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Investigating Thermal Deflection in a Finite Hollow Cylinder Using Quasi-Static Approach and Space-Time Fractional Heat Conduction Equation

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ABSTRACT

This study embarks on an exploration of the thermal deflection characteristics of finite hollow cylinders, employing the space-time fractional heat conduction equation within a quasi-static framework. Heat application is executed on the upper surface of the cylinder, whilst maintaining a zero-temperature condition on the remaining boundaries. Temperature distribution across the cylinder is determined using the integral transform technique, a method ensuring precision in the computation of thermal responses. The discourse on thermal deflection is grounded in the principles of fractional diffusion wave theory, a contemporary approach providing deeper insights into heat conduction dynamics. Numerical analyses are presented, illustrating transient and long-range interaction responses of the hollow cylinder under various diffusion scenarios, namely sub-diffusion, normal diffusion, and super-diffusion.

1. INTRODUCTION

Fractional calculus has recently garnered significant attention in various engineering disciplines, including applications in proportional-integral-derivative controllers, fluid mechanics, bio-mathematics, viscoelasticity, electrochemistry, and signal processing. This surge in interest has catalyzed research in non-integer calculus. The concept of fractional-order calculus, while intriguing, presents substantial challenges in understanding its physical interpretations. Podlubny [1] has contributed to this field with a discussion on the geometric and physical exposition of fractional integration. A notable advantage of fractional differential equations is their nonlocal property, which offers a broader scope of application compared to traditional methods.

Riemann-Liouville's introduction of fractional derivatives has been instrumental in the evolution of fractional calculus. This concept has been extensively applied in mathematical formulations, offering unique and advantageous approaches. The field of fractional calculus has witnessed considerable research, driven by the interest in various methods of defining and utilizing fractional order derivatives. The adoption of fractional theory is attributed to its ability to represent delayed reactions to physical stimuli observed in nature, a feature not encapsulated by the generalized theory of thermoelasticity, which assumes immediate responses to such stimuli.

Sherief et al. [2] advanced the fractional-order theory of thermoelasticity, marking a significant contribution in this field. Povstenko [3-7] conducted in-depth studies on fractional thermoelasticity, employing the quasi-static theory. Raslan [8] addressed a specific problem related to a thick plate with symmetric temperature distribution. The work of Khobragade and Deshmukh [9] successfully tackled the inverse thermoelastic problem, providing a thorough analysis of quasi-static thermal deflection in circular plates. Deshmukh et al. [10] focused on a thin circular plate containing a heat source, employing a quasi-static methodology to ascertain the thermal deflection. Further contributions in this domain include those by Warbhe et al. [11, 12], who explored various problems within fractional order thermoelasticity using a quasi-static approach. Tripathi et al. [13] examined fractional order thermoelastic deflection in thin circular plates with constant temperature distribution. Additionally, Tripathi et al. [14] solved a problem involving a heat source inducing a fractional order generalized thermoelastic response in a half space, which changed periodically. Recently, Warbhe [15] investigated simply supported rectangular plates, focusing on determining thermal stresses through thermal bending moments, facilitated by a time-dependent fractional derivative. Ezzat et al. [16, 17], El-Sayed and Gaber [18] delved into a range of problems in fractional-order thermoelasticity, expanding the scope of research in this specialized area.

Research on thermoelasticity in fractional-order space-time domains has attracted attention from various scholars. Fil' Shtinskii et al. [19] successfully solved the equation describing heat flow in fractional space and time, further analyzing its thermoelastic behavior in a one-dimensional half-space scenario. Povstenko [20] has contributed significantly to the discourse on space and time fractional diffusion equations. Sherief et al. [21] explored the realm of two-dimensional half-space problems, delving into the novel theory of fractional order thermoelasticity. Hussein [22] focused on fractional order thermoelastic problems in the context of an infinitely long solid circular cylinder. In a similar vein, Zhang and Li [23] examined the transient response of a hygrothermoelastic cylinder, grounding their analysis in fractional diffusion wave theory. These studies collectively underscore the growing interest and diversity in research approaches within the field of fractional-order thermoelasticity.

In this study, a mathematical formulation is presented to describe heat flow in materials characterized by spatial and temporal variations. This formulation employs a time fractional differential operator to encapsulate memory effects, while the space fractional differential operator is used to model long-range interactions. The practical applications of fractional calculus have motivated the development of a mathematical model that integrates the space-time fractional differential operator. This model is constructed using a quasi-static approach to examine its thermoelastic effects.

The focus of the current study is a hollow cylinder subject to arbitrary temperature gradients, with the application of space-time fractional order derivatives. The problem is addressed using the integral transform technique. The discussion centers on thermal deflection, analyzed through the lens of time and space fractional order parameters, and these parameters are elucidated graphically. The mathematical model is specifically tailored for pure copper material and has undergone testing using Mathcad Prime 1.0 version. This approach provides a comprehensive understanding of the thermoelastic behavior of materials under fractional order space-time conditions.

2. FORMULATION OF THE PROBLEM

Consider a two-dimensional heat conduction equation in a hollow cylinder in the space-time domain, taking into account fractional order effects with dimensions $b \le r \le c$; $0 \le z \le h$, $r>0$, $t>0$. Maintained temperature zero at the inner boundary, outer boundary and on the lower surface, respectively. The temperature in terms of $\frac{Q(t)\delta(r)}{2\pi r}$ is prescribed at the upper surface. The solution to the problem is obtained by using the integral transform technique.

The definition of Caputo type fractional derivative is given by Povstenko [20]:

$$\frac{\partial^\alpha f(t)}{\partial t^\alpha} = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\tau)^{n-\alpha-1} \frac{d^n f(\tau)}{d\tau^n} d\tau, \quad n-1<\alpha<n \tag{1}$$

The Laplace transform for Caputo derivative is given as Warbhe et al. [12]:

$$L\left\{\frac{\partial^\alpha f(t)}{\partial t^\alpha}\right\} = s^\alpha f^*(s) - \sum_{k=0}^{n-1} f^{(k)}(0^+) s^{\alpha-1-k}, \quad n-1<\alpha<n \tag{2}$$

where, s is the Laplace transform parameter.

The formula of finite Riesz fractional derivative is defined in El-Sayed and Ezzat [24] as:

$$\frac{\partial^\beta \varphi(z)}{\partial z^\beta} = \frac{I_{0+}^{2-\beta} \frac{d^2\varphi}{dz^2} + I_{-}^{2-\beta} \frac{d^2\varphi}{dz^2}}{2\cos\frac{(2-\beta)\pi}{2}} \text{ for } \beta = 2 \tag{3}$$

where, $I_{0+}^{\omega}\varphi(z) = \frac{1}{\Gamma(\omega)} \int_0^z (z-\zeta)^{\omega-1} \phi(\zeta) d\zeta$, $I_{-}^{\omega}\varphi(z) = \frac{1}{\Gamma(\omega)} \int_z^\infty (\zeta - z)^{\omega-1} \phi(\zeta) d\zeta$, are the Riemann-Liouville fractional integrals, $\omega>0$.

The space-fractional derivative of order β is defined in Povstenko [25] as pseudo-differential operator with the following rule for the Fourier transform:

$$F\left\{\frac{d^\beta \varphi(z)}{d|z|^\beta}\right\} = -|\xi|^\beta F\{\varphi(z)\}, \tag{4}$$

where, ξ is the Fourier transform variable.

The equation of space-time fractional-order heat conduction with arbitrary temperature for a hollow cylinder in the domain defined as $b \le r \le c$, $0 \le z \le h$:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^\beta T}{\partial z^\beta} = \frac{1}{a}\frac{\partial^\alpha T}{\partial t^\alpha}, \quad 0<\alpha\le 2, 1<\beta<2, r>0, t>0 \tag{5}$$

with boundary conditions

$$T = 0, \text{ at } r = b, 0 \le z \le h, \tag{6}$$

$$T = 0, \text{ at } r = c, 0 \le z \le h, \tag{7}$$

$$T = 0, \text{ at } z = 0, b \le r \le c, \tag{8}$$

$$T = \frac{Q(t)\delta(r)}{2\pi r}, \text{ at } z = h, b \le r \le c, \tag{9}$$

and initial conditions

$$T = 0, \text{ when } t = 0, \quad 0<\alpha<1, \tag{10}$$

$$\frac{\partial T}{\partial t} = 0 \text{ when } t = 0, 1<\alpha<2, \tag{11}$$

where, a=thermal diffusivity, r=radius, δ=Dirac-delta function.

The temperature on the surface ($z=h$) chosen in terms of $\frac{\delta(r)}{2\pi r}$ with the inclusion of function $Q(t)$ jumps for $t>0$.

Thermal deflection ω(r, t).

The differential equation satisfies ω(r, t) which is given in reference to Warbhe et al. [12] as:

$$\nabla^4 \omega = -\frac{\nabla^2 M_T}{D(1-\nu)} \tag{12}$$

where, M_T=thermal moment and it is defined as:

$$M_T = a_t E \int_0^h z.T(r,z,t) dz \tag{13}$$

and D=flexural rigidity of the cylinder defined as:

$$D = \frac{Eh^3}{12(1-\nu^2)} \tag{14}$$

where, E=Young's modulus, a_t=coefficient of the linear thermal expansion and v=Poisson's ratio.

As the hollow cylinder is fixed on the inner and outer edges, we have:

$$\omega = 0 \text{ at } r = b \text{ and } r = c. \tag{15}$$

3. SOLUTION OF THE PROBLEM

3.1 Temperature distribution function

On applying Hankel transform, finite Fourier Sine transform, and Laplace transform technique and their inversions respectively; to the system of Eqs. (5) to (11), one obtains the temperature distribution as:

$$T(r,z,t) = \frac{2a}{h}\sum_{p=1}^{\infty}\sum_{m=1}^{\infty} K_0(\eta_p, r)\sin(\xi_m z)\frac{1}{\xi_m}(-1)^{m+1} \times [1-\cos(\xi_m h)]\left(\frac{m\pi}{2}\right)^{\beta-1} \times \left(\int_0^t t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]Q(t-\tau)d\tau\right) \tag{16}$$

where, Finite Hankel transforms and its inversion over the spatial variable r, in the range $b \le r \le c$ defined in the study of Sneddon [26] as: $\overline{T}(\eta_p, z,t) = \int_{r=b}^{c} r.K_0(\eta_p, r)\overline{T}(r,z,t)\,dr$, $T(r,z,t) = \sum_{p=1}^{\infty} K_0(\eta_p, r)\,\overline{T}(\eta_p, z,t)$.

The finite Fourier Sine transform and its inversion are defined in Povstenko [25] as: $F\{\overline{T}(\eta_p, z,t)\} = \overline{\overline{T}}(\eta_p, \xi_m, t) = \int_0^h \overline{T}(\eta_p, z,t)\sin(\xi_m z)\,dz$, $F^{-1}\{\overline{\overline{T}}(\eta_p, \xi_m, t)\} = \overline{T}(\eta_p, z,t) = \frac{2}{h}\sum_{m=1}^{\infty}\overline{\overline{T}}(\eta_p, \xi_m, t)\sin(\xi_m z)$, and $\xi_m = \frac{m\pi}{h}$, $m = 1,2,3,\ ....$

The Laplace transform is defined as: $L\left[\overline{\overline{T}}(\eta_p, \xi_m, t)\right] = \overline{\overline{T}}^{*}(\eta_p, \xi_m, t) = \int_0^{\infty} e^{-st}\,\overline{\overline{T}}(\eta_p, \xi_m, t)\,dt$.

* denotes the Laplace transform.

Also, $L^{-1}\left[\frac{1}{s^{\alpha}+a\left(\eta_p^2+\xi_m^{\beta}\right)}\right] = t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]$, $E_{\alpha,\alpha}(.)$ is the generalized Mittag - Leffler function.

The normalized eigen function $K_0(\eta_p, r)$ is defined as: $K_0(\eta_p, r) = \frac{\pi}{\sqrt{2}}\frac{\eta_p . J_0(\eta_p c).Y_0(\eta_p c)}{\left[1-\frac{J_0^{\,2}(\eta_p c)}{J_0^{\,2}(\eta_p b)}\right]^{\frac{1}{2}}}\left[\frac{J_0(\eta_p r)}{J_0(\eta_p c)} - \frac{Y_0(\eta_p r)}{Y_0(\eta_p c)}\right]$, and the roots η_1, η_2, ... are obtained from equation $\frac{J_0(\eta b)}{J_0(\eta c)} - \frac{Y_0(\eta b)}{Y_0(\eta c)} = 0$.

3.2 Thermal deflection

Using Eq. (16) in Eq. (13), we have obtained the thermal moment as:

$$M_T = -a_t\,E\sum_{p=1}^{\infty}\sum_{m=1}^{\infty} 2h\,K_0(\eta_p, r)\frac{1}{\xi_m^2}\cos(\xi_m h) \times [1-\cos(\xi_m h)]\left(\frac{m\pi}{2}\right)^{\beta-2}(-1)^{m+1} \times \left(\int_0^t t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]Q(t-\tau)d\tau\right) \tag{17}$$

Assuming the solution of Eq. (12), this satisfies the Eq. (15), as:

$$\omega(r,t) = \sum_{p=1}^{\infty} C_p(t)\left[\frac{J_0(\eta_p r)}{J_0(\eta_p c)} - \frac{Y_0(\eta_p r)}{Y_0(\eta_p c)}\right] \tag{18}$$

Now,

$$\nabla^4\omega = \left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)^2\sum_{p=1}^{\infty} C_p(t)\left[\frac{J_0(\eta_p r)}{J_0(\eta_p c)} - \frac{Y_0(\eta_p r)}{Y_0(\eta_p c)}\right] \tag{19}$$

Using results $\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)J_0(\eta_p r) = -\eta_p^2 J_0(\eta_p r)$ and $\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)Y_0(\eta_p r) = -\eta_p^2\,Y_0(\eta_p r)$, in Eq. (19), we have:

$$\nabla^4\omega = \sum_{p=1}^{\infty} C_p(t)\,\eta_p^4\left[\frac{J_0(\eta_p r)}{J_0(\eta_p c)} - \frac{Y_0(\eta_p r)}{Y_0(\eta_p c)}\right]$$

$$\nabla^2 M_T = a_t\,E\sum_{p=1}^{\infty}\sum_{m=1}^{\infty} 2h\,\eta_p^2\,K_0(\eta_p, r)\frac{1}{\xi_m^2}\cos(\xi_m h) \times [1-\cos(\xi_m h)]\left(\frac{m\pi}{2}\right)^{\beta-2}(-1)^{m+1} \times \left(\int_0^t t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]Q(t-\tau)d\tau\right). \tag{20}$$

Substituting Eqs. (19), (20) into Eq. (12), and simplifying, one obtains:

$$C_p(t) = -\frac{a_t\,E}{D(1-v)} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty} 2h\frac{1}{\xi_m^2}\cos(\xi_m h)\left[1-\cos(\xi_m h)\right]\left(\frac{m\pi}{2}\right)^{\beta-2}(-1)^{m+1} \times \frac{1}{\eta_p}\frac{\pi}{\sqrt{2}}\frac{J_0(\eta_p c).Y_0(\eta_p c)}{\left[1-\frac{J_0^{\,2}(\eta_p c)}{J_0^{\,2}(\eta_p b)}\right]^{\frac{1}{2}}} \times \left(\int_0^t t^{\alpha-1}E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right]Q(t-\tau)\,d\tau\right) \tag{21}$$

Substituting Eq. (21) into Eq. (18), the thermal deflection obtained as:

$$\omega(r,t) = -\frac{a_t E}{D(1-\nu)} \sum_{p=1}^{\infty}\sum_{m=1}^{\infty} 2h \frac{1}{\xi_m^2} \cos(\xi_m h)\left[1-\cos(\xi_m h)\right]$$
$$\times \left(\frac{m\pi}{2}\right)^{\beta-2} (-1)^{m+1} \frac{1}{\eta_p} \frac{\pi}{\sqrt{2}} \frac{J_0(\eta_p c).Y_0(\eta_p c)}{\left[1-\frac{J_0^{\,2}(\eta_p c)}{J_0^{\,2}(\eta_p b)}\right]^{\frac{1}{2}}} \left[\frac{J_0(\eta_p r)}{J_0(\eta_p c)} - \frac{Y_0(\eta_p r)}{Y_0(\eta_p c)}\right] \tag{22}$$
$$\times \left(\int_0^t t^{\alpha-1} E_{\alpha,\alpha}\left[-a\left(\xi_m^{\beta}+\eta_p^2\right)t^{\alpha}\right] Q(t-\tau)\,d\tau\right)$$

4. NUMERICAL COMPUTATIONS

To prepare the mathematical model for different parameters and functions for a copper (pure) material to discuss the fractional-order thermal deflection, we choose the following values defined (see reference Warbhe et al. [12]) as, b=1 m, c=2 m, z=0.4 m, w=5, t=5 sec., ν=0.35, h=0.4, τ=4.5 sec., μ=26.67 GPa, a=112.34×10^{-6} m^{2}s^{-1}, a_t=16.5×10^{-6} K.

The distribution of the function jumps time t>0 for that we set the function $Q(t)$, as $Q(t)=e^{-wt}$, t>0, w>0.

5. FIGURES

Temperature-time dependence for β=1.75 and various values of α is presented in Figure 1. For the distinct values of α increases, it is observed that the impact of the region of heat disturbances is fluctuating throughout the region 1≤r≤2, which indicates the non-uniform pattern with concerning to radius. It is observed that for a small value of α the temperature equilibrium is faster and interpolating the classical heat conduction equation when α=0, α=1, α=2 which represent the liberalized heat conduction equation, diffusion equation, wave equation, it is shown that the sub-diffusion (0<α<1) implies an anomalous heat conduction with the convergent thermal conductivity and super-diffusion (1<α<2) implies an anomalous heat conduction with divergent thermal conductivity. When α=1, classical heat conduction occurs, which is prescribed as Fourier law of heat conduction.

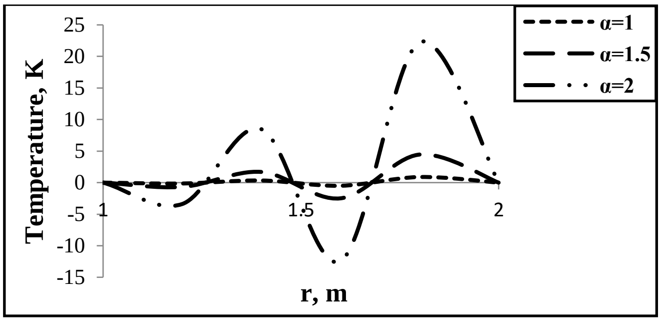

Figure 1. Effect of temperature on r for β=1.75 and distinct values of α

Temperature-space dependence for α=2 and various values of β=1,1.5,2 is presented in Figure 2, which shows the interpolation of the classical heat conduction equation. When α=1, β=2 Eq. (1) becomes diffusion equation and when α=2, β=2 the Eq. (1) becomes wave equation. The classical heat equation which indicates the temperature is in the form of a wave equation.

Thermal deflection time dependence for β=1.75 and various values of α is presented in Figure 3. From the graph, it is observed that thermal deflection radial direction shows weak, moderate, super conductivity for the different values of α=1,1.5,2, which predict the memory impact on the hollow cylinder.

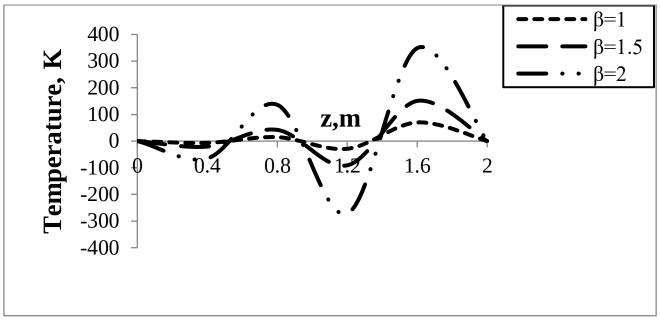

Figure 2. Effect of temperature on z for α=2 and distinct values of β

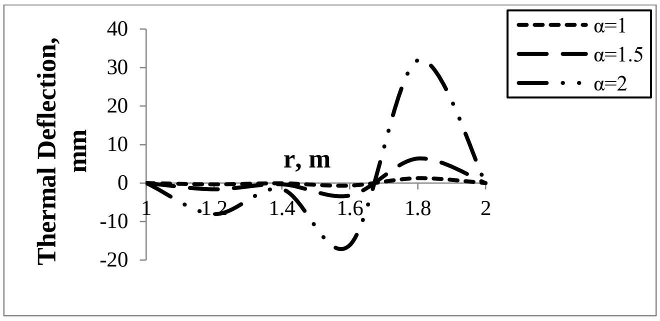

Figure 3. Dependence of thermal deflection on r for β=1.75 and distinct values of α

Thermal deflection space dependence for α=2 and various values of β is presented in Figure 4. Thermal deflection is affected due to the inclusion of the jump function at the upper surface of the cylinder. Therefore, slow variation occurs in the region 0<z<1.5 and the peak of positive thermal deflection is observed in the region 1.5<z<2 with the distinct spatial fractional parameter β.

The variation of thermal deflection depends not only on the values of temperature in the neighbourhood of the selected point, but also depends on its value in a remote point. Therefore, the significant differences in thermal deflection distribution take place only for the values of a spatial variable. When α=1 and β=2, the fractional heat conduction equation turns into Fourier law of heat equation.

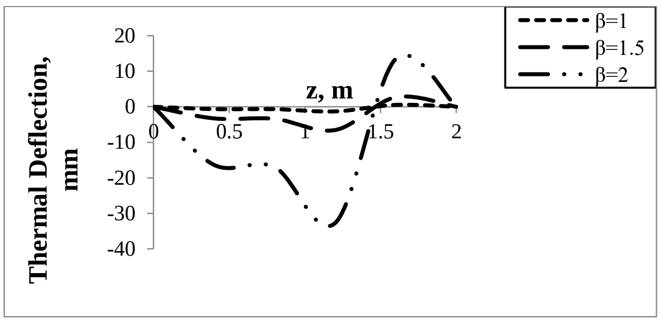

Figure 4. Dependence of thermal deflection on z for α=2 and distinct values of β

6. CONCLUSIONS

In this study, we consider the space-time heat conduction equation with the fractional order parameter in a hollow cylinder. Here, the operator $\frac{\partial^\alpha}{\partial t^\alpha}$ represent Caputo fractional derivative which predict memory effects whereas the fractional parameter $\frac{\partial^\beta}{\partial t^\beta}$ is finite Riesz fractional derivative predicted the long range interaction.

In this problem considering a finite hollow cylinder and discuss the thermal deflection behavior with the help of temperature distribution in terms of space and time fractional order using a quasi-static approach. The temperature maintained zero at the inner and outer radii whereas the arbitrary temperature defined in Eq. (9) is prescribed on the upper surface of the hollow cylinder. As per as the condition (10) and (11) must be needed to reflect sub diffusion, normal diffusion and super diffusion it means which interpolates the classical heat conduction equation. With the help of temperature distribution the thermal deflection has been studied and illustrated graphically.

The fractional-order theory foresees a delayed response to physical stimuli, while the space fractional differential operator effectively accounts for long-range interactions, aligning with observations in the natural world. To sum up, the outcomes detailed in this article are expected to be of value to researchers in the field of material sciences, as well as to material designers and those dedicated to advancing the theory of thermoelasticity through a quasi-static approach that incorporates fractional calculus.

The main aim to study this problem is to interpolate the classical heat conduction equation for the different conductivity. It is observed that graphically, the variation of thermal deflection shows weak, moderate and superconductivity for different space-time fractional parameters with fixed time t=5.

No one studied the space-time heat conduction equation in a finite length hollow cylinder and the impact of thermal stresses. Therefore, we claim that this is new and novel contribution to this field.

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NOMENCLATURE

r	radius, *m*
z	thickness, *m*
b	inner radius of the disk, *m*
c	outer radius of the disk, *m*
a	thermal diffusivity, m^2s^{-1}
a_t	coefficient of linear thermal expansion, $\frac{1}{K}$
$T(r, z, t)$	temperature distribution function, *K*
$\omega(r, t)$	thermal deflection, *mm*
μ	Lamé constant
v	Poisson ratio
E	Young's modulus

Greek symbols

α	fractional order parameter for time
β	fractional order parameter for space