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Fuzzy Anti-Magic Labeling on Comb and Twing Graphs

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ABSTRACT

Fuzzy labeling in graph theory is important because it enhances the modelling of uncertainties, gives relationships a granular representation, makes network models more robust, enables clustering and community detection, contributes to decision-making, enables machine learning and data mining, and has applications in different fields where complex structures need to be expressed with degrees of membership or uncertainty. In this paper, we proved that the twing and comb graphs admit fuzzy labeling by providing an algorithm. Also, we show that the twing and comb graphs reveal vertex and edge anti-magic labeling. Fuzzy end vertex, fuzzy bridge, degree, strong degree, and strong edge are mostly related to connectivity, so they may be applied to networking. Because of that, we have derived some properties related to the fuzzy end vertex, fuzzy bridge, degree, strong degree, and strong edge have been discussed.

1. INTRODUCTION

In graph theory, labeling is a fundamental concept that studies connections and relationships between objects represented by vertices, or nodes, interconnected by edges, or links. In graph labeling, vertices, edges, or both are assigned numerical or symbolic labels to convey specific information or encode graph properties. Depending on the graph's context and application, these labels can represent distances, colors, weights, or any other relevant characteristics.

Fuzzy graph labeling can be used in many areas, such as decision-making, pattern recognition, image processing, and network analysis when inaccuracies and ambiguities need to be taken into account. Graph-based models are often characterized by uncertainty, which can be captured and quantified using fuzzy graph labeling.

Gallian [1] discussed various graph labeling in the dynamic survey of graph labeling. The fuzzy graph (FG) is more noticeable when there is ambiguity on vertices and edges, and the FG is more prominent. Rosenfield [2] had a salient advancement in the mathematical system for haziness in vertices and edges. Fuzzy Labeling graphs (FLG) have applications in coding theories, circuit designs, X-rays, astronomy, communication networks, etc. The notion of fuzzy labeling (FL), magic fuzzy labeling graph discussed by Gani and Subahashini [3], Nagoor Gani and Akram [4]. Ameenal Bibi and Devi [5], Bibi and Devi [6] studied vertex graceful fuzzy labeling and fuzzy anti-magic labeling of some graphs. Sujatha et al. [7, 8] proved some outcomes on graceful FL, magic FL, and Triangular anti-magic FL in an algorithmic approach for some unique graphs. Shanmugapriya and Hemalatha [9] have explored fuzzy edge magic total labeling for some family of graphs and discussed the application of finding the strength of relationship between two persons. Shanmugapriya and Hemalatha [10] have obtained fuzzy vertex magic total labeling of generalized Peterson graph and demonstrated application about electricity passing through transformers. Mahdi et al. [11] discussed about medical images using fuzzy convolutional neural networks. Saibavani and Parvathi [12] explained the power domination in different graphs with applications. Sujatha et al. [13] illustrated the anti-magic labeling on some triangular fuzzy graphs. Rashmanlou et al. [14] discussed the new operations on bipolar FGs. Akram and Waseem [15] explained the notion of metric in m-polar FG and disscussed properties.

A graph is a Fuzzy Edge Anti-Magic labeling graph if a Fuzzy Edge Anti-Magic (FEAM) labeling is defined on it. A graph is a Fuzzy Vertex Anti-Magic labeling if a Fuzzy Vertex Anti-Magic (FVAM) labeling is defined on it. In this paper, we established FEAM labeling and FVAM labeling for fuzzy comb and fuzzy twing graphs. This study may be used in the field of social networking, Road networking and etc. The above results can be expanded with fuzzy interval, triangular fuzzy number and trapezoidal fuzzy number.

2. PRELIMINARIES

"An FG $G(\mu, \rho)$ is a couple of functions $\mu{:}V\rightarrow[0, 1]$ & $\rho{:}V\times V\rightarrow[0, 1]$ where $\forall u,v\in V$, then $\rho(u, v)\leq\min\{\mu(u), \mu(v)\}$." [3]

"A graph $G(\mu, \rho)$ is known as FLG, if $\mu{:}V\rightarrow[0, 1]$ &

ρ:$V\times V\rightarrow[0, 1]$ is one-one correspondence in which the grade of membership of edges and vertices are different and $\rho(u,v)<\min\{\mu(u),\mu(v)\}\forall u,v\in V$." [3]

The vertex degree of u is defined by $d(u)=\sum_{u\neq v,v\epsilon V}\rho(u,v)$.

Example 2.1

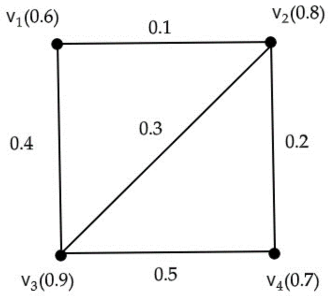

Figure 1. Fuzzy labeling graph

From Figure 1, the degree of the vertex is defined as: $d(v_1)$=0.4+0.1=0.5, $d(v_2)$=0.1+0.3+0.2=0.6, $d(v_3)$=0.4+0.3 +0.5=1.2, $d(v_4)$=0.5+0.2=0.7.

"The strength of P is defined as $\wedge_{d=1}^{n}\mu\,(v_{d-1},v_d)$. i.e., the strength of a path is defined to be the weight of the weakest edge of the path." [3]

"A strongest path joining any two nodes u, v is a path corresponding to maximum strength between u and v. The strength of the strongest path is denoted by $\mu^{\infty}(u, v)$." [3]

An edge (u, v) of an FG is known as strong edge if $\rho^{\infty}(u, v)=\rho(u, v)$.

"A strong neighborhood of $u\in V$ is $N_s(u)=\{v\in V$: edge (u,v) is strong$\}$." [5]

"Strong degree of a vertex is defined as the total membership value of all strong edges incident at that vertex. It is named by $d_s(v)$. Ie, $d_s(v)=\sum_{u\in N_s(v)}\rho(v,u)$." [5]

"An edge is called a fuzzy bridge of G if its removal reduces the strength of connectedness between some pair of vertices in G." [3]

"If a vertex has almost one strong neighbour in G, it is said to as a fuzzy end vertex of $G(\mu, \rho)$." [3]

Example 2.2

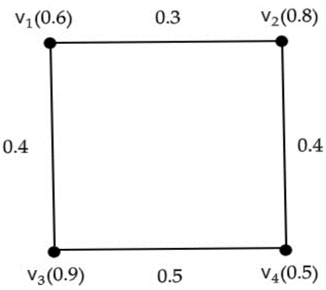

Figure 2. Fuzzy graph

In Figure 2, the possible paths between v_3 and v_4 are v_3v_4, $v_3v_1v_2v_4$ and the strength of the corresponding paths are 0.5 and 0.3.

$$\rho^{\infty}(v_3, v_4)=0.5$$

If we remove the edge v_3v_4 then the strength of the path will be reduced to $\rho'^{\infty}(v_3, v_4)=0.3$.

Therefore, the edge v_3v_4 is a fuzzy bridge, strong edge and v_3, v_4 strong neighbours.

An FG is called Fuzzy Edge Anti-Magic (FEAM) labeling if $\mu(u)+\rho(u,v)+\mu(v)\forall u,v\in V$ are all distinct and is denoted by Am_0.

A Fuzzy Vertex Anti-Magic (FVAM) labeling in "a graph G is a 1-1 correspondence $f: E(G)\rightarrow\{(1,2,3\ldots,|E(G)|)\}$ in which for any two different vertices v and w, the total of the labels on edges incident to 'v' distinct from the total of labels on edges incident to w." [8]

An FG which admits FEAM labeling is known as the FEAM graph. An FG which admits FVAM labeling is known as the FVAM graph.

3. COMB GRAPH

A graph is obtained by joining every vertex of a path with one pendent edge is known as comb graph CP_n.

A comb graph is called a fuzzy comb graph if FL exists. Fuzzy comb graph is called FEAM comb graph if FEAM labeling exists. And also, fuzzy comb graph is called FVAM comb graph if FVAM labeling exists.

In this section, the FEAM comb and the FVAM comb graph have been proved with the use of the proposed algorithm through the theorem.

Algorithm 3.1

FL of vertices and edges CP_n with $n\geq1$, where n is a path length and z→(0, 1]

Step 1: Input values for n and z.

n=int (input ("Enter the value for n:"))

z=float (input ("Enter the value for z:"))

Step 2: Calculate the maximum number of vertices

m=2n+2.

Step 3: Initialize empty lists for vertex and edge labels

vertex_labels = []

edge_labels = []

Step 4: Generate vertex labels using the given formula

for d =1 to m

{

$\mu(v_d)=(2n+d+1)z, 1\leq d\leq 2n+2$

}

Step 5: Generate edge labels using the given formulas

for d =1 to m

if d =1

{

$\rho(v_d,v_{d+1})=max\{\mu(v_d),\mu(v_{d+1})\}-min\{\mu(v_d),\mu(v_{d+1})\}$

}

if 2≤ d ≤n

{

$\rho(v_d,v_{d+1})=max\{\mu(v_d),\mu(v_{d+1})\}-min\{\mu(v_d),\mu(v_{d+1})\}+(d-1)z$

}

for d =1 to m

if 1≤d≤n+1

{

$\rho(v_d,v_{n+1+d})=max\{\mu(v_d),\mu(v_{n+1+d})\}-min\{\mu(v_d),\mu(v_{n+1+d})\}+(n+1-d)z$

}

Step 6: Output the vertices and edges labels

Print ("vertex Labels:")

Print ("Edge Labels:")

Theorem 3.1

Let $CP_n:(\mu, \rho)$ be the fuzzy labeled comb graph for all $n \geq 1$ then CP_n admits FEAM labeling.

Proof:

Let $z \to (0, 1]$ such that:

$$z = \begin{cases} \frac{1}{10^2}; 0 < n \leq 24 \\ \frac{1}{10^{k+3}}; 24 < n \leq 24 + \sum_{\substack{t=0 \\ 0 \leq i \leq k}}^{i} 225 \times 10^t, k = 0 \\ \frac{1}{10^{k+4}}; 24 + \sum_{\substack{t=0 \\ 0 \leq i \leq k}}^{i} 225 \times 10^t < n \leq 24 \\ \quad + \sum_{\substack{t=0 \\ 0 \leq i \leq k+1}}^{i} 225 \times 10^t, k = 0,1,2 \cdots \end{cases}$$

Using algorithm 3.1, we have the membership value of vertices and edges defined as follows:

$$\mu: V \to [0,1] \ni \mu(v_d) = (2n + d + 1)z \ \forall \ v_d \in V, \quad 1 \leq d \leq 2n + 2$$

$\rho: V \times V \to [0, 1]$ such that

$$\rho(v_d, v_{d+1}) = max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\}, d = 1$$

$$\rho(v_d, v_{d+1}) = max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\} + (d - 1)z, 2 \leq d \leq n$$

$$\rho(v_d, v_{n+d+1}) = max\{\mu(v_d), \mu(v_{n+d+1})\} - min\{\mu(v_d), \mu(v_{n+1+d})\} + (n + 1 - d)z, 1 \leq d \leq n + 1$$

To prove the FEAM labeling of the comb graph We show that all the anti-magic constants of the comb graph are different in the following three cases:

Case 1: When d=1 and $\forall n$

Now, $Am_0(CP_n) = \mu(v_d) + \rho(v_d, v_{d+1}) + \mu(v_{d+1}) = (2n + d + 1)z + max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\} + (d + 2n + 2)z = (4n + 2d + 4)z$

Case 2: When $2 \leq d \leq n$ & $\forall n$

Now, $Am_0(CP_n) = \mu(v_d) + \rho(v_d, v_{d+1}) + \mu(v_{d+1}) = (2n + d + 1)z + (d - 1)z + max\{\mu(v_d), \mu(v_{d+1})\} - \min\{\mu(v_d), \mu(v_{d+1})\} + (d + 2n + 2)z = (4n + 3d + 3)z$

Case 3: When $2 \leq d \leq n$ & $\forall n \geq 2$

Now, $Am_0(CP_n) = \mu(v_d) + \rho(v_d, v_{n+1+d}) + \mu(v_{n+1+d}), = (2n + d + 1)z + max\{\mu(v_d), \mu(v_{n+d+1})\} - min\{\mu(v_d), \mu(v_{n+d+1})\} + (n + 1 - d)z = (7n + d + 5)z$

Hence, in Case 1, Case 2, and Case 3, we have proved that all the anti-magic constants of the comb graph are different.

Thus, fuzzy labeled comb graph admits FEAM labeling.

Example 3.1 Figure 3 illustrates the FEAM comb graph of path length 24.

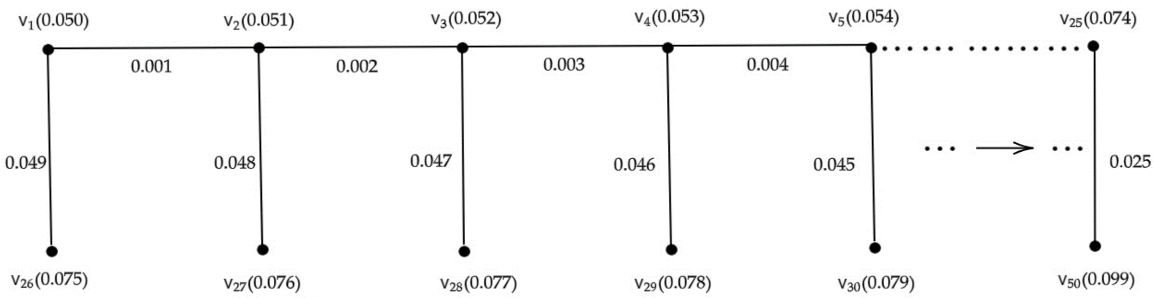

Figure 3. CP_{24}- FEAM comb graph

Algorithm 3.2

FL of vertices and edge of CP_n with $n \geq 1$, where n is a path length and $z \to (0,1]$

Step 1: Input values for n and z.
n=int (input ("Enter the value for *n*:"))
z=float (input ("Enter the value for *z*:"))

Step 2: Calculate the maximum number of vertices
m=2*n*+2.

Step 3: Initialize empty lists for vertex and edge labels
vertex_labels =[]
edge_labels =[]

Step 4: Generate vertex labels using the given formula
for d=1 to *m*
{
$\mu(v_d) = (2n + d + 1)z, 1 \leq d \leq 2n + 2$
}

Step 5: Generate edge labels using the given formulas
for d =1 to m
if d =1
{
$\rho(v_d, v_{d+1}) = max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\}$
}
if d =1
{
$\rho(v_d, v_{2n+2}) = min\{\mu(v_d), \mu(v_{2n+2})\} + nz$
}
for d =1 to m
if $0 \leq d \leq n-2$
{
$\rho(v_{n+3+d}, v_{n+4+d}) = max\{\mu(v_{n+3+d}), \mu(v_{n+4+d})\} - min\{\mu(v_{n+3+d}), \mu(v_{n+4+d})\} + (2n - d)z$
}
for d =1 to m
if $0 \leq d \leq n-1$
{
$\rho(v_{3+d}, v_{2n+2-d}) = min\{\mu(v_{3+d}), \mu(v_{2n+2-d})\} + (2n + 2)z$
}

Step 6: Output the vertices and edges labels
Print ("vertex Labels:")
Print ("Edge Labels:")

Theorem 3.2

Let CP_n:(μ, ρ) be the fuzzy labeled comb graph for all $n\geq 1$ then CP_n admits FVAM labeling.

Proof:

Given CP_n:(μ, ρ) be the fuzzy labeled comb graph.

To prove that fuzzy labeled comb graph CP_n:(μ, ρ) satisfies the condition of FVAM labeling.

That is to prove that for any two vertices u and v in CP_n, the total of the grade membership on the edges incident at the vertex u is distinct from the total of the grade membership on the edges incident at the vertex v.

Let $z\rightarrow(0, 1]$ such that

$$z = \begin{cases} \frac{1}{10^2}; 0 < n \leq 24 \\ \frac{1}{10^{k+3}}; 24 < n \leq 24 + \sum^{i}_{\substack{t=0 \\ 0\leq i\leq k}} 225 \times 10^t \text{ where } k = 0 \\ \frac{1}{10^{k+4}}; 24 + \sum^{i}_{\substack{t=0 \\ 0\leq i\leq k}} 225 \times 10^t < n \leq 24 \\ \quad + \sum^{i}_{\substack{t=0 \\ 0\leq i\leq k+1}} 225 \times 10^t \text{ where } k = 0,1,2\cdots \end{cases}$$

Using algorithm 3.2, we have membership value of vertices and edges defined as follows:

$$\mu: V \rightarrow [0,1] \ni \mu(v_d) = (2n + d + 1)z, \quad 1 \leq d \leq 2n + 2 \tag{1}$$

$$\rho(v_d, v_{d+1}) = max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\} \tag{2}$$

$$\rho(v_d, v_{2n+2}) = min\{\mu(v_d), \mu(v_{2n+2})\} + nz, d = 1 \tag{3}$$

$$\begin{aligned} &\rho(v_{n+3+d}, v_{n+4+d}) \\ &= max\{\mu(v_{n+3+d}), \mu(v_{n+4+d})\} \\ &- min\{\mu(v_{n+3+d}), \mu(v_{n+4+d})\} \\ &+ (2n - d)z, 0 \leq d \leq n - 2 \end{aligned} \tag{4}$$

$$\rho(v_{3+d}, v_{2n+2-d}) = min\{\mu(v_{3+d}), \mu(v_{2n+2-d})\} + (2n + 2)z, 0 \leq d \leq n - 1 \tag{5}$$

Also, sum of the edge labels incident at:

$$v_d = Wt(v_d) = \sum_{u\in N(v_d)} \rho(u, v_d) \tag{6}$$

when, $N(v_d)$ be the neighbourhood vertices of v_d for all d=1 to $2n$+2 and $Wt(v_d)$ be the weight of v_d.

From equation number Eq. (1), Eq. (2), Eq. (3), Eq. (4), Eq. (5) and Eq. (6) for any two vertices v_p and v_q with $p\neq q$, $Wt(v_p)$ and $Wt(v_q)$ have distinct values.

Therefore, FVAM labeling is allowed for comb graph $CP_n \forall n\geq 1$.

Example 3.2 Figure 4 illustrates the FVAM comb graph of path length 25.

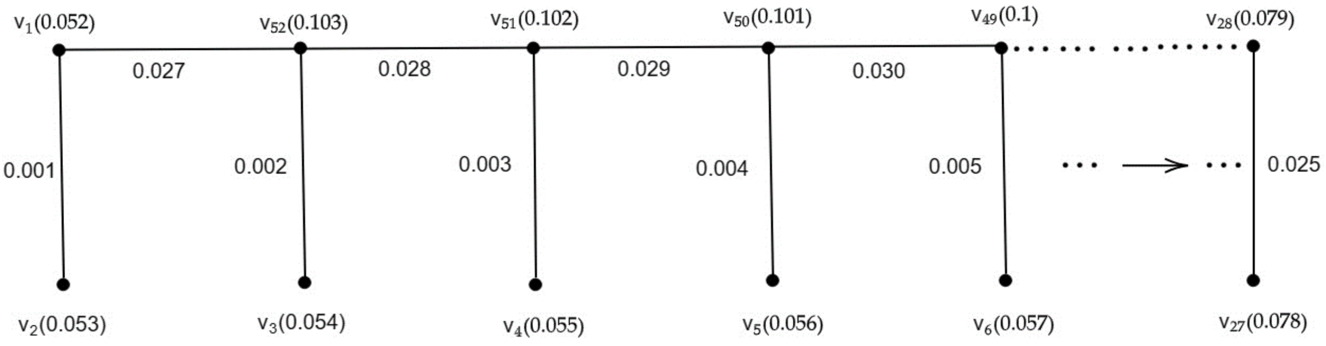

Figure 4. CP_{25} – FVAM comb graph

4. TWING GRAPH

The twing graph is a graph that is formed by joining a couple of terminal edges to every internal vertex of the path. It is named $T(n)$.

A twing graph is called fuzzy twing graph if FL exists. Fuzzy twing graph is called FEAM twing graph if FEAM labeling exists. And also, a fuzzy twing graph is called FVAM twing graph if FVAM labeling is exists.

In this section, the FEAM twing graph and the FVAM twing graph have been proved with the use of the proposed algorithm through the theorem.

Algorithm 4.1

FL of vertices and edges of $T(n)$ with $n\geq 1$, where n is the number of internal vertices in the path and $z\rightarrow(0,1]$

Step 1: Input values for n and z.

n=int (input ("Enter the value for n:"))

z=float (input ("Enter the value for z:"))

Step 2: Calculate the maximum number of vertices

m=3n+2.

Step 3: Initialize empty lists for vertex and edge labels

vertex_labels =[]

edge_labels =[]

Step 4: Generate vertex labels using the given formula

for d =1 to m

{

$\mu(v_d) = (3n + d + 1)z, 1 \leq d \leq 3n + 2$

}

Step 5: Generate edge labels using the given formulas

for d =1 to m

if d =1

{

$\rho(v_d, v_{n+2d}) = max\{\mu(v_d), \mu(v_{n+2d})\} - min\{\mu(v_d), \mu(v_{n+2d})\} - nz$

}

if $2 \le d \le n+1$

{

$\rho(v_d, v_{n+d}) = max\{\mu(v_d), \mu(v_{n+d})\} - min\{\mu(v_d), \mu(v_{n+d})\} + dz$

}

if $n+2 \le d \le 2n+1$

{

$\rho(v_d, v_{d+1}) = max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\} + (d-n-1)z$

}

for d =1 to m

if $n+2 \le d \le 2n+1$

{

$\rho(v_d, v_{n+1+d}) = max\{\mu(v_d), \mu(v_{n+1+d})\} - min\{\mu(v_d), \mu(v_{n+1+d})\} + (d-1)z$

}

Step 6: Output the vertices and edges labels

Print ("vertex Labels:")

Print ("Edge Labels:")

Theorem 4.1

Let $T(n)$:(μ, ρ) be the fuzzy labelled twing graph then $T(n)$ admits FEAM labeling.

Proof:

Let $z \to (0, 1]$ such that:

$$z = \begin{cases} \frac{1}{10^2}; 0 < n \le 16 \\ \frac{1}{10^{k+2}}; 16 < n \le 16 + \sum_{\substack{t=1 \\ 1 \le i \le k}}^{i} 15 \times 10^t \; where\; k = 1 \\ \frac{1}{10^{k+3}}; 16 + \sum_{\substack{t=1 \\ 1 \le i \le k}}^{i} 15 \times 10^t < n \le 16 \\ + \sum_{\substack{t=1 \\ 1 \le i \le k+1}}^{i} 15 \times 10^t \; where\; k = 1,2,3 \cdots \end{cases}$$

The membership values of the vertices and edges are defined as follows using algorithm 4.1.

$$\mu: V \to [0,1] \ni \mu(v_d) = (3n + d + 1)z \; \forall\, v_d \in V, \quad 1 \le d \le 3n + 2$$

ρ:$V \times V \to [0, 1]$ such that:

$$\rho(v_d, v_{n+2d}) = max\{\mu(v_d), \mu(v_{n+2d})\} - min\{\mu(v_d), \mu(v_{n+2d})\} - nz, d = 1$$

$$\rho(v_d, v_{n+d}) = max\{\mu(v_d), \mu(v_{n+d})\} - min\{\mu(v_d), \mu(v_{n+d})\} + dz, \quad 2 \le d \le n + 1$$

$$\rho(v_d, v_{d+1}) = max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\} + (d - n - 1)z, \quad n + 2 \le d \le 2n + 1$$

$$\rho(v_d, v_{n+1+d}) = max\{\mu(v_d), \mu(v_{n+1+d})\} - min\{\mu(v_d), \mu(v_{n+1+d})\} + (d - 1)z, \quad n + 2 \le d \le 2n + 1$$

To prove the FEAM labeling of the twing graph We show that all the anti-magic constants of the twing graph are different in the following four cases:

Case 1: When *d* =1

Now, $Am_0[(T(n)] = \mu(v_d) + \rho(v_d, v_{n+2d}) + \mu(v_{n+2d}) = (3n + d + 1)z + max\{\mu(v_d), \mu(v_{n+2d})\} - min\{\mu(v_d), \mu(v_{n+2d})\} - nz + (3n + n + 2d + 1)z = (7n + 3d + 3)z$

Case 2: When $2 \le d \le n+1$

Now, $Am_0[(T(n)] = \mu(v_d) + \rho(v_d, v_{n+d}) + \mu(v_{n+d}) = (3n + d + 1)z + max\{\mu(v_d), \mu(v_{n+d})\} - min\{\mu(v_d), \mu(v_{n+d})\} + dz + (3n + n + d + 1)z = (8n + 3d + 2)z$

Case 3: When $n+2 \le d \le 2n+1$

Take $Am_0[(T(n)] = \mu(v_d) + \rho(v_d, v_{d+1}) + \mu(v_{d+1}) = (3n + d + 1)z + max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\} + (d - n - 1)z + (3n + d + 1 + 1)z = (5n + 3d + 3)z$

Case 4: When $n+2 \le d \le 2n+1$

Take $Am_0[(T(n)] = \mu(v_d) + \rho(v_d, v_{n+1+d}) + \mu(v_{n+1+d}) = (3n + d + 1)z + max\{\mu(v_d), \mu(v_{n+1+d})\} - min\{\mu(v_d), \mu(v_{n+1+d})\} + (d - 1)z + (3n + n + 1 + d + 1)z = (8n + 3d + 3)z$

Hence, in Case 1, Case 2, Case 3, and Case 4, we have proved that all the anti-magic constants of the twing graphs are different.

Thus, fuzzy labeled twing graph $T(n)$ admits FEAM labeling.

Theorem 4.2

Let $T(n)$:(μ, ρ) be the fuzzy labeled twing graph then $T(n)$ admits FVAM labeling.

Proof:

Given $T(n)$:(μ, ρ) be the fuzzy labeled twing graph.

To prove that fuzzy labeled twing graph $T(n)$:(μ, ρ) satisfies the condition of FVAM labeling.

That is to prove that for any two vertices u and v in $T(n)$, the total of the grade membership on the edges incident at the vertex u is distinct from the total of the grade membership on the edges incident at the vertex v.

The membership values of the vertices and edges are defined as follows using algorithm 4.1.

Let $z \to (0, 1]$ such that:

$$z = \begin{cases} \frac{1}{10^2}; 0 < n \le 16 \\ \frac{1}{10^{k+2}}; 16 < n \le 16 + \sum_{\substack{t=1 \\ 1 \le i \le k}}^{i} 15 \times 10^t \; where\; k = 1 \\ \frac{1}{10^{k+3}}; 16 + \sum_{\substack{t=1 \\ 1 \le i \le k}}^{i} 15 \times 10^t < n \le 16 \\ + \sum_{\substack{t=1 \\ 1 \le i \le k+1}}^{i} 15 \times 10^t \; where\; k = 1,2,3, \cdots \end{cases}$$

$$\mu: V \to [0,1] \ni \mu(v_d) = (3n + d + 1)z \; \forall\, v_d \in V, \quad 1 \le d \le 3n + 2 \tag{7}$$

ρ:$V \times V \to [0, 1]$ such that:

$$\rho(v_d, v_{n+2d}) = max\{\mu(v_d), \mu(v_{n+2d})\} - min\{\mu(v_d), \mu(v_{n+2d})\} - nz, d = 1 \tag{8}$$

$$\rho(v_d, v_{n+d}) = max\{\mu(v_d), \mu(v_{n+d})\} - min\{\mu(v_d), \mu(v_{n+d})\} + dz, \quad 2 \le d \le n + 1 \tag{9}$$

$$\rho(v_d, v_{d+1}) = max\{\mu(v_d), \mu(v_{d+1})\} - min\{\mu(v_d), \mu(v_{d+1})\} + (d - n - 1)z, \quad n + 2 \le d \le 2n + 1 \tag{10}$$

$$\rho(v_d, v_{n+1+d}) = max\{\mu(v_d), \mu(v_{n+1+d})\} \\ -min\{\mu(v_d), \mu(v_{n+1+d})\} + (d-1)z, \\ n+2 \leq d \leq 2n+1 \tag{11}$$

Also, sum of the edge labels incident at:

$$v_d = Wt(v_d) = \sum_{u \epsilon N(v_d)} \rho(u, v_d) \tag{12}$$

when, $N(v_d)$ be the neighbourhood vertices of v_d for all $d = 1$ to m and $Wt(v_d)$ be the weight of v_d.

From equation number Eq. (7), Eq. (8), Eq. (9), Eq. (10), Eq. (11) and Eq. (12) for any two vertices v_p and v_q with $p\neq q$, $Wt(v_p)$ and $Wt(v_q)$ have distinct values.

Hence twing graph $T(n)$ for all $n\geq1$ admits FVAM labeling.

Example 4.1 Figure 5 illustrates the FEAM and FVAM twing graph of T(17).

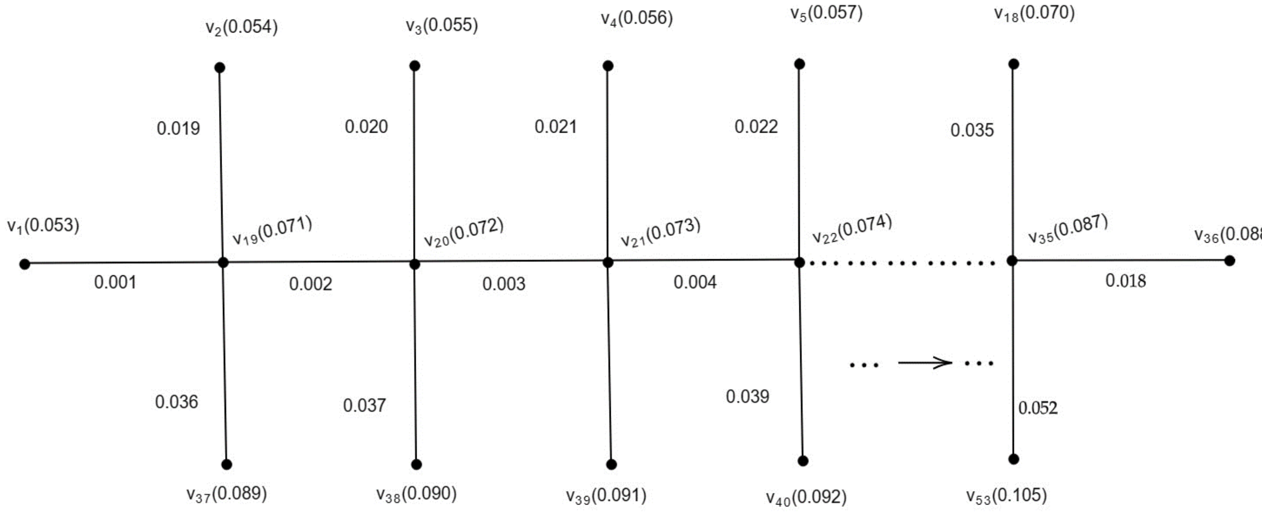

Figure 5. T(17) - FEAM and FVAM twing graph

5. PROPERTIES

·Every FEAM comb graph must have precisely one couple of vertices whose degrees and strong degrees are same.

·In FEAM and FVAM comb graph all the edges are fuzzy bridges and strong edges.

·For any FEAM and FVAM comb graph, $d_s(v)=d(v) \forall\, v \in V$.

·For any FVAM comb graph, $d(u)\neq d(v)$ & $d_s(u)\neq d_s(v)$ for any pair of vertices $u, v \in V$.

·Every FEAM and FVAM comb graph has $\frac{2n+2}{2}$ fuzzy end nodes for all n.

·In FEAM and FVAM twing graph all the edges are fuzzy bridges and strong edges.

·For any FEAM and FVAM twing graph $n\geq1$, $d_s(v)=d(v) \forall v \in V$.

·For any FEAM and FVAM twing graph $n\geq1$, $d(u)\neq d(v)$ for any pair of vertices $u, v \in V$.

·For any FEAM and FVAM twing graph $n\geq1$, $d_s(u)\neq d_s(v)$ for any pair of vertices $u, v \in V$.

·Every FEAM and FVAM twing graph has $2n$+2 fuzzy end nodes for all n.

6. CONCLUSIONS

Fuzzy graph labeling has more applications in different areas, particularly social networking, control of traffic signals, road networking, and finding the strength of relationships between the persons. In this paper, we have obtained fuzzy labeling for twing and comb graphs by providing an algorithm. We have proved the FEAM and FVAM labeling using the proposed algorithm for twing and comb graphs and discussed some properties corresponding to degree of vertex, strong degree, strong edge, fuzzy bridge, and fuzzy end nodes in the FEAM and FVAM twing and comb graphs. It may be applied to find the strength of the relationship between the persons. The FEAM and FVAM labeling of the diamond graph and the lilly graph is still open.

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