X}{N}$

Table 2. Normalization expression for some multicriteria problems

Multicriteria Problem	Benefit	Cost
Bi-objective optimization of identical parallel machine scheduling with flexible maintenance and job release times [19]	$\frac{C_{SM} - (\sum_i^n P_i + R_{min})}{R_{max} - R_{min}} + \frac{T_{SM}}{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d}$	$\frac{\sum_i^n P_i + R_{max} - C_{SM}}{nR_{max} - nR_{min}} + \frac{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d - T_{SM}}{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d}$
A new heuristic for m-machine flowshop scheduling problem with bicriteria of makespan and maximum tardiness [23]	$\frac{C_{SM} - (\sum_i^n P_i + R_{min})}{R_{max} - R_{min}} + \frac{T_{SM}}{C_{max} - d_{min}}$	$\frac{\sum_i^n P_i + R_{max} - C_{SM}}{nR_{max} - nR_{min}} + \frac{C_{max} - d_{min} - X_{SM}}{C_{max} - d_{min}}$
Generating bicriteria schedules for correlated parallel machines involving tardy jobs and weighted completion time [16]	$\frac{N_{t_{SM}}}{N} + \frac{C_{SM} - (P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \ldots + \sum_i^n P_i + nR_{min})}{nR_{max} - nR_{min}}$	$\frac{X}{N} + \frac{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - C_{SM}}{nR_{max} - nR_{min}}$
Minimization of total tardiness and total flowtime on single machine with non-zero release dates [4]	$\frac{T_{SM}}{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d} + \frac{F_{SM} - \sum_i^n P_i}{P_1 + \sum_i^2 P_i + \sum_i^3 P_i + \ldots + \sum_i^{n-1} P_i + nR_{max} - nR_{min}}$	$\frac{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d - T_{SM}}{\sum_{i=1}^1 P_i + \sum_{i=1}^2 P_i + \sum_i^3 P_i \ldots \sum_i^n P_i + nR_{max} - \sum_{i=1}^n d} + \frac{nR_{max} + P_1 + \sum_i^2 P_i + + \sum_i^3 P_i + \ldots + \sum_i^n P_i - nR_{min} - X}{P_1 + \sum_i^2 P_i + + \sum_i^3 P_i + \ldots + \sum_i^{n-1} P_i + nR_{max} - nR_{min}}$

4.2 Normalized LCOF

The extreme values and the X_{SM} for each of the criteria in a multicriteria problem will be substituted into the LCOF Eq. (3). The normalized LCOF can be used for effective comparison of different solution.

$$N_{LCOF} = \alpha N_X + \beta N_Y + \gamma N_z \quad (66)$$

where:

N_{LCOF} is the normalized total composite function.
N_X is the normalized value of criterion X.
N_y is the normalized value of criterion Y.
N_z is the normalized value of criterion Z.

Table 2 shows the N_{LCOF} equations for some multicriteria scheduling problems found in the literature.

5. CONCLUSION

The complexity of multicriteria scheduling problems increased with dynamic constraint imposition. The use of LCOF as well as the need for normalization though established in the literature but the appropriate normalization equation for different criteria is a missing link. This work closed the gap by establishing the normalization equation for numerous scheduling performance measures. Some existing multi-criteria problems found in the literature was also explored. The work will open further research for exploring LCOF for Multi Criteria Decision Making (MCDM) with dynamic constraint.

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