International Journal of Heat and Technology
Vol. 42, No. 1, February, 2024, pp. 29-38
Journal homepage: http://iieta.org/journals/ijht

Numerical Investigations of Natural Convection in a Cubical Enclosure with Various Protuberance Shapes

Fattouma Bouguerne[1], Samir Rahal[2], Abdelhafid Brima[3], Djamel Batache[1], Yousef Belloufi[3,4*], Noureddine Moummi[3]

[1] Mechanical Engineering Department, Mechanical Structures and Materials Laboratory, University of Batna2, Fesdis-Batna 05078, Algeria
[2] Mechanical Engineering Department, LESEI Laboratory, University of Batna2, Fesdis-Batna 05078, Algeria
[3] Mechanical Engineering Laboratory LGM, University of Biskra, Biskra, Biskra 07000, Algeria
[4] Department of Drilling and Mechanics of Oil Fields, Faculty of Hydrocarbons, Renewable Energies and Earth Sciences and Universe, University Kasdi Merbah Ouargla, Ouargla, Ouargla 30000, Algeria

Corresponding Author Email: Yousef_belloufi@yahoo.fr

https://doi.org/10.18280/ijht.420104

Received: 8 October 2023
Revised: 24 January 2024
Accepted: 3 February 2024
Available online: 29 February 2024

Keywords:
enclosure, protuberance, heat transfer, rough surface, natural convection

ABSTRACT

Heat transfer by convection is widely used in many engineering applications such as in heat exchangers, combustion devices or gas processing. In Buildings, the use of rough surfaces allows enhancing heat transfer rates. Practically, in this study, a rough surface is obtained by using protuberances on the vertical left wall. In this perspective, the originality of this research is to study the influence of the protuberance geometric shapes on the heat transfer using 2D numerical simulations as a tool investigation. The homotopic transformation is used to reach a flat plate. The system of equations with the boundary conditions is solved using the finite volume method. A simple algorithm is chosen for the integration of algebraic equations. The governing equations are figured out using ANSYS FLUENT commercial software with SIMPLE algorithm in order to solve pressure-velocity coupling. The numerical simulations have been performed for different shapes of the protuberances (battlement, triangular and sinusoidal). The boundary conditions are based on a uniform heat flux applied to the vertical wall. On the other hand, the horizontal walls are subject to the adiabatic condition. It can be noticed that the effect of the wavy geometry induces a noticeable improvement of the heat transfer rate compared to an enclosure without protuberances. It can also conclude that the wavy configuration exhibits a Nusselt average slightly higher than that of the square cavity, particularly the triangular configuration, by approximately 20%.

1. INTRODUCTION

Natural convection in enclosures is critical in industrial applications such as electronic chip cooling, bioclimatic in buildings, thermal performance of solar collectors, geophysical flows. The protuberance is a key solution to improve the heat transfer. Numerous experimental and numerical studies highlighted the cavity's natural convection.

De Vahl Davis [1] initially presented the second-order centered finite differences approach for a laminar flow benchmarks with Rayleigh's numbers up to 106. Ostrach [2] provided a review of the evolution of heat transport mode. They concluded that the conduction process of heat transfer takes over when the number of Rayleigh is too low. Noorshahi et al. [3] studied an enclosure with adiabatic sidewalls, an isothermal flat cooled top surface, a bottom surface with corrugations maintains a consistent heat rate. He demonstrated that as wave amplitude increases, the pseudo-conduction zone also grows. Natural convection in two-dimensional square enclosure with a periodic array of heated roughness components at the bottom was numerically investigated. Bhavnani and Burgles [4] used interferometric analysis to examine the influence of transverse roughness components on laminar heat transfer by natural convection. Roughness at various spacing heights was investigated. They showed that the heat transferred does not depend only on the surface exchange but it is also strongly connected to the shape and the distance between protuberances.

Shakerin et al. [5] conducted a study on square-sectioned ribs installed on the heated wall. It used finite difference approach to solve the non-dimensional unstable Boussinesq equations. Using a dye and a Mach Zander interferometer, an experimental investigation was carried out. Despite the fact that the increase in surface area is around 32%, the average Nusselt number increased by about 12 percent with one rib. These findings imply that the plate's thermal performance is significantly influenced by the roughness spacing.

Das and Mahmud [6] statistically investigated buoyancy-induced flow and heat transmission in a wavy enclosure with both two straight and two wavy walls. They discovered that the wavelength to amplitude ratio has an impact on the flow field and the rate of local heat transfer.

In a two-dimensional cavity with one corrugated wall and three flat walls with a sinusoidal temperature profile, Dalal and Das [7] explored natural convection. The temperature is maintained constant along three walls, including the corrugated one. The experiments were conducted for various Rayleigh numbers, amplitudes and inclination degrees. They employed three configurations with one, two and three undulations. The developed results reveal that in the cavity, the inclination angle has an influence on heat transmission rate. The corrugated wall's average Nusselt number is noticeably large for low Rayleigh numbers as amplitude increases. On the other hand, adding more corrugations is not advantageous.

Saidi et al. [8] gathered a variety of information regarding numerical and experimental studies. They discovered that the vortex's presence lowered the overall heat exchange between the cavity's wave-like wall and the working fluid. A wide area of uniform temperature is created at the cavity's bottom by the vortex acting as a thermal screen.

The impact of hot inclined wavy wall is numerically analyzed and reported by Adjlout et al. [9]. For one and three undulations, tests were carried out with various inclinations, Rayleigh's number and amplitudes. It has been observed that the local heat transfer has a wavelike pattern, when the mean Nusselt number is compared to that of the square cavity.

On the other hand, Yao [10] theoretically examined the natural convection. It has been found that the heat transfer rate between a flat plate and wavy surface was consistently lower, along a vertical wavy surface.

The impact of the thermal radiation and the inclination angle of the wavy plate on thermal-fluid fields was statistically investigated by Mayouf [11]. They discovered that the surface's waves disrupt the flow. While the heat transfer rate is larger in the crests, the velocity values are decreased by the higher amplitude wave length ratio. In a 2D numerical simulation by Agrouaz et al. [12], they took the cavity's inclination with a wavy bottom wall and the Rayleigh number into consideration. The heat transfer rate has been increased with the variation of the inclination angle of the cavity for a constant value of Rayleigh number.

In order to manage the interaction between surface radiation and natural convection in the encountered irregular geometries in electronic component cooling, El Moutaouakil et al. [13] developed a CFD model employing discrete ordinate approaches with the blocked zone idea. They demonstrated that the mean radius and amplitude of the corrugated wall have more influence on the rate of overall heat transfer than the number of corrugations. They demonstrated that the mean radius and amplitude of the corrugated wall have a greater impact on the rate of overall heat transfer than the number of corrugations.

In rectangular cavity with vertical surfaces that have mixed boundary conditions, Said and Trupp [14] investigated the laminar free convection in two dimensions using Gauss-Seidel ADl and the SOR method to solve the energy equations.

For cavity aspect ratio of 0.5 to 15, over the modified Rayleigh number is ranging from 690 to 1.3×10^5, steady-state air solutions (Pr=0.7) have been found. It has been noticed that the average number of Nusselt reached a peak at an aspect ratio of roughly 1.5 for a particular heat flux. Inversion behaviour was also visible at the transition between the laminar boundary layer and asymptotic states in this region. The experimental data corroborate the acquired results.

The heat transfer coefficient in BIPV air channel is investigated statistically by Nghana et al. [15]. Furthermore, using experimental data, the developed CFD model has been reasonably validated. The study looked at variables such channel inclination angle (θ), dimensionless rib pitch (p/e), dimensionless rib pitch (e/D), and rib form. The heat flow varies from $100W/m^2$ to $1000W/m^2$ in each condition. A vertical gutter with a triangular rib that has dimensions of p/e=8.84 and e/D=0.16 maximizes the coefficient of heat transfer through natural convection on the heated wall. The enhanced cross ribs' heat transfer contributes in reduction of heat gain up to 35.5 percent in summer.

Nayak et al. [16] investigated the impact of various corrugated baffle patterns on double diffusion natural convection in a C-shaped cavity filled with hybrid nanofluid. The finite element method is used to solve the governing equations. It noticed according to the results of their experiments an increase of the Rayleigh number, buoyancy ratio, amplitude of wavy baffle, streamlines and Lewis number of 141.65 percent, 245.64 percent, 25.4 percent, 23.91 percent and 69.77 percent respectively. The velocities, heat and mass transfer rates and streamlines increase for any wavy baffles amplitude. Ra amplification efficiently reduces local Nusselt. At low Ra, the Bejan numbers increase and the average entropy decrease, whereas at high Ra, the reverse phenomenon is observed.

In a corrugated square enclosure with a circular obstruction, Ganesh et al. [17]. investigated in the thermal and hydraulic properties of a multi-walled carbon nanofluid (MWCNT) based on Casson (sodium alginate). A corrugated adiabatic top wall and two cooled vertical walls were made up the square enclosure. The bottom wall's top portion had a flat, adiabatic structure, and the middle section had a heated corrugated structure. Three different models of obstacles cold, hot and adiabatic is analysed numerically to solve the issue by using the finite element method. Analysis is done on the effects of the Rayleigh number ($10^3 \leq Ra \leq 10^6$), Casson parameter ($0.001 \leq \beta \leq 0.1$), radiation parameter ($1 \leq Rd \leq 4$) and volume percentage of the nanoparticles ($0.01 \leq \varphi \leq 0.1$). It noticed that the increasing of the Rayleigh number, volume percent MWCNT, Casson parameter and the radiation parameter for all types of obstacles causes the increase of the Nusselt number along the heated corrugated wall. They established that the average Nusselt number decreases in the presence of cold (or hot) impediments

The heat transfer and buoyancy-driven convection in a hollow with a corrugated wall filled with nanofluid were numerically examined by Ali et al. [18]. A heat-generating cylindrical blocking is placed in the middle of the cavity. It is partially cooled from corrugated walls and heated from its bottom wall while the other walls are adiabatic. The cavity is impregnated with a transverse magnetic field. They developed a thermal conductivity model of a hybrid nanofluid including the Brownian motions of different nanoparticles. The finite element approach is used to solve the non-dimensional governing equations. The results showed that fluid circulation should be intensified to enhance the Rayleigh number and the heater's length. The heat transfer rate is rapidly increased for increasing Rayleigh number and volume fraction of hybrid nanoparticles, but an opposite trend is observed for higher Hartmann number. The existence of the roughness of the hollow walls and a heat-generating cylinder affect flow circulation and temperature diffusion. In addition, a corrugated cavity with a heat-generating obstruction has a stronger force of fluid motion than a smooth cavity without a blockage. A shorter length and heating radius of the heat-generating cylinder ensure maximum heat transfer. The roughness of the cavity also affects how quickly heat is transferred there.

Kadari et al. [19] looked into laminar natural convection in a square hollow with a horizontal fin attached to hot wall and a cold wavy wall. The influence of fin length, number of undulations, thermal conductivity ratio and fin position were also studied. They found that the temperature distribution is significantly affected by the size of the heat source, and that when the local heat source is enlarged, both the average and local Nusselt numbers increase.

Djaomazava et al. [20] studied numerically the natural convection in a vertical channel, where one of its walls has sinusoidal protuberances. They used the explicit finite difference method to discretize the equations governing the transfers within the channel. The effect of the heat flux density and the corrugation shape ratio delivered to the base of these undulations is investigated. They concluded that the adjustment of the protuberance's amplitude causes an increase in the heat exchange surfaces inside this channel and, as a result, an intensification of convection.

Rahmani et al. [21] conducted a numerical and experimental study. A numerical simulation was devoted to examine the turbulent natural convection in a vertical enclosure with sinusoidal protuberances on one of its vertical walls. The results show that the local Nusselt numbers vary with amplitudes, which explains how pure conduction locally affects the modified wall.

Mahdi et al. [22] examined natural convection inside a corrugated enclosure. ANSYS 16.0 was used to solve governing equations assuming two-dimensional solution for steady laminar flow. Three different examples of (Ra) for a heat transfer problem within a sinusoidal cavity with one hot baffle in rectangular geometry are examined.

In their proposal to include correction terms to the evolution equations, Yang et al. [23] used two modified lattice Boltzmann Bhatnagar-Gross-Krook (LBGK) models for the incompressible Navier-Stokes equations and convection-diffusion equations. The LBGK model's stability can be increased by using this change, which allows the value of the dimensionless relaxation time to be maintained within a reasonable range. Even though, some gradient operators are part of the correction terms, it is still possible to compute them quickly using computational techniques like the current LBGK models, which still have inherent parallelism. The lattice Boltzmann method's distinguishing feature. According to numerical investigations of stationary Poiseuille flow and unsteady Womersley flow, the modified LBGK model has a second-order spatial convergence rate, and the compressibility. Additionally, they performed some simulations of natural convection in a square cavity to evaluate the stability of the present models, and they found that the outcomes are consistent with the literature even when Rayleigh is extremely high ($Ra=10^{12}$).

Jain and Bhargava [24] discussed the effect of the undulating surface in a square cavity in natural convection and the influence of the Rayleigh number, the length and height of the heat source as well as the undulation and amplitude on the rate of the transfer. They estimated the results in the form of isotherms and streamlines.

Rao and Barman [25] investigated natural convection in an undulated cavity representing the cooling of a heat source. The right side wall, which is corrugated, is maintained at a fixed ambient temperature, while a partial heat source with constant heat flux is placed on the right wall and the other walls are kept adiabatic. They found that convection inside the corrugated cavity filled with fluid-structured porous media depends on ε only at high Ra. Strong convection inside the cavity is also observed when the surface roughness increases.

Saleh et al. [26] studied, a novel approach to enhancing the cooling efficiency of CPUs is presented, addressing the challenge of high temperatures generated during intense computational processes. A new heat sink design incorporating convex-parabolic fins was developed to mitigate overheating issues, which can hinder task performance. The heat sink was coupled with a hybrid nanofluid for effective heat dissipation, and computational simulations were conducted using the finite volume method to analyze the cooling performance at varying Reynolds and fin numbers. Results revealed a significant improvement of 74% in cooling efficiency with the utilization of a greater number of fins. Additionally, the maximum temperature of the heat sink decreased proportionally with an increase in convex-parabolic fins, ensuring an optimal surface temperature for uninterrupted CPU operation. These findings offer valuable insights for the initial design and optimization of heat sinks, particularly for enhancing the cooling efficiency of high-performance processors like the Core i7.

The aim of this study is to carry out, numerically in a two-dimension approach, the effect of rough surfaces (protuberances) on the vertical walls on heat transfer in an enclosure. The vertical walls of the enclosure are subject to a uniform heat flux. However, the horizontal walls are supposed to be adiabatic. The rough surfaces are based on three different geometrical profiles: sinusoidal, triangular, and battlements. The homotopic transformation is used to reach a flat plate. The system of equations with the boundary conditions is solved using the finite volume method. A simple algorithm is chosen for the integration of algebraic equations. The originality of this work is examined by means of numerical simulations of the thermal-fluid fields, the effect of protuberance geometries and imposed fluxes positions in order to find the best configuration of the enclosure. The results are examined qualitatively by visualization and quantitatively through profiles of dynamic and thermal fields.

2. MATHEMATICAL FORMULATION

In this section, we describe the mathematical formulation of natural convection. First, we develop the governing equations along with their corresponding boundary conditions. Then, we present the equations and boundary conditions in their dimensionless forms. Finally, we determine the heat transfer rate in terms of the Nusselt number.

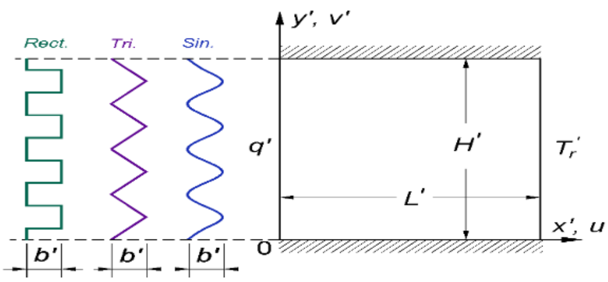


Figure 1. Physical model and boundary conditions

A two-dimensional corrugated cavity with the dimensions (H) and (L) is the configuration considered in this investigation (Figure 1). The fluid is supposed to be Newtonian with a constant viscosity (μ) and thermal conductivity (k). The state equation $\rho = \rho_r[1 - \beta(T' - T_r')]$ is used to model how density (ρ) varies with temperature (T'), where (β) is the thermal expansion coefficient.

The equations of continuity, Navier-Stokes and energy are used in this study with the Boussinesq approximation are given below:

$$\nabla.\vec{V}' = 0 \tag{1}$$

$$\rho_r \frac{\partial \vec{V}'}{\partial t'} + \rho_r(\vec{V}'.\nabla\,\vec{V}') = -\nabla P' + \mu\nabla^2\vec{V}' + \rho_r[1-\beta(T'-T_r')]\,\vec{g} \tag{2}$$

$$\frac{\partial T'}{\partial t'} + (\vec{V}'.\nabla)T' = \frac{k}{\rho_r C_p}\nabla^2 T' \tag{3}$$

All of the enclosure's boundaries are subjected to the no-slip condition. The corrugated left surface is exposed to a constant heat flux q' and the temperature of the flat right surface is considered to be constant at T_r'. On the other two horizontal sides of the cavity, adiabatic condition is applied. Hence, we now have:

$$u' = v' = 0;\ -k\,dT'/dx' = q' \quad \text{at} \quad x' = x_p'(y') \tag{4}$$

$$u' = v' = 0;\ T' = T_r' \quad \text{at} \quad x' = L' \tag{5}$$

$$u' = v' = 0;\ dT'/dy' = 0 \quad \text{at} \quad y' = 0, H' \tag{6}$$

The governing equations are nondimensionalised using L' for the length, L'^2/ν for the time, ν/L' for the velocity, $\rho_r(\nu/L')^2$ for the pressure, $\Delta T = q'L'/k$ for the temperature. Using these scales, the dimensionless form of the governing equations can be written as follows:

$$\nabla.\vec{V} = 0 \tag{7}$$

$$\frac{\partial \vec{V}}{\partial t} + \vec{V}.\nabla\vec{V} = -\nabla P + \nabla^2\vec{V} + \frac{Ra}{\Pr}T\,\vec{j} \tag{8}$$

$$\frac{\partial T}{\partial t} + (\vec{V}.\nabla)T = \frac{1}{\Pr}\nabla^2 T \tag{9}$$

where, $\vec{j}$ is the vertical coordinate, $Pr = \nu/\alpha$ is the Prandtl number and $Ra = g\beta q' L'^4/\nu\alpha$ is the Rayleigh number.

The nondimensional boundary conditions at the cavity's walls are:

$$u = v = 0;\ dT/dx = -1 \quad \text{at} \quad x = x_p(y) \tag{10}$$

$$u = v = 0;\ T = 0_r \quad \text{at} \quad x = 1 \tag{11}$$

$$u = v = 0;\ dT/dy = 0 \quad \text{at} \quad y = 0, A \tag{12}$$

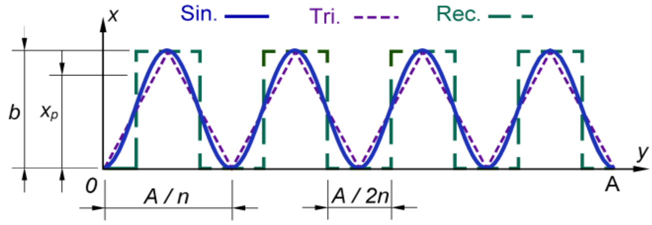


Figure 2. Dimensions and shapes of different protuberance used in this study

In the above equations, $x_p(y)$ is the corrugation's dimensionless height (Figure 2). Different shapes of protuberances were considered in this study, namely sinusoidal, triangular and battlement. For a smooth surface x_p=0, a sinusoidal corrugation discrete values $x_p(y)$ are obtained using a sinusoidal equation:

$$x_p(y) = b\left[1 - \cos(2\pi n\ y/A)\right] \tag{13}$$

where, A is the cavity's aspect ratio, b is the amplitude of corrugation and n is the number of corrugations in the enclosure.

To determine the heat transfer rates in terms of local number of Nusselt, the following expression can be used as follows:

$$Nu = \frac{h\,L'}{k} = \frac{1}{T(0,y) - T(1,y)} \tag{14}$$

The mean number of Nusselt can be evaluated in terms of average temperature T_m of the heated corrugated surface by the following relation:

$$Nu_m = \frac{1}{T_m} \tag{15}$$

3. NUMERICAL SIMULATION

The governing equations are figured out using ANSYS FLUENT commercial software with SIMPLE algorithm in order to solve pressure-velocity coupling. Structured meshing grid is chosen with a refinement in the region of the protuberances. The relaxation factors for velocity and temperature are respectively 0.1 and 0.8 [9, 11]. It is required that the absolute residual values of y-velocity, x-velocity, energy and continuity to be lower than 10^{-8}.

3.1 Validation and verification of the developed model

Table 1. Comparison of Nusselt number with previous works for A=1 and Pr=0.7

Ra	**10^3**	**10^4**	**10^5**	**10^6**	**10^7**
Present results	1.118	2.244	4.518	8.827	16.539
de Vahl Davis [1]	1.118	2.243	4.519	8.8	–
Yang et al. [23]	1.115	2.247	4.544	8.813	16.260

The results obtained by de Vahl Davis [1] and Yang et al. [23] are used to validate the results of this work. Clearly, the current results and that reported in the literature are in good agreement. Table 1 shows that 1.7% is the relative differences in terms of average number of Nusselt. Figure 3 shows the results of developed model compared to the numerical results of Said and Trupp [14] and the experimental results of Rubel and Landis [27]. This work's results are in good accord with the findings provided by the aforementioned authors.

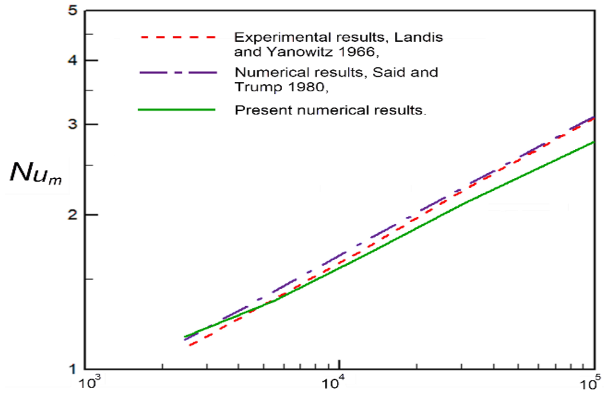


Figure 3. Comparison of average Nusselt number obtained in the present investigation with those published previously

3.2 Mesh dependency analysis

The cavity considered is meshed with a structured, refined and uniform mesh space which allows capturing all the gradients that occur in contact with the walls of the cavity. Throughout the numerical study, it is important to test the independence of the results obtained from the mesh selections before interpreting them. Five different types of mesh were considered but finally the chosen mesh is shown in Figure 2.

Table 2 shows results obtained using various sizes of the mesh for Ra=10^6, A=1, Pr=0.7 and for five sinusoidal corrugations with b=0.07. It is obvious that better outcomes are obtained with finer mesh sizes. Considering results in Table 2, 4370 elements have been used for the analysis of this study.

Table 2. Average Nusselt number as a function of grid size for $Ra=10^6$, $b=0.07$, $n=5$, $A=1$ and $Pr=0.7$

Number of Elements	1360	2220	4370	10440	21580
Nu_m	1.118	2.244	4.518	8.827	16.539

4. RESULTS AND DISCUSSIONS

Five parameters are governed the present problem, namely *q, Pr, b, A and n*. In the current research, the cavity's aspect ratio is maintained at A=1 and q varied from 5 to 100w/m^2 and the Prandtl's number has been set equal to Pr=0.7.

The aim of this investigation is to examine the geometric shape's effect of the protuberances arranged on vertical left wall of the enclosure as well as the effect of flux variation on heat transfer. Temperature and velocity contours are used to display the results in Figures 4, 5, 6 and 7 respectively.

In order to show the influence of the shape protuberance on the various parameters such as Nusselt number, the temperature and the speed, for each shape, the flux varies from 5 to 100w/m^2. In the following section, we will present only a sample of the results in the case of the flux q=100w/m^2.

The static temperature (Figure 4) fields show stratification for all heat density values. The triangle shape indicates a more homogeneity than the other protuberances.

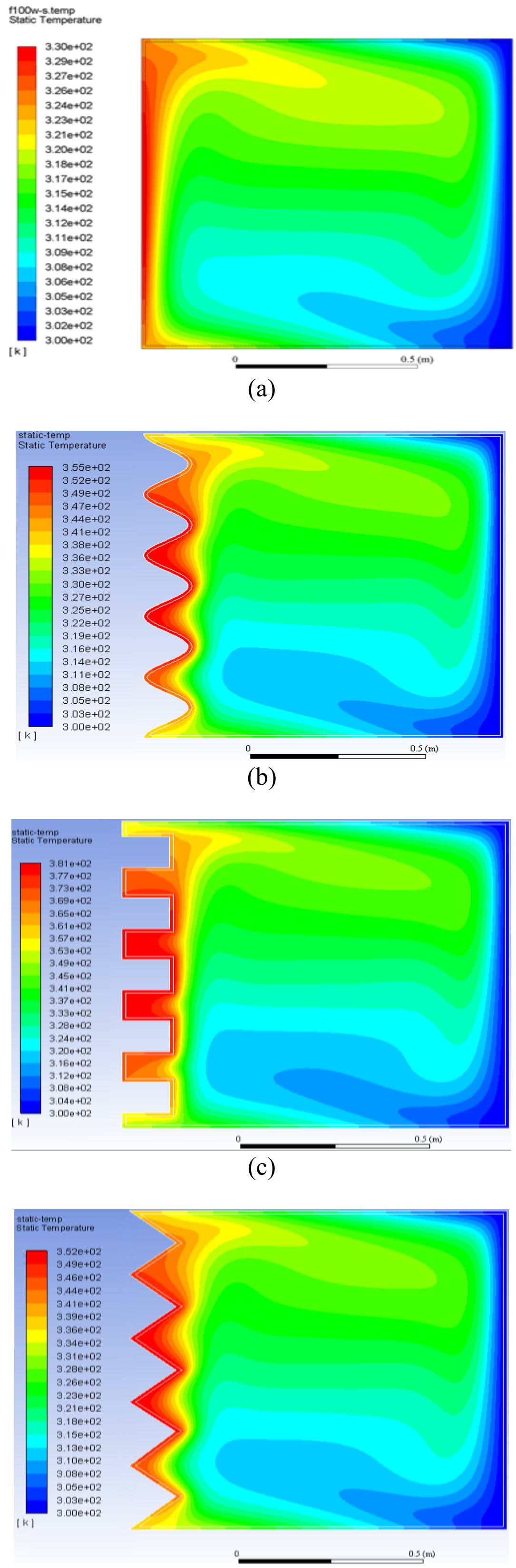


Figure 4. Static temperature without protuberances (a), with different forms (b) sinusoidal, (c) battlement and (d) triangular

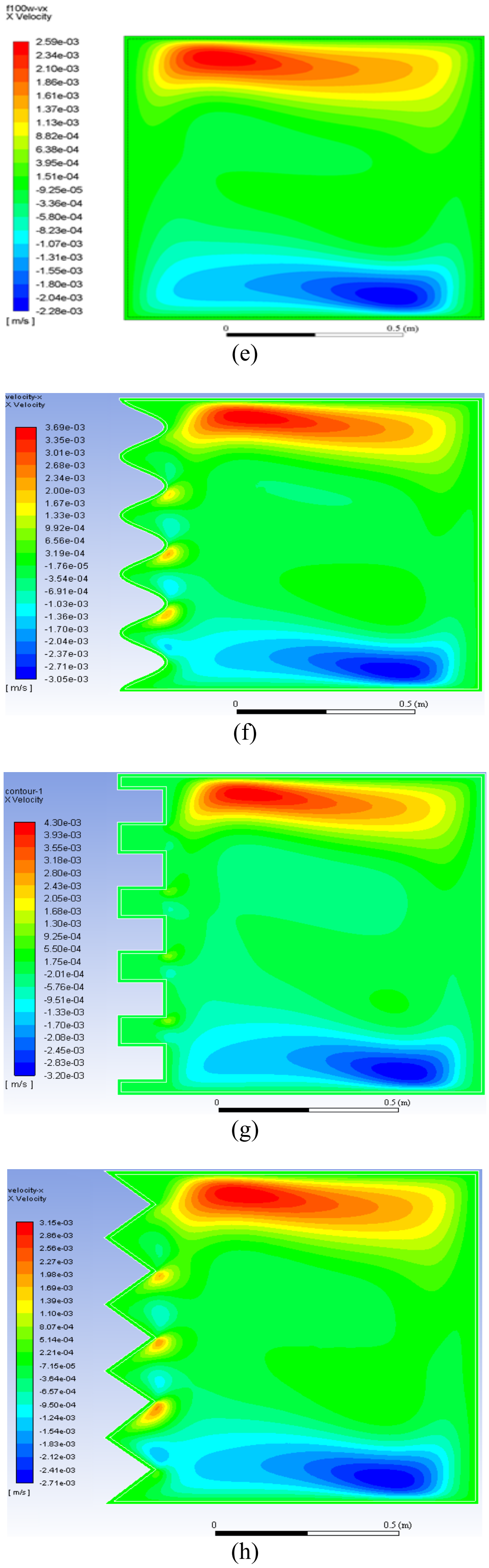


Figure 5. Contours of x velocity without protuberances (e), with different forms sinusoidal (f), battlement (g) and triangular (h)

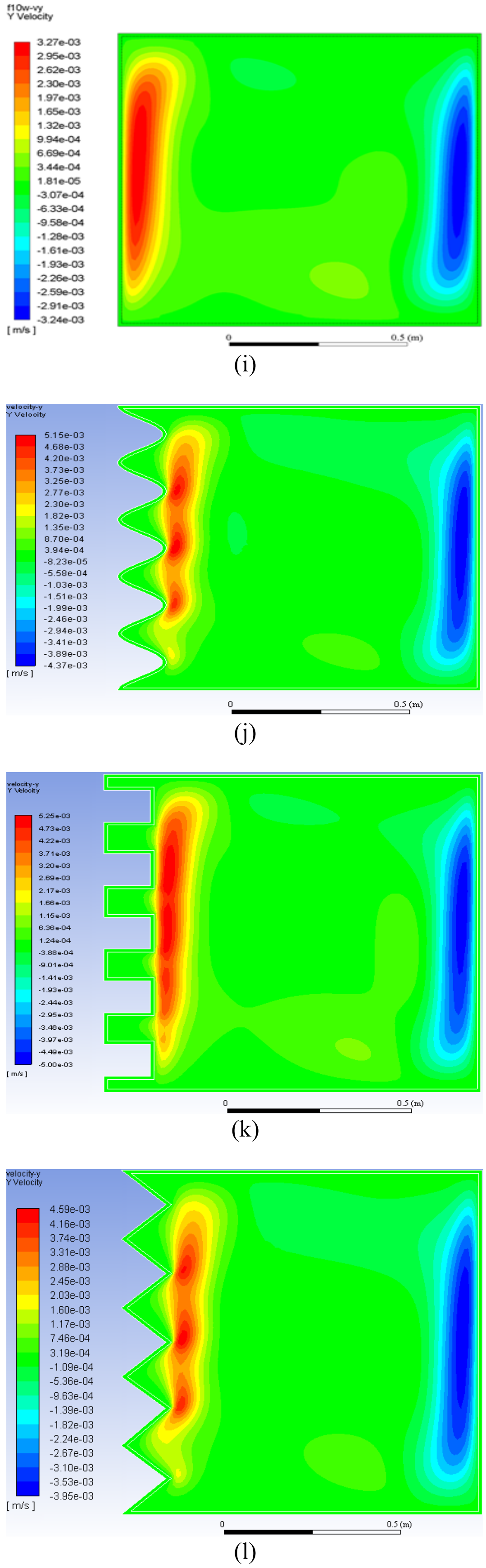
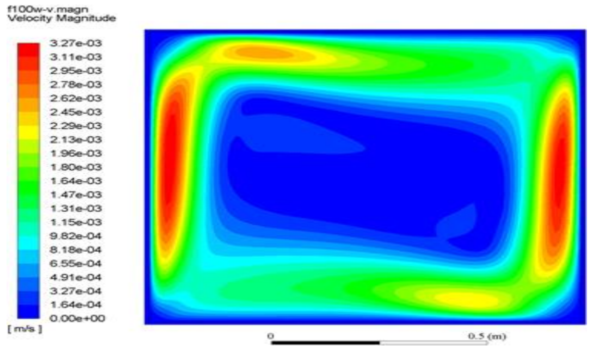
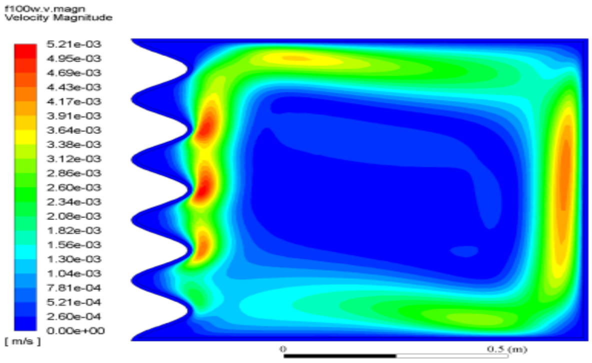


Figure 6. Contours of y velocity without protuberances (i) with different forms; sinusoidal (j), battlement (k) and triangular (l)

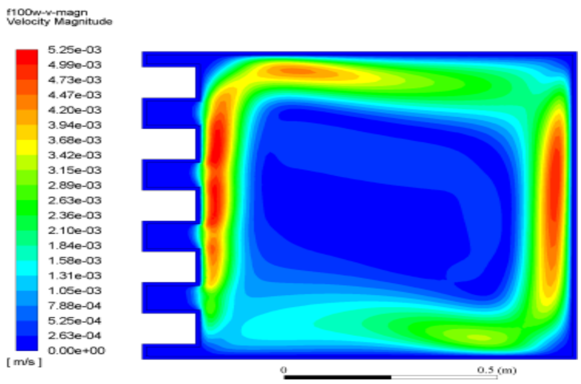
For the dynamical field, it was observed that the stream function values are proportional to the increase in the heat density. Furthermore, the dynamical activity is more pronounced for the triangular protuberances. It is also observed that the circulations are concentrated in the higher zone of the enclosure (Figure 5 and Figure 6).



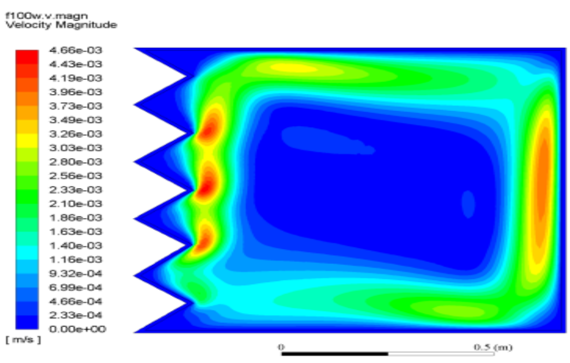
(m)



(n)



(o)



(p)

Figure 7. Velocity magnitude without protuberances (m), and with different forms; sinusoidal (n), battlement (o) and triangular (p)

In the Figure 7, it is evident that positioning protuberances on a cavity wall can markedly enhance heat transfer by modifying fluid flow patterns and improving convective heat transfer. The increased surface area and alterations in flow behavior contribute to more efficient heat dissipation. Additionally, fluid flowing around and over the protuberances undergoes changes in velocity and direction, fostering enhanced mixing. This increased mixing, induced by the presence of protuberances, facilitates improved thermal interaction between the fluid and the surface, consequently enhancing heat transfer. The presence of protuberances disrupts the development of boundary layers near the wall. Boundary layers, which are thin fluid layers near the surface, can impede heat transfer. Disrupting these boundary layers enables fresh, cooler fluid to come into contact with the surface, promoting more effective heat transfer. Moreover, the geometry of protuberances can induce the formation of secondary flow patterns in the fluid. Placing protuberances in a cavity increases the effective surface area, disrupts boundary layers, and encourages secondary flows. Collectively, these effects enhance convective heat transfer, contributing to more efficient heat dissipation from the surface.

Different flow types and bifurcations between these flows were made possible by variations in the flux density. Flow and temperature fields at the free surface go through peaks, with minimums between them. The maxima of the fluid parameters at the free surface are increasing functions of the shape of the protuberances. In general, the transfers developed in a sinusoidal bottom enclosure, triangular and battlement are larger than those obtained in a cavity with horizontal and uniform bottom having the same dimension.

At the ridge, the streamlines move close to the wall just past the ridge. The temperature distribution at this last region exhibits the same characteristics as depicted in Figure 4. This observation demonstrates the drop in thermal boundary layer thickness right after the crest. In the cavity's middle section, the flow stays stratified. Under the effect of undulation, the boundary layer's thickness increases and then decreases. It is noted that the geometrical shape of the cell is influenced by the corrugated wall, as it is clear on various diagrams.

Heat transfer can also be improved by placing protuberances on a wall simply because the total surface area is increased. Finally, it can be concluded that the triangular protuberance remains in the first position, in terms of heat transfer enhancement, followed by the sinusoidal protuberances and finally the battlement protuberances (Figures 8, 9 and 10).

Furthermore, it is observed that high Grashof number (high flux) values have a considerable influence on heat transfer. In other terms, the shape of protuberances surfaces plays an important role compared to the smooth surface (Figure 11).

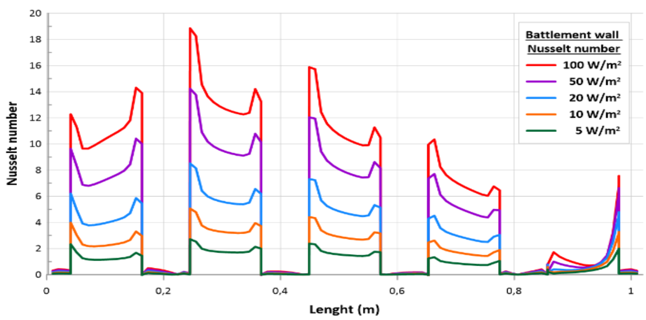


Figure 8. Nusselt number for battlements wall shape

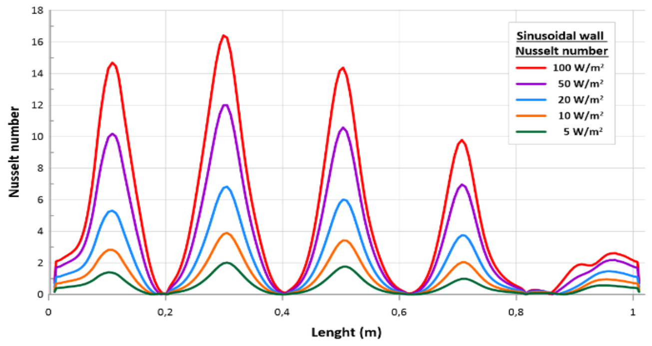


Figure 9. Nusselt number for sinusoidal wall shapes

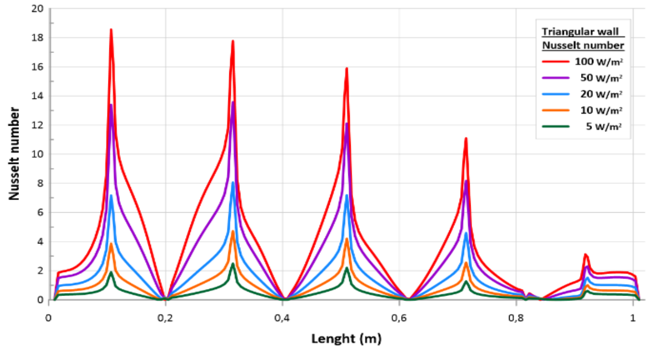


Figure 10. Nusselt number for triangular wall shapes

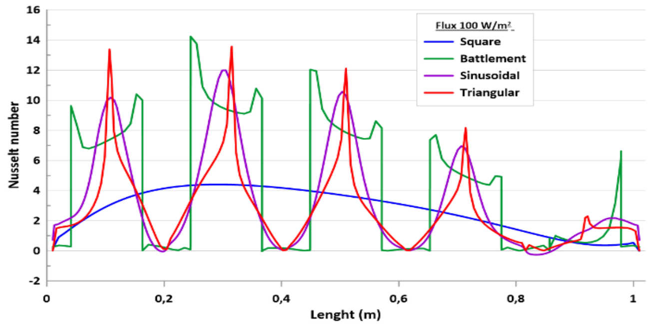


Figure 11. Nusselt number for different wall shapes ($q=100W/m^2$)

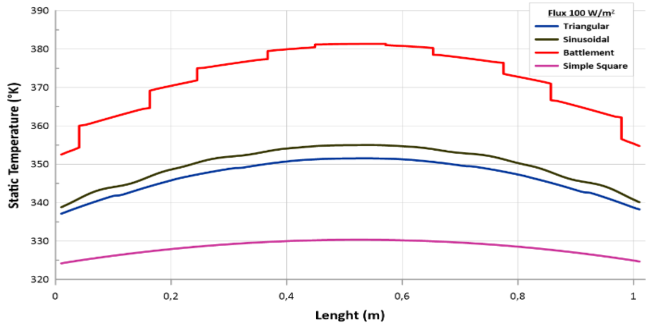


Figure 12. Static temperature for different wall shapes

The steady-state temperature profiles depicted in Figure 12 for various wall shapes, such as battlements or triangular walls, can be influenced by several factors, including material properties, environmental conditions, and the presence of insulation. The thermal conductivity of the material comprising the walls will affect how heat is transferred through them. Materials with higher thermal conductivity facilitate easier heat transfer.

The presence or absence of insulation in the walls can significantly affect the steady-state temperature. Insulation reduces heat transfer and helps maintain a more stable temperature within the structure. Additionally, external factors such as ambient temperature, solar radiation, and wind can play a role in shaping temperature distributions both within and around the structure. The shape of the walls, like battlements or triangular walls, can affect how heat is distributed. Sharp corners and irregular shapes may create areas of higher heat transfer. In Figure 12, it can be seen that the temperature profile is more irregular in the battements wall with a variation about 30°C. However, the simple square is more stable and the temperature remain constant.

5. CONCLUSIONS

In this comprehensive study of the effect of topography on natural convection heat transfer, numerical simulations were used to account for a variety of geometrical features such as sinusoidal, battlement, and triangular protuberances. The study finds a strong link between the shapes of protuberances, heat flux rates, and the flow topology. Among the examined forms, triangular protuberances outperform sinusoidal and battlement shapes in terms of thermal-fluid performance, indicating an increase in natural convection within the enclosure.

The provided studies demonstrate that the presence of protuberances significantly improves heat transfer, with triangular shapes proving to be especially efficient. Surprisingly, the thermal conductivity of these protuberances has little effect on improving heat transport, implying that other aspects are more important. To support the current findings, future research should expand the examination to three-dimensional enclosures, as experimental evidence in the literature is scarce. The results underlines the importance of conducting a more comprehensive analysis that considers the relationship between stream function and heat density in cavities with protuberances. Temperature, thermal conductivity, and fluid flow patterns are recognized as important factors in understanding this relationship, as key components in defining heat transfer processes, particularly in areas of flow separation and recirculation. The presence of protuberances is shown to change flow patterns and influence thermal boundary layer properties. This disruption alters temperature gradients, notably at the leading edge where the thermal boundary layer is thin and downstream where it thickens, which frequently coincides with flow separation and recirculation zones. Understanding these differences in thermal boundary layer thickness is regarded as critical for maximizing heat transmission in systems with cavities and protuberances.

The disruption of boundary layers near the wall enables more effective heat transmission, which is aided by the induction of secondary flow patterns in the fluid due to the geometry of the protuberances.

Thus, this multifaceted investigation of protuberances in cavities provides important insights for enhancing heat transmission mechanisms. The work not only advances our understanding of how varied protuberance shapes affect heat transfer, but it also highlights the complex relationships between flow patterns, thermal boundary layers, and convective heat transfer. These findings have practical significance for bioclimatic thermal management, serving as a platform for future research and implementation in a variety of technical and environmental scenarios.

REFERENCES

[1] de Vahl Davis, G. (1983). Natural convection of air in a square cavity: A bench mark numerical solution. International Journal for Numerical Methods in Fluids, 3(3): 249-264. https://doi.org/10.1002/fld.1650030305

[2] Ostrach, S. (1988). Natural convection in enclosures. Journal of Heat Transfer, 110(4b): 1175-1190. https://doi.org/10.1115/1.3250619

[3] Noorshahi, S., Hall, C., Glakpe, E. (1992). Effect of mixed boundary conditions on natural convection in an enclosure with a corrugated surface. In Proceedings of the ASME-JSES-KSES International Solar Energy Conference, pp. 173-181.

[4] Bhavnani, S.H., Bergles, A.E. (1990). Effect of surface geometry and orientation on laminar natural convection heat transfer from a vertical flat plate with transverse roughness elements. International Journal of Heat and Mass Transfer, 33(5): 965-981. https://doi.org/10.1016/0017-9310(90)90078-9

[5] Shakerin, S., Bohn, M., Loehrke, R.I. (1988). Natural convection in an enclosure with discrete roughness elements on a vertical heated wall. International Journal of Heat and Mass Transfer, 31(7): 1423-1430. https://doi.org/10.1016/0017-9310(88)90251-7

[6] Das, P.K., Mahmud, S. (2003). Numerical investigation of natural convection inside a wavy enclosure. International Journal of Thermal Sciences, 42(4): 397-406. https://doi.org/10.1016/S1290-0729(02)00040-6

[7] Dalal, A., Das, M.K. (2005). Laminar natural convection in an inclined complicated cavity with spatially variable wall temperature. International Journal of Heat and Mass Transfer, 48(18): 3833-3854. https://doi.org/10.1016/j.ijheatmasstransfer.2004.07.051

[8] Saidi, C., Legay-Desesquelles, F., Prunet-Foch, B. (1987). Laminar flow past a sinusoidal cavity. International Journal of Heat and Mass Transfer, 30(4): 649-661. https://doi.org/10.1016/0017-9310(87)90195-5

[9] Adjlout, L., Imine, O., Azzi, A., Belkadi, M. (2002). Laminar natural convection in an inclined cavity with a wavy wall. International Journal of Heat and Mass Transfer, 45(10): 2141-2152. https://doi.org/10.1016/S0017-9310(01)00304-0

[10] Yao, L.S. (1983). Natural convection along a vertical wavy surface. Journal of Heat Transfer. 105(3): 465-468. https://doi.org/10.1115/1.3245608

[11] Mayouf, S.A. (2012). Effect of wavy geometry on natural convection heat transfer, boundary layer flow along a vertical plate with variable wall temperature. Sciences & Technology. A, Exactes Sciences, 37-44.

[12] Agrouaz, Y., Bouhal, T., Jamil, A., Msaad, A.A., Kousksou, T., Mahdaoui, M., El Rhafiki, T. (2015). Effect of inclined wavy surface on heat transfer inside a rectangular cavity: Solar applications. In 2015 3rd International Renewable and Sustainable Energy Conference (IRSEC), Marrakech, Morocco, pp. 1-6. https://doi.org/10.1109/IRSEC.2015.7455006

[13] El Moutaouakil, L., Boukendil, M., Zrikem, Z., Abdelbaki, A. (2020). Natural convection and surface radiation heat transfer in a square cavity with an inner wavy body. International Journal of Thermophysics, 41: 1-21. https://doi.org/10.1007/s10765-020-02688-7

[14] Said, M.N.A., Trupp, A.C. (1980). Laminar free convection in vertical air-filled cavities with mixed boundary conditions. Chemical Engineering Communications, 5(1-4): 93-107. https://doi.org/10.1080/00986448008935956

[15] Nghana, B., Tariku, F., Bitsuamlak, G. (2022). Numerical assessment of the impact of transverse roughness ribs on the turbulent natural convection in a BIPV air channel. Building and Environment, 217: 109093. https://doi.org/10.1016/j.buildenv.2022.109093

[16] Nayak, M.K., Karimi, N., Chamkha, A.J., Dogonchi, A.S., El-Sapa, S., Galal, A.M. (2022). Efficacy of diverse structures of wavy baffles on heat transfer amplification of double-diffusive natural convection inside a C-shaped enclosure filled with hybrid nanofluid. Sustainable Energy Technologies and Assessments, 52: 102180. https://doi.org/10.1016/j.seta.2022.102180

[17] Ganesh, N.V., Al-Mdallal, Q.M., Öztop, H.F., Kalaivanan, R. (2022). Analysis of natural convection for a Casson-based multiwall carbon nanotube nanofluid in a partially heated wavy enclosure with a circular obstacle in the presence of thermal radiation. Journal of Advanced Research, 39: 167-185. https://doi.org/10.1016/j.jare.2021.10.006

[18] Ali, M.M., Akhter, R., Alim, M.A. (2021). Hydromagnetic natural convection in a wavy-walled enclosure equipped with hybrid nanofluid and heat generating cylinder. Alexandria Engineering Journal, 60(6): 5245-5264. https://doi.org/10.1016/j.aej.2021.04.059

[19] Kadari, A., Sad Chemloul, N.E., Mekroussi, S. (2018). Numerical investigation of laminar natural convection in a square cavity with wavy wall and horizontal fin attached to the hot wall. Journal of Heat Transfer, 140(7): 072503. https://doi.org/10.1115/1.4039081

[20] Djaomazava, G., Randriazanamparany, M.A., Rakotomanga, F.D.A. (2019). Numerical study of natural convection in a vertical channel, one of whose walls has sinusoidal protuberances. International Journal of Heat and Engineering, 8(1): 1-6.

[21] Rahmani, K., Bentebbiche, A., Draoui, B. (2013). Numerical investigation of turbulent natural convection for a cavity having sinusoidal protuberances on a vertical wall. Journal of Applied Fluid Mechanics, 6(4): 491-499. https://doi.org/10.36884/jafm.6.04.21278

[22] Mahdi, Q.A., Hamza, M.K., Kadhim, A.J., Jailawi, R.M. (2021). Study of natural convection inside corrugated enclosure containing heated thin rectangle plate, 44(2): 22-35.

[23] Yang, X., Shi, B., Chai, Z. (2014). Generalized modification in the lattice Bhatnagar-Gross-Krook model for incompressible Navier-Stokes equations and convection-diffusion equations. Physical Review E, 90(1): 013309. https://doi.org/10.1103/PhysRevE.90.013309

[24] Jain, S., Bhargava, R. (2023). Natural convection flow on a bent wavy vertical enclosure filled with power-law nanofluid simulated by Element Free Galerkin method. Mathematics and Computers in Simulation, 205: 970-986. https://doi.org/10.1016/j.matcom.2022.10.033

[25] Rao, P.S., Barman, P. (2021). Natural convection in a wavy porous cavity subjected to a partial heat source.

International Communications in Heat and Mass Transfer, 120: 105007. https://doi.org/10.1016/j.icheatmasstransfer.2020.105007
[26] Saleh, M.S., Belloufi, Y., Boutera, Y., Chamkha, A.J., Sahraoui, A., Hroub, Q. (2023). A new design for a heat sink within a convex-parabolic microchannel filled with hybrid nanofluid for cooling Core I7 CPU. Journal of Thermal Analysis and Calorimetry, 148(20): 11315-11324. https://doi.org/10.1007/s10973-023-12404-w
[27] Rubel, A., Landis, F. (1969). Numerical study of natural convection in a vertical rectangular enclosure. The Physics of Fluids, 12(12): II-208. https://doi.org/10.1063/1.1692438

NOMENCLATURE

A	cavity's aspect ratio [H'/L']
b'	amplitude of corrugation [m]
C_p	fluid's specific heat [$J/(kg\ K)$]
Gr	number of Grashof [$g\beta' q' L'^4/\nu^2$]
H'	cavity's height [m]
L'	cavity's width [m]
k	thermal conductivity [$W/(m.K)$]
n	number of corrugations in the cavity
P'	pressure in the cavity [N/m^2]
q'	heat flux [W/m^2]
$\Pr$	Prandtl number [ν/α]
Ra	Rayleigh number [$g\beta' q' L'^4/\nu\alpha$]
t'	time [s]
T'	temperature [K]
x'	horizontal coordinate [m]
y'	vertical coordinate [m]
$\vec{V}'$	velocity [m/s]
x'_p	height of sinusoidal corrugation from a reference point

Greek symbols

α	fluid's thermal diffusivity [m^2/s]
β	coefficient of thermal expansion [K^{-1}]
ν	fluid's kinematic viscosity [m^2/s]
μ	fluid's dynamic viscosity [$N\ s/m^2$]
ρ	fluid's density [kg/m^3]

Subscripts

m	mean value
1	value at $x=1$
0	value at $x=0$
r	reference state
'	refers to dimensional variable

Abbreviations

LBGK	Lattice Boltzmann Bhatnagar-Gross-Krook