IIETA International Information and Engineering Technology Association
Advancing the World of Information and Engineering

Mathematical Modelling of Engineering Problems
Vol. 11, No. 1, January, 2024, pp. 255-262
Journal homepage: http://iieta.org/journals/mmep

Fuzzy Mathematical Approach for Solving Multi-Objective Fuzzy Linear Fractional Programming Problem with Trapezoidal Fuzzy Numbers

Karthick Sivakumar, Saraswathi Appasamy*

Department of Mathematics, College of Engineering and Technology, SRM Institute of Science and Technology, Kattankulathur, Chengalpattu 603203, Tamil Nadu, India

Corresponding Author Email: saraswaa@srmist.edu.in

https://doi.org/10.18280/mmep.110128

Received: 27 July 2023
Revised: 15 September 2023
Accepted: 22 October 2023
Available online: 30 January 2024

Keywords:
trapezoidal fuzzy number, fuzzy mathematical approach, fractional programming, optimal solution, multi objective problem, uncertainty, optimization software

ABSTRACT

This research article presents an innovative approach to address a LFPP under trapezoidal fuzzy environment. The suggested method considers all the objective function coefficients, resources, and technological coefficients as fuzzy trapezoidal numbers, taking into account their inherent uncertainty. To handle this ambiguity effectively, we apply the component-wise optimization method. The proposed method seeks an optimal solution for each fuzzy fractional objective function. The problem attains optimal solution, using the solution the given objective function is transformed from its fractional representation to its corresponding linear form. With the aid of fuzzy programming methodology, the component-wise optimization technique, solves the problem using the lingo software or other optimization software. Previous research studies have successfully addressed the issue of solving problems involving triangular fuzzy numbers by employing mathematical approach. In this study, we have taken a step further by extending our methodology to accommodate trapezoidal fuzzy numbers. This expansion allows us to tackle problems that involve a broader range of fuzzy data representations, thus enhancing the applicability and versatility of our approach.

1. INTRODUCTION

Fuzzy Linear Fractional Programming (FLFP) is like a useful tool in the world of math problem-solving. It comes in handy when we're dealing with complicated real-life problems that aren't straightforward because of uncertainty and confusion. FLFP helps us figure things out when we're not entirely sure about the details.

Imagine you have to make decisions about how to use limited resources for a project or manage money when you're not sure about all the costs. FLFP steps in to help you make these decisions in a practical way.

FLFP started back in the 1970s when smart folks combined two things: fuzzy logic (which deals with handling vague information) and linear fractional programming (a way to make allocation decisions). This mix created FLFP, a versatile tool for solving real-world problems.

Many different fields focus on optimizing various ratios, such as production planning aiming to maximize output ratios. In healthcare and hospital planning, optimization involves ratios like nurses per patients or nurses per room.

In the study, the method works not just for triangles but also for shapes that look like trapezoids.A FMP Approach for solving a MOLFPP that always yields an optimal solution was presented in Chakraborty and Gupta [1].A fuzzy set theoretic approach for solving MOLFPP was proposed in Dutta et al. [2].Initial approach to solving LFPP was introduced by Charnes and Cooper [3].

Deb and De [4] discussed fuzzy LFP problems converted into crisp problems to obtain the optimal solution. Dharmaraj and Appasamy [5] applied a modified Gauss elimination technique for separable fuzzy nonlinear programming. The researchers [6-9] solved Linear fractional programming problems under different methods.

Ebrahimnejad and Tavana [10] came up with a new way to solve FLP problems, inspired by Pramy [11].

Ganesan and Veeramani [12] proposed a novel fuzzy arithmetic technique specifically designed for symmetric trapezoidal fuzzy numbers. These works collectively contribute to the advancement of fuzzy mathematical methods in solving various types of LP and FLP problems.

Guzel's [13] research introduced a method to tackle LFPP by transforming them into LPP, optimizing the weighted sum of all the objective functions. Guzel and Sivri [14] proposed a method based on Taylor series, and Luhandjula [15] used a linguistic variable approach to solve MOLFPP. Hasan and Acharjee [16] devised a technique to address LFP problems by transforming them into LPP. Kumar et al. [17] came up with a clever technique for finding the best fuzzy solution for FLFP problems with equality constraints. Kannan et al. [18] addressed the Linear Diophantine Fuzzy Shortest Path Problem in Network Analysis.

Maleki et al. [19] employed the Roubens approach to solve fuzzy LPP problems effectively. Prakash and Appasamy [20] worked on solving the Optimal Solution for Fully Spherical Fuzzy Linear Programming Problem. Ramík [21] introduced a class of FLP problems and established the concepts of feasible and α-efficient solutions.

Saberi Najafi and Edalatpanah [22] pointed out changes needed to improve the approach from [15]. Researchers found ways to solve problems with triangular fuzzy numbers using a mathematical approach in the past. Saraswathi [23] tried fuzzy-trapezoidal dematel approach method for solving decision making problems under uncertainty.

Swarup [24] tackled LFPP using a solution method based on the simplex method. In many branches of science and engineering, LPP’s are essential. Veeramani and Sumathi [25] solved fuzzy LFP problems with coefficients are triangular fuzzy numbers using the Fuzzy mathematical approach. So, we extend the thoery to solving Fuzzy LFP Problems where the coefficients are TFN. Wan and Dong [26] worked with trapezoidal fuzzy numbers in a linear programming problem. Zadeh initially suggested the idea of a fuzzy set [27].

The remainder of this article is structured as follows: Section 2 contains some fundamental definitions and notations. The general form of the FFLP using the new technique is provided in Section 3.A summary of the algorithm in Section 4. Section 5 contains numerical example was resolved using the suggested methodology to produce the best result. The last section is when the conclusion is reached.

2. PRELIMINARIES

2.1 Definition [27]

If X is a universal set and $x \in X$, then a fuzzy set $\tilde{A}$ defined as a collection of ordered pairs,

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)), x \in X\}$$

where, $\mu_{\tilde{A}}$ is called the membership function that maps X to the membership space M.

2.2 Definition [10]

A fuzzy number $\tilde{A}$=(a, b, c, d) is said to be a trapezoidal fuzzy number if its membership function is given as:

$$\tilde{A}(x) = \begin{cases} \dfrac{x-a}{b-a}, a \le x \le b \\ 1, b \le x \le c \\ \dfrac{x-d}{d-c}, c < x \le d \\ 0, \text{ otherwise} \end{cases}$$

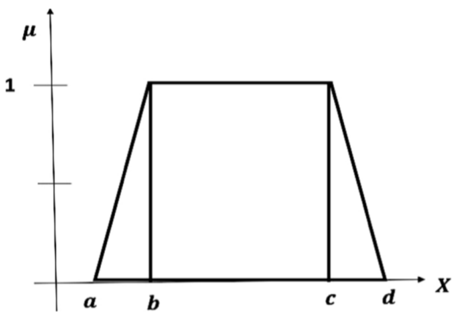

Figure 1. Trapezoidal fuzzy number

The representation of the membership function given in the Figure 1.

2.3 Definition [4]

Let $\tilde{A}$=(a, b, c, d) and $\tilde{B}$=(e, f, g, h) be two non-negative trapezoidal fuzzy numbers then:

i. $\tilde{A}+\tilde{B}$=(a, b, c, d)+(e, f, g, h)=(a+e, b+f, c+g, d+f).
ii. $\tilde{A}-\tilde{B}$=(a, b, c, d)-(e, f, g, h)=(a-h, b-g, c-f, d-c).
iii. $\tilde{A}*\tilde{B}$=(a, b, c, d)*(e, f, g, h)=(α, β, γ, δ).
iv. Where α = min (ae,ah,de,dh), β = min (bf, bg, cf, cg). γ=max(bf, bg, cf, cg), δ=max(ae, ah, de, dh).
v. $\frac{\tilde{A}}{\tilde{B}}$=(ah, bg, cf, de).

2.4 Definition [9]

A ranking function is a function: F(R)→R which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A}$=(a, b, c, d) is a trapezoidal fuzzy number then $\tilde{A} = \frac{a+b+c+d}{4}$.

To understand why these definitions are important, it's crucial to see how they form the basis of our study. These definitions create a shared language and structure for our analysis. As we move forward in the upcoming sections, you'll see how these specific definitions are fundamental to the theory behind our research, setting the stage for the methods, experiments, and results that will come next.

3. LINEAR FRACTIONAL PROGRAMMING PROBLEM

Linear fractional programming (LFP) is a valuable mathematical optimization tool utilized across numerous disciplines such as operations research, economics, engineering, and finance. It focuses on optimizing a ratio of two linear functions while adhering to a set of linear constraints, which makes it a linear objective function. LFP is particularly relevant in practical situations where balancing competing objectives is a critical part of the decision-making process. This type of optimization is frequently employed when decision-makers are faced with the need to navigate through trade-offs among various goals.

One of the key motivations behind linear fractional programming is its ability to model and solve problems involving resource allocation, portfolio optimization, and efficiency maximization. By allowing decision-makers to express preferences and constraints in the form of ratios, linear fractional programming provides a flexible framework to address complex decision-making situations.

In summary, linear fractional programming is a valuable tool for tackling real-world problems where decision-makers must navigate the delicate balance between competing objectives. Its relevance extends to diverse fields, making it an important topic in optimization theory and practice. In this context, understanding the general form of a linear fractional programming problem becomes crucial for both researchers and practitioners seeking to address multi-objective optimization challenges effectively.

A general form of linear fractional programming problem defined as:

$$\text{Max}Z(x) = \frac{N(x)}{D(x)} = \frac{c^t x + \alpha}{d^t x + \beta}$$
subject to
$$x \in S = \{x \in R^n : Ax \leq b, x \geq 0\} \tag{1}$$

where, $A \in R^{m*n}$, $c, d \in R^n$ and $\alpha, \beta \in R$; For some values of x, $D(x)$ may be zero; So, we consider $\{Ax \leq b, x \geq 0, D(x)>0\}$.

3.1 Theorem [3]

Consider the following LFPP:

$$\begin{gathered} Maximize \frac{N(x)}{D(x)} \\ \text{subject to } Ax \leq b \\ x \geq 0 \end{gathered} \tag{2}$$

Then the problem given by Eq. (2) is equivalent to problem given by Eq. (3),

where Eq. (3) is obtained from Eq. (2) by using the transformation $t=\frac{1}{D(x)}$, $y=tx$ and the denominator of the objective function is restricted to be lesser than 1.

$$\begin{gathered} \text{Maximize } tN(\tfrac{y}{t}) \\ \text{Subject to } A\tfrac{y}{t} - b \leq 0 \text{ (3)} \\ tD(\tfrac{y}{t}) \leq 1 \\ y \geq 0, t > 0. \end{gathered} \tag{3}$$

3.2 Theorem [1]

Let for some $x \in S$, $N(x)>0$ and if Eq. (2) reaches a maximum at $x=x^*$, then Eq. (3) reaches a maximum at a point $(t, y)=(t^*, y^*)$, where $\frac{y^*}{t^*}=x^*$ and objective function at these points are equal [1].

3.3 Theorem [5]

A solution $z^*=\frac{N(x^*)}{D(x^*)}$ is said to be an optimal solution of Eq. (2) if it is a solution of $F(z^*, x^*)=0$, where $F(z^*, x^*)=\max\{N(x)-z^*D(x), x \in S\}$.

4. FUZZY LINEAR FRACTIONAL PROGRAMMING PROBLEM

This part of the text outlines an approach to tackle problems involving Multi-Objective Fuzzy Linear Fractional Programming (MOFLFP). Within this framework, crucial elements such as the costs in the objective functions, the resources at disposal, and the coefficients related to technology are depicted using trapezoidal fuzzy numbers. This depiction is used to address the MOFLFP challenge, where key parameters are enveloped in uncertainty.

$$\begin{gathered} Max\tilde{z} = \frac{\sum \tilde{c}_i \tilde{x}_i + \tilde{p}}{\sum \tilde{d}_i \tilde{x}_i + \tilde{q}} \\ \text{Subject to} \\ \sum_{i=1}^{n} \tilde{a}_{ij} \tilde{x}_i \leq \tilde{b}_j \text{, j=1, 2, ..., m} \\ \tilde{x}_i \geq 0\text{, i= 1, 2, ..., n.} \end{gathered} \tag{4}$$

We assume that, $\tilde{c}_i$, $\tilde{p}$, $\tilde{d}_i$, $\tilde{q}$, $\tilde{b}_i$, $\tilde{a}_{ij}$ are trapezoidal fuzzy numbers for each j=1, ..., m and i= 1, ..., n. Therefore,

$$\begin{gathered} Max\tilde{z} = \frac{\sum (c_i^1, c_i^2, c_i^3, c_i^4)\tilde{x}_i + (p_1, p_2, p_3, p_4)}{\sum (d_i^1, d_i^2, d_i^3, d_i^4)\tilde{x}_i + (q_1, q_2, q_3, q_4)} \\ \text{Subject to} \\ \sum (\tilde{a}_{ij}^1, \tilde{a}_{ij}^2, \tilde{a}_{ij}^3, \tilde{a}_{ij}^4)\tilde{x}_i \leq (\tilde{b}_j^1, \tilde{b}_j^{\,2}, \tilde{b}_j^{\,3}, \tilde{b}_j^{\,4}) \text{, j=1, 2..., m} \\ \tilde{x}_i \geq 0\text{, i= 1, 2..., n.} \end{gathered} \tag{5}$$

Since component-wise optimization is a strategy for simplifying complex optimization problems by breaking them into independent components, each optimized separately. This approach allows for the use of different techniques for each component and is often used in fields like machine learning and engineering design. The key challenge is integrating the solutions of individual components to address the overall problem effectively.

By using Component-wise optimization, the problem (5) reduce to an equivalent MOLP problem as follows:

$$\begin{gathered} \text{Max } \tilde{Z}_1 = \frac{\sum c_i^1 x_i + p^1}{\sum d_i^4 x_i + q^4} \\ \text{Max } \tilde{Z}_2 = \frac{\sum c_i^2 x_i + p^2}{\sum d_i^3 x_i + q^3} \\ \text{Max } \tilde{Z}_3 = \frac{\sum c_i^3 x_i + p^3}{\sum d_i^2 x_i + q^2} \\ \text{Max } \tilde{Z}_4 = \frac{\sum c_i^4 x_i + p^4}{\sum d_i^1 x_i + q^1} \\ \text{Subject to} \\ \sum \tilde{a}_{ij}^1 \tilde{x}_i \leq \tilde{b}_j^1 \\ \sum \tilde{a}_{ij}^2 \tilde{x}_i \leq \tilde{b}_j^{\,2} \\ \sum \tilde{a}_{ij}^3 \tilde{x}_i \leq \tilde{b}_j^{\,3} \\ \sum \tilde{a}_{ij}^4 \tilde{x}_i \leq \tilde{b}_j^{\,4} \\ \tilde{x}_i \geq 0\text{, i= 1, 2..., n.} \end{gathered} \tag{6}$$

which is a MOLFP problem.

In summary, the Charnes and Cooper transformation is a valuable technique for converting LFP problems into standard LP problems, making them more amenable to solution using LP methods. This transformation simplifies the optimization process and allows you to find the optimal solution for LFP problems efficiently.

Using Charnes and Cooper transformation we get:

$$\begin{gathered} \text{Max } \tilde{Z}_1 = \sum c_i^1 y_i + p^1 t \\ \text{Max } \tilde{Z}_2 = \sum c_i^2 y_i + p^2 t \\ \text{Max } \tilde{Z}_3 = \sum c_i^3 y_i + p^3 t \\ \text{Max } \tilde{Z}_4 = \sum c_i^4 y_i + p^4 t \\ \text{Subject to} \\ \sum d_i^4 y_i + q^4 t \leq 1 \\ \sum d_i^3 y_i + q^3 t \leq 1 \\ \sum d_i^2 y_i + q^2 t \leq 1 \\ \sum d_i^1 y_i + q^1 t \leq 1 \\ \sum \tilde{a}_{ij}^1 \tilde{y}_i - \tilde{b}_j^1 t \leq 0 \\ \sum \tilde{a}_{ij}^2 \tilde{y}_i - \tilde{b}_j^{\,2} t \leq 0 \\ \sum \tilde{a}_{ij}^3 \tilde{y}_i - \tilde{b}_j^{\,3} t \leq 0 \\ \sum \tilde{a}_{ij}^4 \tilde{y}_i - \tilde{b}_j^{\,4} t \leq 0 \\ z_j, t \geq 0\text{, i = 1, 2, ..., n, j = 1, 2, ..., m.} \end{gathered} \tag{7}$$

Solving the transformed MOLPP for each objective function we get:

$$
\begin{gathered}
\text{Max } \lambda \\
\sum c_i^1 y_i + p^1 t - z_1^* \lambda \geq 0 \\
\sum c_i^2 y_i + p^2 t - z_2^* \lambda \geq 0 \\
\sum c_i^3 y_i + p^3 t - z_3^* \lambda \geq 0 \\
\sum c_i^4 y_i + p^4 t - z_4^* \lambda \geq 0 \\
\sum d_i^4 y_i + q^4 t \leq 1 \\
\sum d_i^3 y_i + q^3 t \leq 1 \\
\sum d_i^2 y_i + q^2 t \leq 1 \\
\sum d_i^1 y_i + q^1 t \leq 1 \\
\sum \tilde{a}_{ij}^1 \tilde{y}_i - \tilde{b}_j^1 \, t \leq 0 \\
\sum \tilde{a}_{ij}^2 \tilde{y}_i - \tilde{b}_j^{\,2} t \leq 0 \\
\sum \tilde{a}_{ij}^3 \tilde{y}_i - \tilde{b}_j^{\,3} t \leq 0 \\
\sum \tilde{a}_{ij}^4 \tilde{y}_i - \tilde{b}_j^{\,4} t \leq 0 \\
z_j, t, \lambda \geq 0, \text{ i = 1, 2, ..., n, j = 1, 2, ..., m.}
\end{gathered} \tag{8}
$$

Solving Eq. (8) we obtain y_i, i=1, 2, …, n and t and using the transformation $x_i = \frac{y_i}{t}$, we get the optimal value of Z_S, s=1, 2, ..., k.

5. ALGORITHM

This article introduces a structured approach designed to enhance the efficacy of finding optimal solutions in scenarios where traditional crisp optimization methods are not adequate. The presented algorithm applies the concepts of fuzzy mathematics to adeptly navigate the uncertainty and imprecision that typically characterize real-world decision-making scenarios. In this paper, we explore the foundational concepts and procedural steps of this algorithm, showcasing its capability to adeptly resolve intricate FLFP (Fuzzy Linear Fractional Programming) challenges.

Summarized below is the method proposed to address the FLFP problem:

a) Frame the issue within the context of FLFP.
b) Decompose the trapezoidal fuzzy linear fractional objectives into four distinct linear fractional objectives through a component-wise optimization process. In a parallel fashion, convert trapezoidal fuzzy constraints into four definitive constraints composed of exact coefficients.
c) Apply the Charnes and Cooper technique to transform the Multi-Objective Linear Fractional Programming (MOLFP) problem into a Multi-Objective Linear Programming (MOLP) problem.
d) Isolate and determine the optimal solution for each objective.
e) Implement the fuzzy programming method detailed in Eq. (8) to ascertain the optimal resolution for Eq. (7).
f) Utilize the derived values of 'y' and 't' to compute the vector 'x', enabling the calculation of the optimal values for each trapezoidal fuzzy objective.
g) Convert the Multi-Objective Linear Fractional Programming Problem (MOLFPP) into a singular objective Linear Programming Problem (LPP) by maximizing P (Ni(x) - Zi*Di(x)), where each trapezoidal fuzzy objective's optimal solution is employed.
h) Use component-wise optimization to simplify the LPP into a Multi-Objective Linear Programming Problem (MOLPP).
i) Resolve each objective function on its own to procure the optimal solution, denoted as Q_i for each, and collate these solutions into the optimal set S.
j) Within the set S, identify the minimal possible value for each objective function.
k) Employ a linear membership function to solve the derived crisp LPP, thereby securing an efficient solution.

The linear membership function utilized is delineated as follows:

$$
\mu_F(x) = \left\{ \begin{array}{l} 0, F < P_i \\ \dfrac{F - P_i}{Q_i - P_i}, P_i < F < Q_i \\ 1, F > Q_i \end{array} \right\}
$$

The algorithm is shown in Figure 2 in the form of flow chart. The algorithm offers a valuable tool for tackling optimization challenges in situations where uncertainty and imprecision are present. By embracing the power of fuzzy mathematics and trapezoidal fuzzy numbers, this approach provides a systematic and practical way to address real-world decision-making problems. It extends the applicability of optimization techniques to a wide range of domains, including finance, engineering, and logistics, where decisions often rely on incomplete or uncertain information.

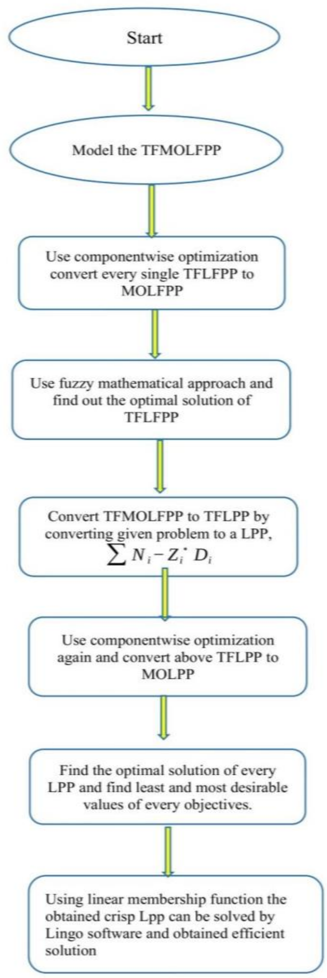

Figure 2. Flowchart of the method

Figure 2 illustrates the step-by-step process of the method explained. It provides a visual representation of how the method works and the sequence of actions involved.

6. NUMERICAL EXAMPLE

The numerical case study concerns the application of Linear Fractional Programming (LFP) in situations where trapezoidal fuzzy numbers are used to represent uncertainty and imprecision. The primary objective is to find an optimal solution while taking into account the inherent vagueness associated with these fuzzy numbers. The outcome of this optimization process is a solution characterized by fuzziness or uncertainty, known as a "fuzzy optimal solution." This approach is valuable for decision-making in scenarios where precise numerical data may be lacking or when dealing with problems influenced by uncertain factors.

$$\begin{gathered}
\text{Max} Z^1 = \frac{(4,7,10,12)x_1+(8,10,14,15)x_2+(2.5,4,7.5,11.5)x_3+(2,3,4,6)}{(10,14,20,22)x_1+(20,23.5,27.5,29)x_2+(18,20,25,28)x_3+(5,10,18,20)} \\
\text{Max } Z^2 = \frac{(20,22,24,28)x_1+(18,21,25,30)x_2+(14,17,19,25)x_3+(1,3,6,10)}{(14,16,19,23)x_1+(18,21,25,27)x_2+(15,20,25,30)x_3+(10,15,20,23)} \\
\text{Subject to the constraints} \\
(10,17,19,25)x_1 + (14,16,22,24)x_2 + (20,25,27,30)x_3 \le (40,43,45,47) \\
(0.03,0.06,0.07,0.09)x_1 + (0.05,0.07,0.08,0.1)x_2 + (0.02,0.05,0.06,0.07)x_3 \le (0.7,0.8,0.9,1) \\
(4,6,10,13)x_1 + (0,5,10,15)x_2 + (8,11,14,20)x_3 \le (25,30,35,40) \\
x_1, x_2, x_3 \ge 0.
\end{gathered} \tag{9}$$

Step 1: Here we consider first objective function and the constraints,

$$\begin{gathered}
\text{Max } z_1 = \frac{4x_1+8x_2+2.5x_3+2}{22x_1+29x_2+28x_3+20} \\
\text{Max } z_2 = \frac{7x_1+10x_2+4x_3+3}{20x_1+27.5x_2+25x_3+18} \\
\text{Max } z_3 = \frac{10x_1+14x_2+7.5x_3+24}{14x_1+23.5x_2+20x_3+10} \\
\text{Max } z_4 = \frac{12x_1+`15x_2+11.5x_3+6}{10x_1+20x_2+18x_3+5} \\
\text{Subject to the constraints} \\
10x_1 + 14x_2 + 20x_3 \le 40 \\
19x_1 + 22x_2 + 27x_3 \le 45 \\
25x_1 + 24x_2 + 30x_3 \le 47 \\
0.03x_1 + 0.05x_2 + 0.02x_3 \le 0.7 \\
0.06x_1 + 0.07x_2 + 0.05x_3 \le 0.8 \\
0.07x_1 + 0.08x_2 + 0.06x_3 \le 0.9 \\
0.09x_1 + 0.1x_2 + 0.07x_3 \le 1 \\
4x_1 + 0x_2 + 8x_3 \le 25 \\
6x_1 + 5x_2 + 11x_3 \le 30 \\
10x_1 + 10x_2 + 14x_3 \le 35 \\
13x_1 + 15x_2 + 20x_3 \le 40 \\
x_1, x_2, x_3 \ge 0.
\end{gathered} \tag{10}$$

Step 2: Use Charnes and Cooper transformation

$$\begin{gathered}
\text{Max } z_1 = 4y_1 + 8y_2 + 2.5y_3 + 2 \\
\text{Max } z_2 = 7y_1 + 10y_2 + 4y_3 + 3 \\
\text{Max } z_3 = 10y_1 + 14y_2 + 7.5y_3 + 24 \\
\text{Max } z_4 = 12y_1 + 15y_2 + 11.5y_3 + 6 \\
\text{Subject to the constraints} \\
22y_1 + 29y_2 + 28y_3 + 20t \le 1 \\
20y_1 + 27.5y_2 + 25y_3 + 18t \le 1 \\
14y_1 + 23.5y_2 + 20y_3 + 10t \le 1 \\
10y_1 + 20y_2 + 18y_3 + 5t \le 1 \\
10y_1 + 14y_2 + 20y_3 - 40t \le 0 \\
17y_1 + 16y_2 + 25y_3 - 43t \le 0 \\
19y_1 + 22y_2 + 27y_3 - 45t \le 0 \\
25y_1 + 24y_2 + 30y_3 - 47t \le 0 \\
0.03y_1 + 0.05y_2 + 0.02y_3 - 0.7t \le 0 \\
0.06y_1 + 0.07y_2 + 0.05y_3 - 0.8t \le 0 \\
0.07y_1 + 0.08y_2 + 0.06y_3 - 0.9t \le 0 \\
0.09y_1 + 0.1y_2 + 0.07y_3 - t \le 0 \\
4y_1 + 0y_2 + 8y_3 - 25t \le 0 \\
6y_1 + 5y_2 + 11y_3 - 30t \le 0 \\
10y_1 + 10y_2 + 14y_3 - 35t \le 0 \\
13y_1 + 15y_2 + 20y_3 - 40t \le 0 \\
y_1, y_2, y_3 \ge 0 \text{ and } t > 0.
\end{gathered} \tag{11}$$

Step 3: when each objective is individually maximized using Lingo, the resulting solutions are as follows:

$$\begin{gathered}
z_1 = 0.23 \\
z_2 = 0.2940 \\
z_3 = 0.4091 \\
z_4 = 0.4654 \\
\text{Maximize } \lambda \\
\text{Subject to} \\
4y_1 + 8y_2 + 2.5y_3 + 2t - 0.23\lambda \ge 0 \\
7y_1 + 10y_2 + 4y_3 + 3t - 0.2940\lambda \ge 0 \\
10y_1 + 14y_2 + 7.5y_3 + 24t - 0.4091\lambda \ge 0 \\
12y_1 + `15y_2 + 11.5y_3 + 6t - 0.4654\lambda \ge 0 \\
22y_1 + 29y_2 + 28y_3 + 20t \le 1 \\
20y_1 + 27.5y_2 + 25y_3 + 18t \le 1 \\
14y_1 + 23.5y_2 + 20y_3 + 10t \le 1 \\
10y_1 + 20y_2 + 18y_3 + 5t \le 1 \\
10y_1 + 14y_2 + 20y_3 - 40t \le 0 \\
17y_1 + 16y_2 + 25y_3 - 43t \le 0 \\
19y_1 + 22y_2 + 27y_3 - 45t \le 0 \\
25y_1 + 24y_2 + 30y_3 - 47t \le 0 \\
0.03y_1 + 0.05y_2 + 0.02y_3 - 0.7t \le 0 \\
0.06y_1 + 0.07y_2 + 0.05y_3 - 0.8t \le 0 \\
0.07y_1 + 0.08y_2 + 0.06y_3 - 0.9t \le 0 \\
0.09y_1 + 0.1y_2 + 0.07y_3 - t \le 0 \\
4y_1 + 0y_2 + 8y_3 - 25t \le 0 \\
6y_1 + 5y_2 + 11y_3 - 30t \le 0 \\
10y_1 + 10y_2 + 14y_3 - 35t \le 0 \\
13y_1 + 15y_2 + 20y_3 - 40t \le 0 \\
y_1, y_2, y_3 \ge 0 \text{ and } t > 0.
\end{gathered} \tag{12}$$

Solving we get:

$y_1 = 0.000952$, $y_2 = 0.02$, $y_3 = 0$, $\lambda = 0.99$, $t = 0.01$

Using the transformation $y=t^*x$, we solve get:

$$x_1 = 0.0952, x_2 = 2, x_3 = 0$$

Therefore:

$$\text{Max } Z^1 = (0.4, 0.4057, 0.4399, 0.4637)$$

Here we consider 2^{ND} objective and the constraints:

$$\text{Max } Z^2 = \frac{(20,22,24,28)x_1+(18,21,25,30)x_2+(14,17,19,25)x_3+(1,3,6,10)}{(14,16,19,23)x_1+(18,21,25,27)x_2+(15,20,25,30)x_3+(10,15,20,23)}$$

$$\text{Max } z_1 = \frac{20x_1+18x_2+14x_3+1}{23x_1+27x_2+30x_3+23}$$
$$\text{Max } z_2 = \frac{22x_1+21x_2+17x_3+3}{19x_1+25x_2+25x_3+20}$$
$$\text{Max } z_3 = \frac{24x_1+25x_2+19x_3+6}{16x_1+21x_2+20x_3+15}$$
$$\text{Max } z_4 = \frac{28x_1+30x_2+25x_3+10}{14x_1+18x_2+15x_3+10}$$

Subject to the constraints

$$\begin{gathered} 10x_1 + 14x_2 + 20x_3 \le 40 \\ 17x_1 + 16x_2 + 25x_3 \le 43 \\ 19x_1 + 22x_2 + 27x_3 \le 45 \\ 25x_1 + 24x_2 + 30x_3 \le 47 \\ 0.03x_1 + 0.05x_2 + 0.02x_3 \le 0.7 \\ 0.06x_1 + 0.07x_2 + 0.05x_3 \le 0.8 \\ 0.07x_1 + 0.08x_2 + 0.06x_3 \le 0.9 \\ 0.09x_1 + 0.1x_2 + 0.07x_3 \le 1 \\ 4x_1 + 0x_2 + 8x_3 \le 25 \\ 6x_1 + 5x_2 + 11x_3 \le 30 \\ 10x_1 + 10x_2 + 14x_3 \le 35 \\ 13x_1 + 15x_2 + 20x_3 \le 40,\ x_1, x_2, x_3 \ge 0. \end{gathered} \tag{13}$$

Step 4: Use Charnes and Cooper transformation

$$\begin{gathered} \text{Max } z_1 = 20y_1 + 18y_2 + 14y_3 + t \\ \text{Max } z_2 = 22y_1 + 21y_2 + 17y_3 + 3t \\ \text{Max } z_3 = 24y_1 + 25y_2 + 19y_3 + 6t \\ \text{Max } z_4 = 28y_1 + 30y_2 + 25y_3 + 10t \\ \text{Subject to the constraints} \\ 23y_1 + 27y_2 + 30y_3 + 23t \le 1 \\ 19y_1 + 25y_2 + 25y_3 + 20t \le 1 \\ 16y_1 + 21y_2 + 20y_3 + 15t \le 1 \\ 14y_1 + 18y_2 + 15y_3 + 10t \le 1 \\ 10y_1 + 14y_2 + 20y_3 - 40t \le 0 \\ 17y_1 + 16y_2 + 25y_3 - 43t \le 0 \\ 19y_1 + 22y_2 + 27y_3 - 45t \le 0 \\ 25y_1 + 24y_2 + 30y_3 - 47t \le 0 \\ 0.03y_1 + 0.05y_2 + 0.02y_3 - 0.7t \le 0 \\ 0.06y_1 + 0.07y_2 + 0.05y_3 - 0.8t \le 0 \\ 0.07y_1 + 0.08y_2 + 0.06y_3 - 0.9t \le 0 \\ 0.09y_1 + 0.1y_2 + 0.07y_3 - t \le 0 \\ 4y_1 + 0y_2 + 8y_3 - 25t \le 0 \\ 6y_1 + 5y_2 + 11y_3 - 30t \le 0 \\ 10y_1 + 10y_2 + 14y_3 - 35t \le 0 \\ 13y_1 + 15y_2 + 20y_3 - 40t \le 0,\ y_1, y_2, y_3 \ge 0 \text{ and } t > 0. \end{gathered} \tag{14}$$

Step 5: Maximizing each objective separately using lingo, gives the following solutions:

$$\begin{gathered} z_1 = 0.58, z_2 = 0.67, z_3 = 0.77, z_4 = 0.94 \\ \text{Maximize } \lambda \text{ S.T.} \\ 20y_1 + 18y_2 + 14y_3 + t - 0.58\lambda \ge 0 \\ 22y_1 + 21y_2 + 17y_3 + 3t - 0.67\lambda \ge 0 \\ 24y_1 + 25y_2 + 19y_3 + 6t - 0.77\lambda \ge 0 \\ 28y_1 + `30y_2 + 25y_3 + 10t - 0.94\lambda \ge 0 \\ 23y_1 + 27y_2 + 30y_3 + 23t \le 1 \\ 19y_1 + 25y_2 + 25y_3 + 20t \le 1 \\ 16y_1 + 21y_2 + 20y_3 + 15t \le 1 \\ 14y_1 + 18y_2 + 15y_3 + 10t \le 1 \\ 10y_1 + 14y_2 + 20y_3 - 40t \le 0 \\ 17y_1 + 16y_2 + 25y_3 - 43t \le 0 \\ 19y_1 + 22y_2 + 27y_3 - 45t \le 0 \\ 25y_1 + 24y_2 + 30y_3 - 47t \le 0 \\ 0.03y_1 + 0.05y_2 + 0.02y_3 - 0.7t \le 0 \\ 0.06y_1 + 0.07y_2 + 0.05y_3 - 0.8t \le 0 \\ 0.07y_1 + 0.08y_2 + 0.06y_3 - 0.9t \le 0 \\ 0.09y_1 + 0.1y_2 + 0.07y_3 - t \le 0 \\ 4y_1 + 0y_2 + 8y_3 - 25t \le 0 \\ 6y_1 + 5y_2 + 11y_3 - 30t \le 0 \\ 10y_1 + 10y_2 + 14y_3 - 35t \le 0 \\ 13y_1 + 15y_2 + 20y_3 - 40t \le 0 \\ y_1, y_2, y_3 \ge 0 \text{ and } t > 0 \end{gathered} \tag{15}$$

Solving we get:

$$y_1 = 0.028,\ y_2 = 0,\ y_3 = 0,\ \lambda = 0.99,\ t = 0.015$$

Using the transformation y=t*x, we solve get:

$$x_1 = 1.86,\ x_2 = 0,\ x_3 = 0$$

Therefore:

$$\text{Max } Z^2 = (0.58, 1.15, 1.5, 2)$$

Step 6: Now we convert MOLFPP into the single objective linear fractional programming problems using Maximize $\sum (N_i(x) - Z_i^* D_i(x))$ subject to the constraints of given original problem.

$$\begin{gathered} \text{Maximize } Z = (-32.2014, -8.298, 9.9202, 27.88)x_1 + \\ (-41.4473, -18.5972, 5.3161, 26.56)x_2 + \\ (-56.4836, -27.4975, -4.614, 20.8)x_3 \\ + (-52.274, -31.9182, -11.307, 8.2) \\ \text{Subject to:} \\ (10,17,19,25)x_1 + (14,16,22,24)x_2 + (20,25,27,30)x_3 \le (40,43,45,47) \\ (0.03,0.06,0.07,0.09)x_1 + (0.05,0.07,0.08,0.1)x_2 \\ + (0.02,0.05,0.06,0.07)x_3 \le (0.7,0.8,0.9,1) \\ (4,6,10,13)x_1 + (0,5,10,15)x_2 + (8,11,14,20)x_3 \le (25,30,35,40) \\ x_1, x_2, x_3 \ge 0. \end{gathered}$$

Step 7: Now we again use component wise optimization and convert LPP to MOLPP given by:

$$\begin{gathered} \text{Max } Q_1(x) = -32.2014x_1 - 41.4473x_2 - 56.4836x_3 - \\ 52.274 \\ \text{Max } Q_2(x) = -8.298x_1 - 18.5972x_2 - 27.4975x_3 - 31.9182 \\ \text{Max } Q_3(x) = 9.9202x_1 + 5.3161x_2 - 4.614x_3 - 11.307 \\ \text{Max } Q_4(x) = 27.88x_1 + 26.56x_2 + 20.8x_3 + 8.2 \end{gathered}$$

subject to the constraints given in Eq. (7)

Step 8: Solving each objective Q_i, i=1,2,3,4 gives the optimal solution as:

$$Q_1 = -52.274, Q_2 = -31.9182, Q_3 = 7.3429, Q_4 = 60.6144.$$

The set S of optimal points is (0,0,0), (1.88,0,0).

Step 9: Using the set S, we find P_i, i=1, 2, 3, 4. So we obtain:

$$P_1 = -112.812, P_2 = -47.518, P_3 = 7.3429, P_4 = 60.6144.$$

Step 10: Now using linear membership function we obtained crisp LPP given by:

$$\begin{gathered} \text{Max } Z=\lambda; \\ -0.5319 * x_1 - 0.6846 * x_2 - 0.93 * x_3 + 1 - \lambda >= 0 \\ -0.5319 * x_1 - 1.192 * x_2 - 1.7626 * x_3 + 1 - \lambda >= 0 \\ 0.5319 * x_1 + 0.285 * x_2 - 0.2474 * x_3 - \lambda >= 0 \\ 0.5319 * x_1 + 0.5067 * x_2 + 0.3968 * x_3 - \lambda >= 0 \\ 10x_1 + 14x_2 + 20x_3 \le 40 \\ 17x_1 + 16x_2 + 25x_3 \le 43 \\ 19x_1 + 22x_2 + 27x_3 \le 45 \\ 25x_1 + 24x_2 + 30x_3 \le 47 \\ 0.03x_1 + 0.05x_2 + 0.02x_3 \le 0.7 \\ 0.06x_1 + 0.07x_2 + 0.05x_3 \le 0.8 \\ 0.07x_1 + 0.08x_2 + 0.06x_3 \le 0.9 \\ 0.09x_1 + 0.1x_2 + 0.07x_3 \le 1 \\ 4x_1 + 0x_2 + 8x_3 \le 25 \\ 6x_1 + 5x_2 + 11x_3 \le 30 \\ 10x_1 + 10x_2 + 14x_3 \le 35 \\ 13x_1 + 15x_2 + 20x_3 \le 40,\ x_1, x_2, x_3, \lambda \ge 0. \end{gathered}$$

Solving we get:

$$x_1 = 0.94,\ x_2 = 0, x_3 = 0,\ \lambda = 0.5$$

Therefore, we get:

Max Z^1= (0.1415,0.2603,0.578,1.2)
Max Z^2= (0.4539,0.6254,0.9507,1.5682).

Here we obtained the optimal solution for the multi objective function without converting to the crisp model. So here we have reduced the vagueness compared to the Pramy's Method [10]. The comparison graph discussed below we obtained the better results for the objective value.

Using the definition 2.5 applying ranking we get:

Max Z=0.54>0.44=Max Z=Pramy's Method the comparison shown in the Figure 3 below:

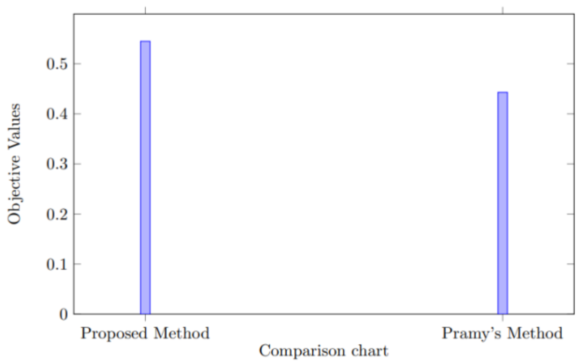

Figure 3. Graphical representation for comparison of proposed method

7. CONCLUSIONS

In this research study, we addressed a linear fractional programming problem by incorporating trapezoidal fuzzy numbers. Our approach involved transforming the problem into four distinct multi-objective optimization problems through component-wise optimization. Subsequently, we further converted the LFPP into a LPP by employing the Charnes and Cooper transformation. Through the application of fuzzy mathematical techniques, we were able to derive an optimal solution for the problem at hand.

This is impressive achievement, as fuzzy optimization problems often involve dealing with imprecise or vague data and preferences, and different methods can lead to varying levels of vagueness reduction and optimization results. This approach underscores the originality and effectiveness of your approach in addressing these challenges.

The algorithm's computational complexity can significantly increase for large-scale problems and high-dimensional fuzzy variables, leading to longer execution times and resource-intensive computations, which may limit its practicality. Like many optimization algorithms, the algorithm's performance relies on parameter selection, such as fuzzification levels and convergence criteria, which can be challenging to determine and require extensive experimentation. The assumption of independence between fuzzy variables and constraints may not hold in real-world scenarios, where interdependencies and correlations among variables are common, potentially limiting its applicability. The research primarily uses trapezoidal fuzzy numbers to handle data uncertainty but does not explore alternative fuzzy number representations or discuss the implications of different modeling choices, reducing its versatility. The algorithm's efficiency in solving large-scale problems with numerous constraints and variables, particularly regarding memory usage and scalability, requires further investigation.

Recognizing these limitations is crucial for researchers and practitioners considering the algorithm's application. Future research should focus on addressing these issues to extend its usability in practical problems involving fuzzy data and uncertainty. Furthermore, our proposed methodology holds promise for potential extensions to tackle Intuitionistic Fuzzy Linear Programming Problems utilizing trapezoidal Intuitionistic Fuzzy numbers. Intuitionistic fuzzy sets introduce an additional degree of freedom compared to classical fuzzy sets. By incorporating IFLPP, our methodology could better capture and model the uncertainty, hesitancy, and ambiguity present in many real-world decision-making scenarios. This extension would render our approach even more applicable to a broader range of practical problems. This future application would allow us to broaden the scope and utility of the proposed approach in solving more complex real life problems.

REFERENCES

[1] Chakraborty, M., Gupta, S. (2002). Fuzzy mathematical programming for multi objective linear fractional programming problem. Fuzzy Sets and Systems, 125: 335-342. https://doi.org/10.1016/S0165-0114(01)00060-4

[2] Dutta, D., Tiwari, R., Rao, J. (1992). Multiple objective linear fractional programming a fuzzy set theoretic approach. Fuzzy Sets and Systems, 52: 39-45. https://doi.org/10.1016/0165-0114(92)90034-2

[3] Charnes, A., Cooper, W.W. (1962). Programming with linear fractional functionals. Naval Research Logistics Quarterly, 9: 181-186. https://doi.org/10.1002/nav.3800090303

[4] Deb, M., De, P.K. (2014). A note on fuzzy multi-objective linear fractional programming problem.

[5] Dharmaraj, B., Appasamy, S. (2023). Application of a modified gauss elimination technique for separable fuzzy nonlinear programming problems. Mathematical Modelling of Engineering Problems, 10(4): 1481-1486. https://doi.org/10.18280/mmep.100445

[6] Chanas, S. (1989). Fuzzy programming in multiobjective linear programming-A parametric approach. Fuzzy Sets and Systems, 29: 303-313. https://doi.org/10.1016/0165-0114(89)90042-0

[7] Dinkelbach, W. (1967). On nonlinear fractional programming. Management Science, 13: 492-498. https://doi.org/10.1287/mnsc.13.7.492

[8] Edalatpanah, S.A. (2019). A data envelopment analysis model with triangular intuitionistic fuzzy numbers. International Journal of Data Envelopment Analysis, 7: 47-58.

[9] Ebrahimnejad, A., Ghomi, S.J., Mirhosseini-Alizamini, S.M. (2018). A revisit of numerical approach for solving linear fractional programming problem in a fuzzy environment. Applied Mathematical Modelling, 57: 459-473. https://doi.org/10.1016/j.apm.2018.01.008

[10] Ebrahimnejad, A., Tavana, M. (2014). A novel method for solving linear programming problems with symmetric trapezoidal fuzzy numbers. Applied Mathematical Modelling, 38: 4388-4395. https://doi.org/10.1016/j.apm.2014.02.024

[11] Pramy, F.A. (2018). An approach for solving fuzzy multi-objective linear fractional programming problems. International Journal of Mathematical, Engineering and Management Sciences, 3(3): 280.

[12] Ganesan, K., Veeramani, P. (2006). Fuzzy linear programs with trapezoidal fuzzy numbers. Annals of Operations Research, 143: 305-315. https://doi.org/10.1007/s10479-006-7390-1

[13] Guzel, N. (2013). A proposal to the solution of multiobjective linear fractional programming problem. Abstract and Applied Analysis. https://doi.org/10.1155/2013/435030

[14] Guzel, N., Sivri, M. (2005). Taylor series solution of multi-objective linear fractional programming problem. Trakya Üniversitesi Fen Bilimleri Dergisi, 6(2): 80-87.

[15] Luhandjula, M.K. (1984). Fuzzy approaches for multiple objective linear fractional optimization. Fuzzy Sets and Systems, 13: 11-23. https://doi.org/10.1016/0165-0114(84)90023-X

[16] Hasan, M.B., Acharjee, S. (2010). Solving LFP by converting it into a single LP.

[17] Kumar, A., Kaur, J., Singh, P. (2011). A new method for solving fully fuzzy linear programming problems. Applied Mathematical Modelling, 35(2): 817-823. https://doi.org/10.1016/j.apm.2010.07.037

[18] Kannan, V., Appasamy, S. (2023). Employing the Bellman-Ford algorithm with score functions to address the linear Diophantine fuzzy shortest path problem in network Analysis. Mathematical Modelling of Engineering Problems, 10(5): 1884-1892. https://doi.org/10.18280/mmep.100542

[19] Maleki, H.R., Tata, M., Mashinchi, M. (2000). Linear programming with fuzzy variables. Fuzzy Sets and Systems, 109(1): 21-33. https://doi.org/10.1016/S0165-0114(98)00066-9

[20] Prakash, Y., Appasamy, S. (2023). Optimal solution for fully Spherical Fuzzy Linear Programming Problem. Mathematical Modelling of Engineering Problems, 10(5): 1611-1618. https://doi.org/10.18280/mmep.100511

[21] Ramík, J. (2005). Duality in fuzzy linear programming: some new concepts and results. Fuzzy Optimization and Decision Making, 4: 25-39. https://doi.org/10.1007/s10700-004-5568-z

[22] Najafi, H.S., Edalatpanah, S.A. (2013). A note on "A new method for solving fully fuzzy linear programming problems". Applied mathematical modelling, 37(14-15): 7865-7867. https://doi.org/10.1016/j.apm.2013.02.039

[23] Saraswathi, A. (2019). A fuzzy-trapezoidal dematel approach method for solving decision making problems under uncertainty. In AIP Conference Proceedings, 2112(1): 020076. https://doi.org/10.1063/1.5112261

[24] Swarup, K. (1965). Letter to the editor linear fractional functionals programming. Operations Research, 13: 1029-1036. https://doi.org/10.1287/opre.13.6.1029

[25] Veeramani, C., Sumathi, M. (2014). Fuzzy mathematical programming approach for solving fuzzy linear fractional programming problem. RAIRO Operations research, 48(1): 109-122. https://doi.org/10.1051/ro/2013056

[26] Wan, S.P., Dong, J.Y. (2014). Possibility linear programming with trapezoidal fuzzy numbers. Applied Mathematical Modelling, 38(5-6): 1660-1672. https://doi.org/10.1016/j.apm.2013.09.006

[27] Zadeh, L.A. (1965). Fuzzy sets. Information and Control, 8(3): 338-353. https://doi.org/10.1016/S0019-9958(65)90241-X

NOMENCLATURE

LPP	Linear Programming Problem
LFPP	Linear Fractional Programming Problem
MOLFPP	Multi Objective Linear Fractional Programming Problem
TFN	Trapezoidal Fuzzy Number
FMP	Fuzzy Mathematical Programming