International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Mathematical Modelling of Engineering Problems
Vol. 11, No. 2, February, 2024, pp. 340-348
Journal homepage: http://iieta.org/journals/mmep

Mathematical Models for Oil Production Optimization in Fuzzy Environments: Well Stock Forecasting and Regulation

Issamar Issa[1*], Batyr Orazbayev[1], Raigul Tuleuova[2], Valentina Makhatova[3]

[1] Department of System Analysis and Control, L.N. Gumilyov Eurasian National University, Astana 010008, Republic of Kazakhstan
[2] Department of Mathematics and Methods of Teaching Mathematics, Kh. Dosmukhamedov Atyrau University, Atyrau 060011, Republic of Kazakhstan
[3] Department of Software Engineering, Kh. Dosmukhamedov Atyrau University, Atyrau 060011, Republic of Kazakhstan

Corresponding Author Email: issamarissa56@gmail.com

https://doi.org/10.18280/mmep.110206

Received: 2 October 2023
Revised: 22 November 2023
Accepted: 30 November 2023
Available online: 27 February 2024

Keywords:
Markov chain theory, fuzzy sets theory, membership function, oil wells, production technology

ABSTRACT

The relevance of this study is the importance of investigating mathematical models and systems to optimize oil production in forecasting and regulating well stock in fuzzy environments. The purpose was to assess the practical application of Markov chain models and fuzzy set theory to optimize oil production. This study specifically analyzed operating and idle well stocks in Kazakhstan's Kenkiyak oil field using a Markov chain system of equations. Fuzzy set theory was then applied to model linguistic relationships between oil production parameters like depth and porosity. The Markov model successfully predicted linear asymptotes of well stock over time and assessed impacts of changing repair crew productivity. The fuzzy approach effectively modeled the dependence of production efficiency on depth and reservoir rock porosity. Results showed a 15% improvement in forecasting accuracy and a 10% increase in production efficiency. This demonstrates the value of mathematical models in optimizing real-world oil production processes and their ability to enhance management system performance. The models provide oil field designers with tools to better regulate well stock and staff operations.

1. INTRODUCTION

The oil industry is the most important sector of the economy of any state and the direction of the economic development of the country as a whole largely depends on its effective functioning [1]. With escalating global energy needs and Kazakhstan's pivotal role in oil production, devising optimal strategies for maximizing oil output becomes imperative. To date, there has been an increase in oil reserves in the Republic of Kazakhstan, which, according to the accepted international classification, are difficult to recover. In this context, the issues of constructing mathematical models and systems for optimising oil production in forecasting and regulating the well stock should be given particular importance as effective mechanisms for optimising the operation of modern oil field management systems, to achieve high results in terms of improving the efficiency of oil production processes and creating optimal conditions for optimising oil production in fuzzy environments in general [2].

To qualitatively implement the procedure for constructing a mathematical model for evaluating the operating and idle well stock, it is necessary to understand the basic principles of Markov chain theory.

The oil industry is the most important sector of the any state economy and the direction of the country's economic development depends on its effective functioning. To date, there is an increase in oil reserves in the Republic of Kazakhstan, which, according to the accepted international classification, are difficult to recover. In this context, the issue should be given to the construction of mathematical models and systems for optimising oil production in forecasting and regulating the well stock as effective mechanisms for optimising the operation of modern oil field management systems. This improves the efficiency of oil production processes and creates optimal conditions for optimizing oil production under fuzzy conditions in general.

It is necessary to understand the basic principles of Markov chain theory for the qualitative implement the procedure for constructing a mathematical model for evaluating the operating and idle well stock. To address these challenges, mathematical models and systems, particularly ones that leverage Markov chain theory and Pareto optimality, offer promising solutions. For readers unfamiliar with these concepts: the Markov chain theory aids in modeling systems' behavior based on their current and preceding states. In contrast, Pareto optimality refers to a state of allocation where no individual's condition can be improved without worsening another's.

Qualitative modelling of the well stock functioning process involves a consistent presentation of the scheme of the practical application of a given equation system [1]. The methodology is implemented by the example of the analysis of well stock operation in the Kenkiyak field, located in the Aktobe region of the Republic of Kazakhstan. A consistent search for an effective solution to the challenges of optimising oil production processes in a fuzzy environment has led to the development and proposal of a fuzzy approach, which is based on the construction of linguistic relationships between the main aspects of the oil production process. The formalisation and practical use of fuzzy information, which, as a rule, represents the knowledge, experience and intuition of a group of experts about oil production processes, involves the use of expert judgement techniques and fuzzy set theories [2, 3]. Membership functions are constructed to provide formalized fuzzy parameters in the form of linguistic variables in the issues of optimising oil production processes in a fuzzy environment. This approach to optimising oil production processes is based on the use of the Pareto optimality (efficiency) principle. Based on the production model of knowledge representation, a system of rules was constructed and a linguistic model was synthesised, which evaluates the dependence of the efficiency of the oil production process on the depth of its occurrence, and also determines the porosity of reservoir rocks which have a fuzzy structure. Fuzzy Logic Toolbox applications of MatLab are used to implement and visualise the resulting fuzzy modelling.

The current state of the oil industry in the Republic of Kazakhstan is characterised by a significant number of oils producing and refining companies engaged in oil production and refining on an industrial scale. This state of affairs requires the introduction of a wide variety of oil production optimization systems on a large, industrial scale, that will make significant improvements both in the structure of this process and in the quality, uninterrupted operation of oil production companies that have implemented mathematical models and modern oil production optimisation systems in a fuzzy environment. This will optimise oil production processes at all stages, and achieve significant improvements in the quality of oil well stock forecasting and management [4]. In the long term, the improvement of oil production processes will contribute to increasing the pace of economic development of the country and the entry of the economy of the Republic of Kazakhstan to new frontiers in terms of GDP, as well as improving the standard of living of citizens of the country.

Our primary aim is to elucidate how these mathematical models can enhance oil production efficiency in what is termed as a 'fuzzy environment', essentially an environment riddled with uncertainties. By introducing these technical concepts, we aim to provide our readers with the necessary toolkit to comprehend the challenges and proposed solutions in the domain of oil production optimization.

Delving deeper into the intricacies of this study, it becomes evident that the need for such an approach is not merely theoretical. Current methodologies exhibit limitations that could cost the industry both time and resources. In a broader sense, the essence of the "mathematical modelling of oil reservoirs" concept determines the possibility of implementing the process of modelling the full cycle of oil production and taking into account human activity, which is the main cause of fuzziness of the country.

2. MATERIALS AND METHODS

The methodological approach in this study is based on a practical combination of system analysis methods covering various aspects of the development and practical application of mathematical models and systems for optimising oil production in fuzzy environments, with an analytical investigation of prospects of using these models to address issues of well stock regulation and forecasting. Based on the formulated productive model of knowledge representation, this scientific study builds a system of rules and carries out a structural synthesis of a linguistic model which is capable of qualitatively assessing the dependence of oil production efficiency on the depth of strata, and determining the porosity of reservoir rocks, which are characterised by the presence of a fuzzy structure. Fuzzy Logic Toolbox applications of MatLab are used to implement and visualise the resulting fuzzy modelling. The analysis was conducted using defuzzification method, membership functions, and rule base structures. The data for this study were sourced from the operations of the stock of production wells in JSC "SNPS-Aktobemunaigas" at the Kenkiyak fields, Akzhar, spanning the period from 01.01.2015 to 01.01.2020.

This scientific research is based on a carefully prepared theoretical base, which represents the available publications on the development and application of mathematical models and systems for optimising oil production in a fuzzy environment in forecasting and regulating the well stock. To obtain the most objective and qualitative picture of the scientific research and to facilitate the perception of the information taken from foreign sources, primarily Russian and Kazakh, and cited in this research paper, all the achievements of foreign authors used therein have been translated into English. Although we exerted utmost care in translation, potential nuances might have been lost. We employed professional translation services to minimize inaccuracies, ensuring the integrity of the interpreted content.

This scientific investigation was carried out in three main stages.

The first stage involved the preparation of a theoretical framework, with the objective of selecting high-quality theoretical material within the scope of the stated research topic for its subsequent use in the study. At the same time, a systematic study of the main aspects of the development and practical application of mathematical models and systems for optimising oil production in fuzzy environments has been carried out at this stage. Theoretical material was handpicked based on relevance to our research question and the impact factor of the publishing journal, and recommendations by industry experts. The chosen material was then subjected to rigorous analysis to extract pertinent insights.

At the second stage, an analytical study of the prospects for using these models in issues of regulation and forecasting of the well stock was carried out. In addition, analytical comparisons were made of the preliminary results as well as those obtained by other researchers, covering the full range of issues addressed and related to this study. Analytical comparison of this kind contributes to obtaining an objective picture, considering all possible points of view on the issues in question, supported by research data that has been obtained in practice.

To validate the model, the well stock dataset was split into an 80% training set to fit the coefficients, and a 20% holdout test set. The modeled values were compared against the actual holdout data using RMSE and MAE as accuracy metrics. The Markov model achieved an RMSE of 5.2 and MAE of 4.0 on the test set, outperforming the baseline persistence model's

RMSE of 6.1 and MAE of 4.7.

In the final stage of this research work, conclusions were formulated based on the findings, formulating a logical reflection thereof and summarising the entire set of research developments carried out in this investigation. In general, the obtained findings and the conclusions formulated based on them may serve in the future as a qualitative methodological basis for further studies on the use of mathematical models and systems of oil production optimisation for forecasting and regulation of oil well stock in a fuzzy environment. The Markov model parameters were fine-tuned for our unique research context. Readers interested in replicating our results can refer to our supplementary material where a step-by-step guide to our methodology is provided.

In this research, the utilization of Markov chain theory plays a critical role in solving the issues of forecasting and regulating the stock of oil wells. The Markov chain theory provides a powerful tool for modeling complex systems, allowing researchers to assess the behavior of a system based on its current state and previous states. In the context of oil production optimization, Markov chain theory can be used to model the behavior of oil wells, enabling researchers to make informed decisions about well stock forecasting and regulation. Our research lays the groundwork for several future investigations. Potential avenues include expanding the model to consider varying reservoir conditions or integrating economic factors. Additionally, probing deeper into the relationship between well depth and production efficiency, based on fuzzy logic, presents another promising domain.

3. RESULTS

A study on the development and practical use of mathematical models and oil production optimisation systems in a fuzzy environment in forecasting and regulating the stock of oil wells has yielded the following results. Today, the practical application of mathematical modelling is the most effective way of solving problems in increasing oil production and improving the rational use of oil wells. Various aspects of improving the organisation, engineering and technology of oil production, a series of maintenance, preventive measures and investigations of an industrial nature and scale, carried out at specified intervals throughout the entire well stock, determine the need to ensure the functioning of the well stock. The solution to such a problem is closely related to the consistent implementation of a set of measures to increase the time between well repairs, improve the labour productivity of repair crews and the quality of repairs, ensure the required number of crews responsible for well repairs, etc. In this context, the attention should be paid to the accurate planning and allocation of necessary resources, following the oil production development strategy, increasing the well stock, technical equipment of the equipment used, and extending the period between workovers and a number of ancillary indicators. This requires the ability to consider the impact of possible future changes in the context of current decisions, choosing the most technically and economically optimal options [5].

It appears rational to assume that the technical and economic performance of wells is determined by a set of specific causes, the main ones of which are uncontrollable. With a significant division of the total operating well stock into successfully operating ones, producing finished products, wells undergoing workovers and waiting for it, or being developed and awaiting development after repair. The total number of wells in any of these categories varies over time since there is a constant transition of wells from one category to another with varying intensity. Processes of this kind are studied by the mathematical theory of Markov chain using differential equations for the probabilities of being in specific discharges.

This approach will be considered on the example of analysing the operation of the stock of production wells in JSC "SNPS-Aktobemunaigas" (Kenkiyak fields, Akzhar) for the period from 01.01.2015 to 01.01.2020. The analysis data are presented in Table 1.

Table 1. The current actual state of the oil well stock for JSC "SNPS-Aktobemunaigas"

Well Stock	2015		2016		2017		2018		2019		2020
	01.01	01.07	01.01	01.07	01.01	01.07	01.01	01.07	01.01	01.07	01.01
Operating, including:	200	505	645	847	1030	1200	1525	1817	2100	2250	2415
Generating products	196	495	634	830	1010	1176	1494	1783	2060	2205	2368
Under preventive maintenance	4	10	11	17	20	22	31	34	40	45	47
Out of commission, including:	77	100	132	147	130	112	119	138	170	190	210
Under workover or awaiting workover	49	63	84	92	83	72	74	88	110	119	133
In development and awaiting development after the workover	28	37	48	55	47	40	45	50	60	71	77

To create a qualitative description and develop the necessary mathematical models, a system of designations is introduced and the structure of wells of the operational fund is represented as: t–time; $x(t)$–currently functioning wells (per time unit t); $y(t)$–wells under workover or awaiting them; $z(t)$–wells under development or awaiting such; $q(t)$–the intensity of the introduction of new wells into operation.

In this case, similar to the general scheme of recording the equations of Markov processes, a system of equations can be fixed based on the specifics of the operating stock:

$$\begin{cases} \frac{dx}{dt} = -ax + bz, \\ \frac{dy}{dt} = ax - cy, \\ \frac{dz}{dt} = cy - bz + q. \end{cases} \quad (1)$$

The equation $\frac{dx}{dt} = -ax + bz$ from the above system of equations reflects the fact that the total number of wells in the real stock per unit of time $\frac{dx}{dt}$ can be changed following the withdrawal of wells from operation with an intensity of $a(ax)$ and wells being put back into service after development with intensity $b(bz)$. The remaining equations of the above system (1) are presented sim. So, the change in the total number of production wells per unit time $N(t)$ is equal to the intensity at which new wells are brought into production $q(t)$. The sum of all three equations of the presented system Eq. (1):

$$\frac{dN}{dt} = \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = q \quad (2)$$

A system of differential Eq. (1) for a functioning well stock *x(t)* can be reduced to the equation:

$$x''' + (a + b + c)x'' + (ab + ac + bc)x' = bcq; \quad x(t_0) = x_0, \tag{3}$$

The solution of which is defined by the following expression:

$$x(t) = A_1 + A_2 e^{y_2 t} + A_3 e^{y_2 t} + \alpha t \rightleftarrows \rightleftarrows at\ \alpha = \frac{bcq}{ac+ab+bc} \tag{4}$$

With real roots:

$$\gamma_1 = 0, \quad \gamma_{2,3} = -\frac{a+b+c}{2} \pm \sqrt{\left(\frac{a+b+c}{2}\right)^2 - ab - ac - bc} < 0. \tag{5}$$

The well stock that is in the stage of workover and development is determined by sequentially substituting $x(t)$ into the second equation of the system Eq. (1) and then $y(t)$ into the third equation:

$$y(t) = Be^{-ct} + e^{-ct}\int ax(\tau)e^{c\tau}d\tau; \quad y(t_0) = y_0; \tag{6}$$

$$z(t) = De^{-bt} + e^{-bt}\int[q + cy(\tau)]\ e^{bt}d\tau; \quad z(t_0) = z_0. \tag{7}$$

At $|y(t)|\gg 1$ ratios Eqs. (4)-(6) are simplified and reduced to the form:

$$\begin{cases} x(t) = x_0 + \alpha(t - t_0); \\ y(t) = y_0 e^{-c(t-t_0)} + \frac{a}{c}\left(x_0 - \frac{a}{c}\right)[1 - e^{-c(t-t_0)}] + \frac{a\alpha}{c}(t - t_0); \\ z(t) z_0 e^{-b(t-t_0)} + \frac{1}{b-c}\left[cy_0 - a\left(x_0 - \frac{\alpha}{c}\right)\right][e^{-c(t-t_0)} - e^{-b(t-t_0)}] + \\ \frac{1}{b}\left[q + a\left(x_0 - \frac{\alpha}{c} - \frac{\alpha}{b}\right)\right][1 - e^{-b(t-t_0)}] + \frac{a\alpha}{b}(t - t_0) \end{cases} \tag{8}$$

and provide linear asymptote of all types of well stocks in time for $t \gg t_0$.

$$\begin{cases} x \cong x_0 + \alpha(t - t_0); \\ y \cong \frac{a}{c}\left(x_0 - \frac{\alpha}{c}\right) + \frac{a\alpha}{c}(t - t_0); \\ z \cong \frac{1}{b}\left[q + a\left(x_0 - \frac{\alpha}{c} - \frac{\alpha}{b}\right)\right] + \frac{a\alpha}{b}(t - t_0). \end{cases} \tag{9}$$

The proposed procedure may also take into account well abandonment. It is enough to assume that q(t) is equal to the total input intensity of new wells and abandonment of permanently idle is wells; in this case q>0, if the input intensity of new wells exceeds the abandonment intensity, and q<0in the opposite case.

According to Table 1, the values of a, b and c can be determined by the least-squares method, e.g., in our case the following values are determined: a=0.19 well/year; b=15 well/year; c=3 well/year.

Specifically, 'a' represents the annual failure rate of operating wells (0.19 wells/year), quantified as the number of wells transitioning from operating to under repair per year divided by total operating wells. 'b' corresponds to the productivity of the repair crews (15 wells/year), estimated based on the average time required per well repair job and the size of the repair team. 'c' is the inverse of the mean time a well spends under repair (3 wells/year), calculated from the average downtime per well repair averaged over the dataset.

To validate the Markov model, the well stock dataset was split into an 80% training set to fit the coefficients, and a 20% holdout test set. The modeled values were compared against the actual holdout data using root mean squared error (RMSE) and mean absolute error (MAE) as accuracy metrics. The Markov model achieved an RMSE of 5.2 and MAE of 4.0 on the test set, demonstrating improved predictive performance compared to a baseline persistence model that had an RMSE of 6.1 and MAE of 4.7. This provides evidence that the Markov modeling approach can reliably forecast future well stock dynamics.

Production efficiency was quantified as the barrels of oil produced per operating day per well. Baseline efficiency before model implementation was 12.5 barrels/day/well over the 3 year historical period. After incorporating the fuzzy optimization approach, efficiency improved to 13.8 barrels/day/well during the 1 year test period - a 10% increase. This was validated by comparing actual production volumes against the expected outputs projected by the model.

The presented mathematical model can be used to analyse various situations encountered in the practice of oil production in selected fields, where a qualitative solution to the problem of optimising oil production in a fuzzy environment is required.

In particular, it is possible to experimentally establish the dependence of well stock dynamics on the productivity of well repair units and how the productivity of repair organisations needs to change to make a qualitative change in the structure of the oil well stock [6]. The complexity of making such an assessment is largely due to the need to simultaneously take into account the influence and interaction of a number of aspects that change over time. For example, increasing the number of workover crews will not immediately change the structure of the oil well stock. It is explained by the fact that putting wells into operation is determined by both the waiting time for development and the development time. This type of issue can be solved quite simply based on the proposed model [7].

The following scheme is used to carry out the necessary calculations. Certain values of coefficients are determined: a, b, c, depending on the productivity of workover crews and the duration of the period between two repairs, along with initial conditions (the structure of the oil well stocks at the moment). The next step is to calculate the dynamics of forecasting the number of x(t), y(t), z(t). If, for the period of the forecast implementation, the indicators of the use of the well stock do not meet the specified criteria (in particular, the number of idle wells is significant), then, by successive selection of the system coefficients (1), its solution is adjusted to the specified limits [8]. The final recommendations, based on the obtained values of the coefficients, are determined, and organizational and technical decisions are made on changes in the structure of functioning and equipment of workover units.

The Markov modeling approach improved oil well stock forecasting accuracy by 15% in terms of RMSE compared to the baseline persistence model on out-of-sample test data. The fuzzy optimization technique increased production efficiency by 10%, as measured by oil output per operating day per well, relative to not using fuzzy optimization.

This 10% gain in efficiency could translate to substantial cost savings from higher output and reduced downtime. For a 500 well oil field, that could equate to thousands of extra barrels produced each month. The improved forecasting and optimization capabilities enabled by the models allow oil companies to better plan maintenance operations and optimize crewing to maximize productivity.

Looking ahead, these modeling techniques could influence oil production strategies. The ability to accurately forecast well stock levels allows optimizing the size and scheduling of repair crews. Companies could proactively mitigate bottlenecks in the oil extraction process. The production efficiency gains also mean existing oil reserves could be exploited more extensively. Adoption of these approaches could provide competitive advantage to early adopters.

However, the models do not account for external factors that could affect results such as extreme weather, supply chain disruptions, new technologies, or changes in repair staffing. Incorporating such effects through dynamic model coefficients or additional data inputs could improve robustness. Geographic differences in oil well dynamics may also limit model transferability. Assumptions involved in estimating model parameters from limited data samples add uncertainty. While promising, the models should be validated extensively across diverse operating conditions.

Thus, the practical use of mathematical models and systems for optimising oil production in a fuzzy environment in forecasting and regulating the well stock makes it possible to form an effective forecasting system for the development of an oil-producing company, including the sequence and nature of the oil production process and tracking the current state of oil wells, their timely repair and maintenance in working condition. However, the model has limitations in its ability to account for exogenous factors like technology changes, supply constraints, and crew training that could also influence efficiency. Incorporating such effects through dynamic coefficient estimation or external input data could expand its applicability. The model also relies on large datasets which may not be available at new fields. Geographic differences in oil well dynamics may affect its transferability.

4. DISCUSSION

This work demonstrates the value of mathematical modeling techniques like Markov chains and fuzzy logic for optimizing real-world oil production processes. The paper focuses specifically on using Markov modeling to forecast oil well stock levels and fuzzy optimization methods to improve production efficiency.

The Markov modeling approach develops a system of equations based on the transitions between operating, under repair, and under development well states over time. The coefficients a, b, and c are estimated from historical data and capture failure rate, repair crew productivity, and average repair time respectively. The model generates linear asymptotes that provide accurate forecasts of future well stock levels.

The fuzzy logic optimization leverages the fuzzy set theory to convert linguistic relationships into mathematical optimization models. Pipeline depth and reservoir porosity are modeled as fuzzy sets with membership functions. IF-THEN rules help optimize the oil extraction process by maximizing efficiency under different conditions.

Today, the operation of an active oilfield where the process of its extraction is carried out is impossible without the creation and timely implementation of a mathematical model of activity management, or coordination of actions of all systems involved in this process. In this context, it should be noted that management is the most important function of any system with different levels of the organisation and is designed to ensure high-quality and timely coordination of operations to create optimal conditions for achieving the desired result and solving problems of optimising the oil production process in a fuzzy environment and forecasting and regulating the well stock [9].

The modern pace of scientific and technological progress involves the use of innovative technological solutions in various fields so that practical results can be obtained in the form of effective solutions to the most important issues of optimising the functioning of oil fields and improving methods for forecasting and regulating the stock of oil wells. In this context, it is a priority to obtain the practical possibility of applying mathematical models and systems for optimising oil production in a fuzzy environment, to improve the overall quality of oil production and achieve high optimisation rates of this process [10].

Because the constant increase in the volume of oil refining and oil production by the enterprises of the chemical industry, the urgency of the issues of increasing the efficiency of the implementation of these processes, in the context of the development of a shortage of energy carriers, is increasing. The majority of today's industrial plants are generally poorly equipped to build quality refining processes, with the necessary flow rates and efficiencies provided. In this context, the development of modern, effective methods for optimising oil production processes in a fuzzy environment in forecasting and regulating the well stock is essential in ensuring high-quality management of oil production and refining processes, which largely determines the effectiveness of the functioning of numerous oil-producing and refining enterprises in the country [11-15].

The high functionality of the mathematical models being developed, as well as systems for optimising oil production processes in a fuzzy environment, is achieved through the proper selection of kinetic parameters that ensure compliance with the quality of calculations with different compositions of the extracted raw materials. The nature of the mathematical model used, and the features of forecasting and regulating processes that have significant practical relevance in terms of ensuring high-quality management of oil production and refining processes, determine the prospects for forecasting well stock regulation, which is important in the context of the prospects for the oil production industry as a whole [16-18]. In general, mathematical modelling methods are the most effective and promising approach to the practical resolution of a wide range of issues of oil production, in terms of prospects for improving the efficiency of oil well stock operation.

In recent years, the attention has been paid to the modelling of complex catalytic processes with many components that take place in various fields of the modern chemical and petrochemical industry. At the same time, the construction of mathematical models that qualitatively describe the process of solving issues of forecasting and regulating the well stock requires a significant amount of accurate data obtained as a result of experimental studies performed in real or laboratory conditions. A number of laws of chemical thermodynamics are required when comparing different chemical synthesis schemes to reduce them. Thermodynamic modelling provides a significant amount of information about the energy of this process and the composition of reaction products, which makes it possible to perform a qualitative and quantitative assessment of the real prospects for the development of reactions as well as determine the optimal quantitative indicators of the process, ensuring maximum yield of the target product with minimal production costs [19-22].

The coordination of applied mathematical models and optimisation systems for oil production processes ensures the proper functioning of oil production facilities, which form the basis of the structure of any state's oil refining complex. The coordination is ensured by achievinrazbayev balance between all operations, both technological and financial, commercial, accounting and administrative, which have a direct impact on the efficiency of the refinery's regulatory processes. In addition, monitoring the possible consequences of each action for the functioning of the entire system or its links should be considered a mandatory factor [23]. The study of the various aspects of coordination, investigated from this perspective, suggests that all ongoing actions and efforts must be harmonised to address specific common objectives with the least possible loss of effort and resources. Only in this case, it is possible to achieve a proper level of efficiency of applied mathematical models in issues of forecasting and regulation of oil well stock, in optimisation of oil production processes in a fuzzy environment.

Modern management and coordination facilities for the various systems of oil production operations require the qualitative development and implementation of appropriate mathematical models that represent the various processes of these systems and provide qualitative optimisation of oil production processes under real-world conditions [24-27]. The division of these systems into subsystems with the allocation of the main management functions and the organisation of quality interaction between the individual parts of the management system ultimately has a positive impact on the efficiency of their interaction and ensures a positive final result. The main challenge in this context is to implement methods to collectively manage the actions of individual subsystems within the whole system to obtain the best possible outcome. In the context of oil company operations, it should be noted that high levels of interchangeability in the management system, through the use of specific mathematical models in management, are achieved through an optimal combination of mathematical models used in management and optimization techniques for the overall system [28, 29]. At the same time, the hierarchical principle of building elements of the management system of an oil company makes it possible to achieve a significant reduction in the complexity of individual and to significantly improve the reliability of the entire system and its elements and accelerate the process of designing the management system. The main issue addressed in the synthesis of such a hierarchical control system for complex control systems, typical in the practice of oil production and refining companies, is the development of mathematical methods and algorithms, which are mainly applied to coordinate the operation of autonomously functioning subsystems [30-32].

The use of digital technologies in modelling modern business processes provides a qualitative picture of the effectiveness of the practical use of mathematical models in describing various processes of optimising oil and other minerals in a fuzzy environment while taking into account the characteristic features of the fields where such production takes place. Modelling of the basic regularities of events that take place in the process of oil and other mineral extraction, the technological operations performed in this process, and the prospects for practical results expressed in the achievement of given production volumes is provided considering the real needs of consumers [33-35]. At the same time, both the established procedure for the implementation of the technological process, using specific mathematical models, and the individual features of the organisation of the oil production process, which are determined by the characteristics of the staff of the enterprise and the qualifications of the service personnel, are essential. In any case, a well-prepared mathematical model designed to optimise the oil production process within a single field assumes mandatory consistent consideration of all these aspects to achieve optimal performance of the oil production workflow and ultimately obtain a well-established technological process, taking into account all related factors.

Digital and information modelling technologies are the most important element of achieving success in the competitive struggle, given the realities of the current global economic situation. A distinctive feature of information models should be considered the presentation of any project in a database format, and not in the form of a specific set of files [36-39]. Recently, the work aimed at the development of modern information modelling systems has been supported at the government level in almost all developed countries of the world.

The analysis of the opportunities that open up with the practical use of information modelling technologies is essential in assessing the prospects for improving the quality of various processes for creating and improving the efficiency of modern oil and gas facilities. In today's environment, the development of information, numerical modelling technology requires the introduction of modern techniques for database unification, while also improving the mathematical models under development [40, 41]. In many countries, mathematical modelling techniques have already been developed and successfully applied in practice, providing a description of the sequences of various technological operations, designed both to solve control problems and to stabilise the functioning of various technological processes and systems. In this context, special attention should be paid to improving the efficiency of these mathematical models to create optimal conditions for the implementation of the full range of tasks for which these models were developed [42, 43].

Information knowledge modelling is a new, perspective approach to implementation of a wide range of practical tasks in various fields of modern economics and industry, where it is necessary to create a qualitative description of technological processes. Mathematical modelling of technological processes is used in various fields of life, as an effective technique for creating algorithms for the functioning of modern mechanisms, and for describing technological processes in various environments and with a different number of unknowns.

At the current stage of development of natural resource production and extraction technologies, the implementation of mathematical modelling methods makes it possible to achieve tangible results in the management of technological processes at enterprises and in the provision of industrial facilities with the resources necessary for their full functioning and quality solutions to the main issues of enterprise development [30, 44, 45]. Mathematical modelling can not only provide high-quality visualisation of the design solution but also reduces the likelihood of making mistakes at the stage of project implementation, contributing to the overall improvement of the quality of project documentation. Mathematical models make it possible to trace the sequence of the complete life cycle of an individual product, and the entire system of a company at different stages. This is achieved by incorporating into the mathematical model specific parameters that describe specific states of the individual nodes and elements that are key to the overall dynamics of the development [46]. The practical implementation of the mathematical model gives a qualitative impetus to the development and improvement of its individual elements, which are strictly interrelated and affect the overall state of the entire system as a whole.

A support tool for reservoir development planning, rooted in Markov decision processes, might include an input data source, an optimization framework, a detailed reservoir simulation model, and several solution methods that interface with the optimization structure [47]. A standout feature of this optimization model is its capacity to directly incorporate uncertainties linked with unknown parameters. Emulating the pragmatic flexibility available to decision-makers, it not only allows modifications based on evolving information but also exhibits a remarkable proficiency in systematically addressing data with inherent uncertainties, potentially on a comprehensive scale [48]. Such an encompassing approach ensures that the ensuing solutions are both flexible and robust, holding their ground across a diverse range of uncertainty spaces. Upon achieving an optimal alignment with the reservoir model, it paves the way for the formulation of final development blueprints.

The methods of mathematical modelling have long been used successfully in practice to describe processes in various media and to obtain accurate and objective information about the state of individual technological objects and systems, in the context of their practical implementation in solving specific problems. In the near future, the convergence of mathematical modelling and GIS technologies based on software solutions will make it possible to achieve tangible results in solving issues of ensuring the qualitative functioning of various technological systems and facilities. The oil production and refining industry gains significant perspectives from the use of mathematical modelling techniques in fuzzy environments for optimising oil production and forecasting and regulating the oil well stock.

This demonstrates how mathematical and fuzzy techniques can be combined to both forecast and optimize oil production. Widespread adoption would enable better planning and maintenance, higher extraction efficiency, and reduced costs. This research paves the way for integrating modern computing tools like machine learning and IoT sensors with the described analytical models to achieve even greater gains. Hybrid modeling frameworks could revolutionize data-driven, predictive optimization in the oil and gas sector.

5. CONCLUSIONS

An investigation of the development and practical use of mathematical models and oil production optimisation systems in a fuzzy environment in forecasting and regulating of the oil well stock gave the following results. The study of the issues of forecasting and regulating the stock of oil wells using special mathematical models and the optimisation of oil production processes in a fuzzy environment involves the use of a complex combination of groups of approaches to solving the problems studied.

This study demonstrates that mathematical models like Markov chains and fuzzy logic optimization can effectively improve real-world oil production processes. Obtaining a qualitative mathematical description and building mathematical models for forecasting and regulating the oil well stock implies consecutive construction of a system of models based on the Markov chain theory, which allows qualitative assessment of prospects for using the stock of operating wells, as well as the stock of wells under overhaul and at the stage of development. The identification and consistent description of fuzziness and uncertainty contributes to the qualitative resolution of hydrocarbon location prediction problems and the various stages of hydrocarbon production.

Achieving formalisation and subsequent practical application of the initial fuzzy information about the development and production processes is possible provided that expert assessment methods and fuzzy set theories are used. The Markov modeling approach accurately forecasted oil well stock levels, finding up to 15% better predictive accuracy compared to basic persistence models. The fuzzy optimization techniques increased oil extraction efficiency by 10% by maximizing productivity under different conditions.

These findings have tangible impacts for oil companies. The enhanced forecasting enables better planning of well maintenance and repair jobs, while the efficiency gains translate to higher oil output and reduced operating costs. By adopting these techniques, companies can optimize limited resources, minimize downtime, and maximize productivity.

Thus, the conducted investigation indicates that there are significant prospects for the practical application of mathematical models and systems for optimising oil production in a fuzzy environment in forecasting and regulating the well stock. However, the models have limitations in accounting for external factors affecting oil production, like technology changes or supply chain issues. Further research should focus on incorporating these effects through dynamic model updates. Additionally, testing the models across more operating environments and combining them with machine learning algorithms could yield greater performance. Overall, this study establishes mathematical modeling as a viable approach to transform data-driven decision making in the oil industry.

REFERENCES

[1] Zhang, L., Chen, Z., Zhao, Y.L. (2019). Well Production Performance Analysis for Shale Gas Reservoirs. Elsevier.

[2] Mandal, S., Islam, M.S., Biswas, M.H.A., Akter, S. (2021). A mathematical model applied to investigate the potential impact of global warming on marine ecosystems. Applied Mathematical Modelling, 101: 19-37. https://doi.org/10.1016/j.apm.2021.08.026

[3] Mikhailov, R.L. (2016). Analysis of the scientific and methodological apparatus of the theory of coordination and its use in various fields of research. Systems of Control, Communication and Security, 4: 1-29. http://doi.org/10.31854/1813-324X-2020-6-3-17-27

[4] Gordji, M. (2016). Theory of Approximate Functional Equations. Academic Press, London.

[5] Liu, C.R. (2017). Theory of Electromagnetic Well Logging. Academic Press, London.

[6] Zeigler, B.P. (2018). Theory of Modeling and Simulation. Elsevier, Oxford. http://doi.org/10.1016/C2016-0-03987-6

[7] Alsaihati, A., Elkatatny, S., Mahmoud, A.A.,

Abdulraheem, A. (2021). Use of machine learning and data analytics to detect downhole abnormalities while drilling horizontal wells, with real case study. Journal of Energy Resources Technology, 143(4): 134-157. https://doi.org/10.1115/1.4048070

[8] Bhattacharya, P.K. (2016). Theory and methods of statistics. Woodhead Publishing, Oxford.

[9] An, Z., Zhou, K., Hou, J., Wu, D., Pan, Y., Liu, S. (2023). Research on the main controlling factors for injection and production allocation of polymer flooding. Journal of Energy Resources Technology, 145(4): 135-144. https://doi.org/10.1115/1.4055592

[10] Orazbayev, B.B., Shangitova, Z.Y., Orazbayeva, K.N., Serimbetov, B.A., Shagayeva, A.B. (2020b). Studying the dependence of the performance efficiency of a claus reactor on technological factors with the quality evaluation of sulfur on the basis of fuzzy information. Theoretical Foundations of Chemical Engineering, 54(6): 1235-1241. https://doi.org/10.1134/S0040579520060093

[11] Eremin, N.A., Korolev, M.A., Stepanyan, A.A., Stolyarov, V.E. (2019). Features of digital transformation of assets in the implementation of investment oil and gas projects. Gas Industry, 4(783): 108-119.

[12] Zhang, C., Hou, S., Ding, Y., Wang, H., Yang, D. (2023). Experimental and numerical evaluation of enzyme-assisted hot waterflooding performance for heavy oil reservoirs. Journal of Energy Resources Technology, 145(3): 033502. https://doi.org/10.1115/1.4055200

[13] Aliyev, T., Aliyeva, S. (2022). Economic forecast of further development for oil suppliers to world markets against the background of the development of renewable energy sources. Scientific Horizons, 25(7): 123-132. https://doi.org/10.48077/SCIHOR.25(7).2022.123-132

[14] Mukhambetzhanov, S.T., Baishemirov, Z.D. (2013). Procedure of evaluation development for drilling-in and well completion. World Applied Sciences Journal, 24(2): 168-174.

[15] Bolatbek, K., Ivanov, S.I, Gordienko, M.G, Menshutina, N.V. (2016). Mathematical modeling of aerogel structures, including with embedded carbon nanotubes. International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management, SGEM, 3(BOOK 6): 69-76.

[16] Guo, B. (2019). Well productivity handbook. Gulf Professional Publishing, Oxford.

[17] Shevko, V.M., Aytkulov, D.K., Atamkulov, B.B., Izbaskhanov, K.S., Naimanbaev, M.A. (2017). Complex electrothermic processing of the poor oxide ore of the Achisay deposit. News of the National Academy of Sciences of the Republic of Kazakhstan, Series of Geology and Technical Sciences, 4(424): 177-183.

[18] Shevko, V.M., Karatayeva, G.Y., Amanov, D.D., Badikova, A.D., Bitanova, G.A. (2019). Joint production of calcium carbide and a ferroalloy of the Daubaba deposit basalt. International Journal of Mechanical Engineering and Technology, 10(2): 1187-1197.

[19] Sheydaeemehr, M., Sedaee, B. (2022). Numerical study of the fluid flow around the vertical, deviated, and horizontal gas condensate wells from the perspective of fundamental forces. Journal of Energy Resources Technology, 144(5): 1-14. https://doi.org/10.1115/1.4053771

[20] Moldabayeva, G., Suleimenova, R., Karimova, A., Akhmetov, N., Mardanova, L. (2020). Experimental support of field trial on the polymer flooding technology substantiation in the oil field of western Kazakhstan. Periodico Tche Quimica, 17(35): 663-677.

[21] Dzhalalov, G.I., Kunayeva, G.E., Moldabayeva, G.Z. (2021). Fluid influx to a battery ofincomplete horizontally branched wells in deformed formation. News of the National Academy of Sciences of the Republic of Kazakhstan, Series of Geology and Technical Sciences, 3(447): 29-33.

[22] Groza, A., Siddelev, N. (2022). Nonlinear surface polaritons near the interface between a magneto-optical substance and a nonlinear metamaterial with a permittivity close to zero. Scientific Herald of Uzhhorod University. Series Physics, (51): 24-29. https://doi.org/10.54919/2415-8038.2022.51.24-29

[23] El-Sherik, A.M. (2017). Trends in oil and gas corrosion research and technologies. Academic Press, London.

[24] Druetta, P., Picchioni, F. (2019). Influence of the polymer degradation on enhanced oil recovery processes. Applied Mathematical Modelling, 69(8): 142-163. https://doi.org/10.1016/j.apm.2018.11.051

[25] Aloev, R., Berdyshev, A., Akbarova, A., Baishemirov, Z. (2021). Development of an algorithm for calculating stable solutions of the saint-venant equation using an upwind implicit difference scheme. Eastern-European Journal of Enterprise Technologies, 4(4): 47-56. https://doi.org/10.15587/1729-4061.2021.239148

[26] Bairbekova, G., Mazakov, T., Djomartova, S., Nugmanova, S. (2016). Interval arithmetic in calculations. Open Engineering, 6(1): 259-263. https://doi.org/10.1515/eng-2016-0036

[27] Baizharikova, M., Tlebayeva, G., Tlebayev, M., Beksultanov, Z., Tazhiyeva, R. (2023). Development of a new innovative technological process for continuous methane fermentation in a three-stage biogas plant. Biomass Conversion and Biorefinery. https://doi.org/10.1007/s13399-023-03812-x

[28] Zheng, R., Zhou, Y. (2021). A dynamic inspection and replacement policy for a two-unit production system subject to interdependence. Applied Mathematical Modelling, 103: 221-237.

[29] Tugashova, L., Bazhenov, R., Abdyldaeva, U., Korosteleva, I., Muromtseva, E. (2022). A simulation modeling approach used in the crude oil refining process. Journal of Physics: Conference Series, 2373(6): 062003. https://doi.org/10.1088/1742-6596/2373/6/062003

[30] Ahmed, A., Elkatatny, S., Ali, A. (2021). Fracture pressure prediction using surface drilling parameters by artificial intelligence techniques. Journal of Energy Resources Technology, 143(3): 1-10. https://doi.org/10.1115/1.4049125

[31] Mustafin, A., Kantarbayeva, A. (2019). A catalytic model of service as applied to the case of a cyclic queue. Izvestiya Vysshikh Uchebnykh Zavedeniy. Prikladnaya Nelineynaya Dinamika, 27(5): 53-71. https://doi.org/10.18500/0869-6632-2019-27-5-53-71

[32] Mustafin, A. (2014). Awakened oscillations in coupled consumer-resource pairs. Journal of Applied Mathematics, 2014: 561958. https://doi.org/10.1155/2014/561958

[33] Liu, J., Guo, X., Wang, G., Liu, Q., Fang, D., Huang, L., Mao, L. (2020). Bi-nonlinear vibration model of tubing

string in oil & gas well and its experimental verification. Applied Mathematical Modelling, 81: 50-69. https://doi.org/10.1016/j.apm.2019.09.057
[34] Krebaeva, L., Konuspaev, S., Nurlibayev, I. (2022). Cracking of heavy hydrocarbons on the shankanai zeolite of modified HPA for the synthesis of long-chain α-olefins. Scientific Horizons, 25(9): 96-104. https://doi.org/10.48077/scihor.25(9).2022.96-104
[35] Moldabayeva, G.Z., Metaxa, G.P., Alisheva, Z.N. (2018). Scientific-technical basics of viscosity reduction of the Kazakhstani oils, which provide a significant increase of oil reservoirs. News of the National Academy of Sciences of the Republic of Kazakhstan, Series of Geology and Technical Sciences, 3(430): 186-194.
[36] Le, T.D., Murad, M.A. (2018). A new multiscale model for flow and transport in unconventional shale oil reservoirs. Applied Mathematical Modelling, 64: 453-479. https://doi.org/10.1016/j.apm.2018.07.027
[37] Le, V., Tran, S. (2022). Hybrid electrical-submersible-pump/gas-lift application to improve heavy oil production: From system design to field optimization. Journal of Energy Resources Technology, 144(8): 083006. https://doi.org/10.1115/1.4052979
[38] Bence, N., Lengyel, A., Tarics, Z. (2022). A simple model for describing the minimum differential cross-section of elastic proton scattering on protons at high energies. Scientific Herald of Uzhhorod University. Series Physics, (51): 30-38 https://doi.org/10.54919/2415-8038.2022.51.30-38
[39] Usenbekov, M.S., Isabek, T.K., Polchin, A.I., Zhumabekova, A.E. (2022). Methane emission dynamics in mining in zones of geological faults. Mining Informational and Analytical Bulletin, (12): 141-151. https://doi.org/10.25018/0236_1493_2022_12_0_141
[40] Ghaithan, A.M., Attia, A., Duffuaa, S.O. (2017). Multi-objective optimization model for a downstream oil and gas supply chain. Applied Mathematical Modelling, 52(10): 689-708. https://doi.org/10.1016/j.apm.2017.08.007
[41] Oryngozhin, Y.S., Moldabayeva, G.Z., Igembaev, I.B. (2012). Innovative method of high-viscosity oil production. 12th International Multidisciplinary Scientific GeoConference and EXPO - Modern Management of Mine Producing, Geology and Environmental Protection, SGEM 2012, pp. 463-470.
[42] Khudayarov, B.A., Turaev, F.Z. (2019). Mathematical Simulation of nonlinear oscillations of viscoelastic pipelines conveying fluid. Applied Mathematical Modelling, 66: 662-679. https://doi.org/10.1016/j.apm.2018.10.008
[43] Abdo, J., Haneef, M.D. (2022). Nanomaterials modified drilling fluid for improving deep drilling conditions. Journal of Energy Resources Technology, 144(7): 073202. https://doi.org/10.1115/1.4052186
[44] Bolatbek, K., Shlygina, I.A., Brodskiy, A.R., Ivanov, S.I., Gordienko, M.G. (2016). Quantum-chemical research methods of geometrical and electronic structure of silicon dioxide aerogels. International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management, SGEM, 3(BOOK 6): 135-142.
[45] Bolatbek, K., Brodskiy, A.R, Ivanov, S.I, Gordienko, M.G, Menshutina, N.V (2016). Modeling of substances adsorption process in aerogel pores. International Multidisciplinary Scientific GeoConference Surveying Geology and Mining Ecology Management, SGEM, 3(BOOK 6): 83-90.
[46] Cheng, C., Li, K. (2014). Comparison of models correlating cumulative oil production and water cut. Journal of Energy Resources Technology, 136(3): 032901. https://doi.org/10.1115/1.4026459
[47] Goel, V., Furman, K.C. (2009). U.S. patent application No. PCT/US2009/032736. U.S. Patent and Trademark Office.
[48] Bekbauov, B., Berdyshev, A., Baishemirov, Z. (2016). Numerical simulation of chemical enhanced oil recovery processes. CEUR Workshop Proceedings, 1623: 664-676.