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An Innovative Method for Attribute Reduction: Weighted Attribute Concepts for Probabilistic Analysis of Decision Attributes

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ABSTRACT

Attribute reduction, a seminal aspect of data analysis, primarily hinges on the indiscernibility matrix. Previous studies have explored the weight of an attribute via various methods, yet achieving optimal reduction remains elusive. This study proposes a novel approach to optimal reduction, leveraging the concept of weighted attributes based on the probability values of core and non-core elements. This approach scrutinizes the accuracy of both core and non-core attributes, thereby enhancing our comprehension of the object's attributes. The weighted attribute concept is derived in light of entropy information and the indiscernibility matrix. A discernibility matrix aids in ascertaining the reduct, whereas entropy information facilitates the analysis of the weight of uncertain data. By deploying decision attributes, we derive the core and its corresponding probabilistic value, establishing an algebraic structure as an ordered pair of objects with associated weight concepts. This structure further enables the investigation of the consistency set and the join (meet) irreducible set employing weighted attribute concepts. Ultimately, optimal reduction is determined by the weight of non-core elements, allowing a comprehensive analysis of the information system and procurement of its essential attributes for decision-making. The proposed concept of weighted attributes is elucidated using a biological dataset.

1. INTRODUCTION

The concept of an indiscernibility relation, along with features of discernibility and indiscernibility matrices, was first introduced by Pawlak [1]. Yao [2] advanced this concept, constructing three regions - positive, negative, and boundary - to illustrate the advantages of the three-way decision rule in the rough set model and decision theory. The relationship between the error level of tolerance and incorrect decision cost was also elucidated.

Wu et al. [3] explored the fundamental notions of formal concept analysis, employing granular concepts. The object-oriented concept lattice's properties were examined, and an algorithm was proposed to probe the concept of lattice and its properties. Li et al. [4] established a multi-granular decision rule for the decomposition of the concept lattice, leading to the generation of a new algorithm for disjoint normal functions.

Ezhilarasi et al. [5] advanced the classification of rough set theory for uncertain datasets using the weighted attribute concept. This concept facilitated analysis of a decision rule's subset, and the results were accentuated by prediction accuracy for the datasets. Kang et al. [6] proposed employing an algebraic structure and lattice structure to investigate knowledge of decision dependency, and the potential applications of decision dependency were discussed in a technology that merges rough set theory and formal concept analysis.

The formal concept of reliability engineering problems was determined by Rocco et al. [7], who offered fundamental views on formal concepts pertaining to the field of reliability. Qian et al. [8] extended formal concepts to three-way decisions, and the characteristics of formal context were investigated using the isomorphism relationship between type I and type II dual interactable.

Huang and Bian [9] applied a formal concept analysis to online tour planning, while Yao and Chen [10] introduced the formal concept analysis for two definable sets and defined an approximation operator. They also examined the interpretation of lattice theory and set theory. Srirekha et al. [11] investigated the projection of a distributive lattice by defining the trivial ordered set and its properties. The lattice homomorphism was analyzed based on an equivalence relation, and its condition was verified.

Jiang et al. [12] analyzed the relationship between generalized weighted averages and its properties using hesitant fuzzy, while Xiao and He [13] discussed the properties of the formal concept lattice. They also calculated the weight for each attribute using the degree of importance in rough set theory and entropy information. Shi and Chen [14] proposed a new method on rough set theory and granular theory, characterizing the effectiveness of weight. Li et al. [15] experimented with a combination of fundamental geographic information data and proposed numerous applications for geo-ontologies merging. Bao et al. [16] generalized a new method on attribute weight for a significant individual degree of each conditional attribute.

Weighted attribute concepts have been extensively applied to attribute reduction. The positive region and degree of individual attributes have been investigated, along with their applications in diverse fields. Combining entropy information with these concepts, a formal concept lattice was defined. In this paper, an algebraic structure for the weighted attribute concept is defined, and attribute reduction by core and non-core elements is investigated. Decision attributes are utilized to estimate the probability of the core, demonstrating the effectiveness of weighted attribute concepts. Optimal reduction features were determined using weighted attribute concepts.

The remainder of this paper is organized as follows: Section 2 discusses the basic knowledge about the formal context that generates the granular. Section 3 provides the attribute reduction obtained from the indiscernibility matrix. In Section 4, the weighted attribute concept is introduced with the knowledge of core and non-core elements. Section 5 demonstrates the significance of the proposed work through a real-time application with the implementation of weighted attribute concepts.

2. PRELIMINARIES

In this section, we analyze the basic information about the formal concept lattice that is implemented in rough set theory. An algebraic structure has been defined in the form of a formal context and analysis has been made on the properties using the condition of the join (meet) operator. An indiscernibility matrix is examined with the knowledge obtained from the reduct and core.

2.1 Formal concept analysis

Definition 2.1.1: Let us consider the formal context (O, At, R) where O and At are the finite non-empty set of objects and attributes respectively, R is a binary relation which is a subset of a cartesian product of object and attributes i.e., ($R\subseteq O\times At$) where the elements are represented as $(g, m)\in R$, $\forall g\in O$, $m\in At$ [8, 11, 12].

Let a pair of (G, M) be granular that define on an operator *. If $G\subseteq O$ and $M\subseteq At$ in formal context (O, At, R) then:

$$G^* = \{m | m \in At, \forall g \in G, gRm\}$$
$$M^* = \{g | g \in O, \forall m \in M, gRm\} [11,12]$$

Definition 2.1.2: Let (O, At, R) be a formal context. For any $G\subseteq O$, $M\subseteq At$, * be an operator on *: $2^G\rightarrow 2^M$ as follows:

$$G^* = \{m \in At | m^* \subseteq G\}$$
$$M^* = \{g \in O | g^* \cap M \neq \emptyset\}$$

Thus (G, M) is the extent and intent of the formal context (O, At, R) [5].

Definition 2.1.3: Let (O, At, R) be a formal context if $G^*=M$ and $M^*=G$ are the set of attributes and objects respectively then the extent and intent of the formal concept are defined as (G, M).

A partially ordered relation is defined in a formal concept lattice ($L(O, At, R)$,$\leq$) with respect to the meet ∧ and join ∨. If for any ordered pairs $(G_1, M_1) \leq (G_2, M_2) \Leftrightarrow G_1 \subseteq G_2 (\Leftrightarrow M_2 \subseteq M_1)$ is represented as:

$$(G_1, M_1) \wedge (G_2, M_2) = (G_1 \cap G_2, (M_1 \cap M_2)^{**})$$
$$(G_1, M_1) \vee (G_2, M_2) = ((G_1 \cup G_2)^{**}, M_1 \cup M_2) [11,13]$$

Definition 2.1.4: Consider (G_1, M_1) and (G_2, M_2) be the two formal context on (O, At, R), If R is a binary relation which can be defined as $(G_1, M_1)\leq(G_2, M_2)$ then there exist a partial relation for $((O, At, R),\leq)$ such that $(G_1, M_1)\leq(G_3, M_3)\leq(G_2, M_2)$. Hence, ($G_1$, M_1) is the son concept of (G_2, M_2) and (G_2, M_2) is the mother concept of (G_1, M_1) which implies $(G_1, M_1)\leq(G_2, M_2)$ [12].

Hence the lattice structure has been analysed using the granular context and this generates the Hasse diagram that depicted the reduction concepts.

2.2 Reduct and core

Definition 2.2.1: Let us consider (G, M) that represents the granular on the formal context, If D is a subset of an attribute M (i.e., $D \subseteq M$) then it satisfies the isomorphic condition $L(G, M, R_D) \cong L(G, M, R)$. Hence D is said to be the consistent set on the granular (G, M). Therefore, if an element $d\in D$ such that $L(G, D - \{d\}, R_{D-\{d\}}) \cong L(G, M, R)$ then D is said to be the reduct and the core can be obtained from the intersection of all the reduct [8].

2.3 Indiscernibility matrix and function

Definition 2.3.1: Let (G, M) be granular in the formal context, if the indiscernibility matrix is defined as IDS=$IDS(g, m)$ then the matrix represents the Cartesian product of the object and attribute where $(g, m) \in (G, M)$. i.e., $IDS\ (g, m) = \{f \in At | R(g_1, m_1) = R(g_2, m_2)\}$.

Definition 2.3.2: The indiscernibility function can be defined as $IDS^M = \wedge\{\vee IDS(g, m) | \forall g \in G, m \in M, IDS(g, m) \neq \emptyset\}$ which indicate disjunction and conjunction operator that can be distinguished to obtain the set of all attributes.

From the above representations, an ordered pair of object and attribute were analysed to obtain the basic knowledge about the formal context and generalized granular concept. Thus, this granular is further extended to determine its reduct and core using the indiscernibility function.

3. GRANULAR REPRESENTATION ON FORMAL LATTICE FOR THE REDUCTION OF ATTRIBUTE USING INDISCERNIBILITY MATRIX

Here, we examine the values V_a={0, 1}, from an information table that describes an indiscernibility relation using these values. Thus, the granular has been represented in a hasse diagram and an indiscernibility matrix is investigated to obtain the reduct.

3.1 Representation of indiscernibility table

Definition 3.1: Consider an information system (O, At, V, R) where the non-empty finite set of Object $G \subseteq O$, the attribute $M \subseteq At$, V is the value of the cartesian product of objects and attributes, and $R \subseteq O \times At$

$$V_a = \begin{cases} 1; if\ gRm \in V \\ 0; if gRm\ \notin V \end{cases}$$

where $g \in G$ and $m \in M$.

Definition 3.2: Let us consider $(O, At, V_a = \{0,1\}, f, R)$ to be the formal context, where *O* and *At* be the non-empty set of objects, and attributes respectively, V_a is the value, $f: O \times At \rightarrow V_a$ is a function and R be the binary relation such that $R \subseteq O \times At$.

Definition 3.3: Let (*O, At, V_a, R*) be a formal context then the value of the indiscernibility relation can be defined as:

$$R_{(i,j)} = \{(1,1) \in O \times At | \forall\ 1 \in V_a, (i,j) \in (O \times At)\}$$

where, throughout this paper, we represent (*i, j*) as {*ij*}.

Definition 3.4: Let (*O, At, V_a, R*) be a formal context, consider $G \subseteq O$ and $M \subseteq At$ then the Cartesian product of object and attribute is defined as:

$$R_{ij} = \{(i,j) \in G \times M | \forall m \in M, R_m(i) = R_m(j)\}$$

Definition 3.5: Let (*G, M, V_a*={1,0}, *R*) be the formal context, where *G* is a set of object {$g_1, g_2, .., g_n$} and *M* is a set of attribute {$m_1, m_2, ... m_k$} then V_a is the value which defined as:

$$V_{ij} = \begin{cases} 1; if\ g_i \in m_j \\ 0; if g_i \notin m_j \end{cases}$$

3.2 Indiscernibility matrix

Definition 3.6: The indiscernibility matrix is defined as $I_{ij} = \{v_{ij} \in G \times M | R_{ij} = R_{kl}\}$. If the matrix satisfies the symmetric condition $I_{ij}=I_{ji}$ then take for consideration as $I_{ii}=M$, otherwise (*i, j*) is a pair of discernible values and $I_{ij} \neq \emptyset$.

$$IDS^{Fun} = \wedge\{\vee R_{ij} | \forall i,j \in O, R_{ij} \neq \emptyset\}$$

Example 3.1: Consider an information table with the 5 object along with the attribute *price, Room, Furniture*, also with decision attributes *Prestigious flat* consisting of "Yes" and "No".

Table 1. Information table

O/At	Price	Room	Furniture	PF
1	High	1	No	Yes
2	High	3	Yes	Yes
3	High	1	No	No
4	High	2	No	Yes
5	Low	2	Yes	No

Table 2. Indiscernibility table

	a	b	c	d	e	f	g
1	1	0	1	0	0	0	1
2	1	0	0	0	1	1	0
3	1	0	1	0	0	0	1
4	1	0	0	1	0	0	1
5	0	1	0	1	0	1	0

From Tables 1 and 2, objects 1 and 3 are considered as inconsistent data. Hence, an indiscernibility matrix has been framed from Figure 1 for the existence of an indiscernibility function (from Figure 2) to obtain the reduct as:

$$IDS^{Flat} = \{M\} \wedge \{a\} \wedge \{a \vee c \vee g\} \wedge \{a \vee g\} \wedge \{f\} \wedge \{d\} = \{a \wedge f\} \vee \{a \wedge d\}.$$

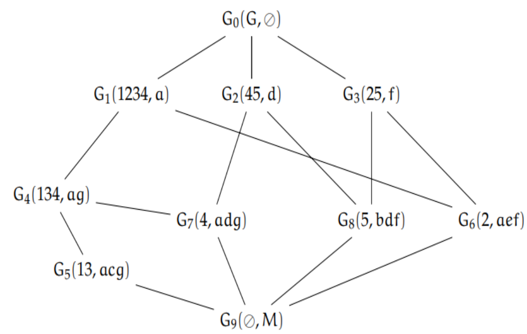

Figure 1. Hasse diagram from Table 2

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & M & a & acg & ag & \emptyset \\ 2 & \emptyset & M & a & a & f \\ 3 & \emptyset & \emptyset & M & ag & \emptyset \\ 4 & \emptyset & \emptyset & \emptyset & M & d \\ 5 & \emptyset & \emptyset & \emptyset & \emptyset & M \end{array}$$

Figure 2. Indiscernibility matrix

Thus, the two reduct are T_1={$a \wedge f$} and T_2={$a \wedge d$} can be represented as hasse diagram (Figure 3).

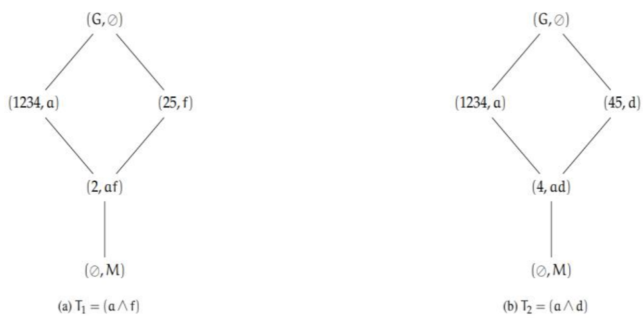

Figure 3. Reduct

4. ATTRIBUTE REDUCTION USING WEIGHTED ATTRIBUTE CONCEPTS

This section highlights the weighted attribute concept using entropy information and the probabilistic values of core and non-core elements. Thus, we introduce an algebraic structure that consists of objects and attributes with their weighted concept. Also, an optimal reduction has been discussed using the weighted attribute concept.

Definition 4.1: Let (O, At, V_a, R) be a formal context, where *at* represents both the conditions and decision attributes (i.e., $At=cds \cup des$). Hence the probability value of the decision attribute is denoted as τ^{des}.

4.1 Entropy information with weighted attribute concepts

Definition 4.2: Let (G, M, V_a, R) be the formal context. If the probability of an object "i" that holds to an attribute "j" then $\alpha(v_{ij})$ is represented as the regular information of attributes provided by the set of an object. Thus, the entropy information $\alpha(v_{ij})$ is defined as:

$$\alpha(v_{ij}) = -\sum p(v_{ij}) log_2 (v_{ij})$$

Definition 4.3: Let (G, M, V_a, R) be the formal context, then the attribute M provides the information about the condition and decision attributes. The probabilistic value of core has been determined with the decision attributes based on the importance of indispensable function which is referred as:

$$core[v_{ij}] = \sum p(core(v_{ij})) * \tau^{des}$$

Definition 4.4: Let (G, M, V_a, R) be the formal context, then the weighted attribute concept has been defined as an average of entropy information and probabilistic value of core with the decision attributes. Hence, the weighted attribute is denoted as:

$$w_i = \frac{core[v_{ij}] * \alpha(v_{ij})}{\{\sum \{core[v_{ij}] * \alpha(v_{ij})\}}$$

Definition 4.5: Let (G, M, R) be a formal concept then $(G,(M, W_i))$ be a join (meet) irreducible ordered pair if $D \subseteq M$ which is a consistency set where the weighted attribute concept can be redefined as the sum of attribute weight from the reduct.

Definition 4.6: Let $T_1, T_2, T_3, \ldots, T_n$ be the reduct of the formal concept $(G,(M, W))$. If $W_i \in T_1, W_j \in T_2$ then there exist the non-core element $w_i \in W_i$ and $w_j \in W_j$ and $w_j \geq w_i$ such that T_2 be the optimal reduct.

Theorem 4.1: Let (G, M, R) be the formal context. If $(G,(M, W_i))$ be the weighted attribute concept then it satisfies $L_0(G,M,R)=L_0(G,(M,W_i))$.

Proof. Since (G, M, R) be a formal concept then there exist $G^* = M$ and $M^* = G$ such that $G \subseteq O$ and $M \subseteq At$ from Definition 2.1.2.

If $(g, m) \in L_0(G, M, R)$ then the element $g^*=m$ and $m^*=g$ generate the weight for the attribute in each join(meet) irreducible ordered pair. Hence $((m^*)^* \in L_0(G, M, R)$ which implies $(g, m) \in L_0(G, (M, W_i))$. Thus, $L_0(G, M, R) \subseteq L_0(G, (M, W_i))$.

Conversely, D is a consistency set and $D \subset M$ this shows that $L_0(G, M, R) \supseteq L_0(G, M, R_D)$ therefore $L_0(G, M, R) \supseteq L_0(G, (M, W_i))$.

Hence $L_0(G,M,R)=L_0(G,(M,W_i))$.

Theorem 4.2: Let (G, M, R) be the formal context. If $(G,(M,W_i))$ be the weighted attribute concept then it satisfies the isomorphic condition as $L_0 (G, M, R) \cong L_0(G, (M, W_i))$.

Proof. Let $\chi : L_0(G, M, R) \to L_0(G, (M, W_i))$ for any $(G, M) \in L_0(G, M, R), \chi(G, M) = (G, M^{**})$. Hence from the above Theorem 4.2 $L_0 (G, M, R) = L_0(G, (M, W_i))$. It is easy to prove the isomorphism mapping. Therefore $L_0(G, M, R)$ and $L_0(G, (M, W_i))$ are isomorphic. i.e., $L_0(G, M, R) \cong L_0(G, (M, W_i))$.

Lemma 4.3: Consider (G, M, R) to be a formal concept. If $(g_i,(m_i,w_i)),(g_j,(m_j,w_j))$ be a join (meet) irreducible element then the ordered pair implies that $g_i \subseteq g_j$, $m_j \subseteq m_j$ and $w_j \leq w_i$.

Lemma 4.4: Let (G, M, R) be a formal context and $D \subseteq M$. If D is consistency set then $(g_i,(m_i,w_i)) \subseteq (g_j,(m_j,w_j))$ which implies $(g_i, m_i) \subseteq (g_j, m_j)$ and $w_j \leq w_i$.

Proof. Let (G, M, R) be a formal context and if D is a consistency set and $D \subseteq M$ then the formal concept of lattice preserves the consistency set of Join (meet) irreducible element. By Definition 4.5 and Lemma 4.3, There exist $(g_i, m_i) \subseteq (g_j, m_j)$, $\forall$ (g_i, m_i), $(g_j, m_j) \in (G,M)$ such that the weight of the attribute concept will be $w_j \leq w_i$. Hence the reduct of a formal concept has a weighted attribute concept that is defined as $(g_i,(m_i,w_i)) \subseteq (g_j,(m_j,w_j))$.

Theorem 4.5: Let (G, M, R) be a formal context, the reduct D is the meet (join) irreducible set if and only if $(g_i,(m_i,w_i))$ be a weighted attribute concept.

Proof. $(i) \Rightarrow (ii)$: If (G, M, R) be a meet irreducible set then there exist a reduct D_i such that $D_i \subseteq M$ from Definition 4.5.

$\Rightarrow$This preserves that there will be an extended set of meet irreducible elements ordered pair $D_i \subset M$ then D_i is a consistent set.

Since D_i is a consistent set there exist an element $(g_i, m_i) \subseteq (g_j, m_j)$ and also $w_j \leq w_i$.

Therefore, By Lemma 4.4: $(g_i,(m_i,w_i)) \subseteq (g_j,(m_j,w_j))$ which preserves the weight of a consistency set.

Hence, $(g_i,(m_i,w_i))$ is a weighted attribute concept.

$(ii) \Rightarrow (i)$: If $(g_i,(m_i,w_i))$ be a weighted attribute concepts then there exist an element (g_i, m_i) in the Reduct of (G, D, R) where $D \subseteq M$.

Therefore, By Lemma 4.4 and Definition 4.5: D is a consistency set with the element (g_i, m_i).

Theorem 4.6: Let C be the core and $m_i, m_j, \ldots, m_n$ be the noncore element existing in the reduct $T_1, T_2, \ldots, T_n$. Suppose C is a consistency set on the lattice L then $C \cap (g_i,(m_i,w_i)) \neq C \cap (g_j,(m_j,w_j))$ if and only if $w_j \geq w_i$ then w_j is an optimal reduct.

Proof. $(i) \Rightarrow (ii)$: Let C be a consistency set then there exist $C \cap (g_i,(m_i,w_i)) \neq C \cap (g_j,(m_j,w_j))$ such that from Theorem 4.5 $(g_i,(m_i,w_i)) \leq (g_j,(m_j,w_j))$

Therefore, $C \cap (g_i,(m_i,w_i)) \leq C \cap (g_j,(m_j,w_j))$, since $g_i \subseteq g_j$ which implies $m_j \supseteq m_i$, $w_j \supseteq w_i$.

Thus, $w_j \geq w_i$, by definition 4.6, w_j is an optimal reduct.

$(ii) \Rightarrow (i)$: Let w_j is an optimal reduct then $w_j \geq w_i$

By Lemma 4.3 and 4.4, the lattice L such that $(g_i,(m_i,w_i)) \leq (g_j,(m_j,w_j))$ which satisfies the result $g_i \subseteq g_j$ which implies $m_j \supseteq m_i$, $w_j \supseteq w_i$.

Hence, By theorem 4.5, the consistency set exist with the core element C such that $C \cap (g_i,(m_i,w_i)) \leq C \cap (g_j,(m_j,w_j))$

Therefore $C \cap (g_i,(m_i,w_i)) \neq C \cap (g_j,(m_j,w_j))$

Hence the result.

From the above study, we analysis the weight of the core and noncore element using the entropy information values. This has been generalized and examined by lemma and theorems. Hence, we can calculate the weighted attribute concept by an example below.

Example 4.1: From Example 3.1, we can obtain the weight of an attribute reduction. Therefore, an optimal reduction was obtained from the comparative study on noncore element. Thus, from Tables 3 and 4 we found that there is no significant difference between the noncore elements. Hence it is not possible to diagonalize the optimal reduction for the information Table 1.

Table 3. Weight for reduct $T_1=\{a, f\}$

Reduct	$p(v_{ij})$	$\alpha(v_{ij})$	$Core[v_{ij}]$	w_i
a	0.8	0.2575	0.48	0.3275
f	0.4	0.5288	0.48	0.6725

Table 4. Weight for reduct $T_2=\{a, d\}$

Reduct	$p(v_{ij})$	$\alpha(v_{ij})$	$Core[v_{ij}]$	w_i
a	0.8	0.2575	0.48	0.3275
d	0.4	0.5288	0.48	0.6725

From Tables 3 and 4 it is observed that the weight of the core and non core element remains same. Hence, it is not possible to determine the optimal reduct using the concept of weighted attribute concepts.

5. AN EMPIRICAL ANALYSATION ON BIOLOGICAL DATA

This section investigates statistical data that describes breast cancer. These data have been classified from the image of the mammograms which has been identified from the test report of the patient survey who undergone the process of a breast cancer test. The weighted attribute concept helps us to determine the optimal reduct and easy to analyze the knowledge of the information table.

Work Rule:

Step 1: Consider an information table with condition and decision attributes.

Step 2: Create an indiscernibility relation table from the information table.

Step 3: Generate a granular hasse diagram from the table.

Step 4: From the granular, define the indiscernibility matrix and find its reduct.

Step 5: Determine the core and non core element.

Step 6: Form the table using entropy value and core values with probability value of decision attributes.

Step 7: Identify the non core element weight and analyse the weighted attribute concepts for study.

Example 5.1: Table 5 shows a mammogram analysis of the effect of detecting breast cancer using machine learning technology. The set of the object obtained from the 5 patient details which is consist of mammogram image {1, 2, 3, 4, 5} and the set of attributes obtained from the extraction of the feature of breast cancer that are collected from the knowledge of mean, standard deviation, smoothness, third moment, uniformity and entropy. From these details, the attribute was defined as {*Contrast, IDM, Dissimilarity*}. The contract is the measure of the image inertia or the local variations present in images as "*a*" (9.1-10.9), "*b*" (11.1-12.9); Inverse Difference Moment (*IDM*)-Homogentiy is the large values of cases which describes the largest element of principal diagonal as "*c*" (3.1-4), "*d*" (4.1-5); Dissimilarity is a directional moment that measures the image contrast that increases the linearly not exponentially as "*e*" (2.4-2.5),"*f*" (2.6-2.7), "*g*" (2.8-2.9); The analyzation of imperfect data can be classified by a new mathematical approach on Rough Set Theory. The Decision data is the specific decision rule of the *action, result, and outcome*.

Table 5. Information table

O/At	Contrast	IDM	Dissimilarity	Decision
1	9.85	4.9	2.4342	Yes
2	10.19	3.9	2.6045	Yes
3	10.73	4.3	2.5495	No
4	10.98	4.1	2.6482	No
5	12.36	3.1	2.9401	Yes

Table 6. Indiscernibility table

	a	b	c	d	e	f	g
1	1	0	0	1	1	0	0
2	1	0	1	0	0	1	0
3	1	0	0	1	1	0	0
4	1	0	0	1	0	1	0
5	0	1	1	0	0	0	1

From Table 6, the structure of the formal concept lattice was defined using the data obtained from Table 5. The hasse diagram and indiscernibility matrix constructed in Figures 4 and 5. Therefore the indiscernibility function can be obtain attribute reduction without the loss of information the reduct can be constructed based on join and meet irreducible context.

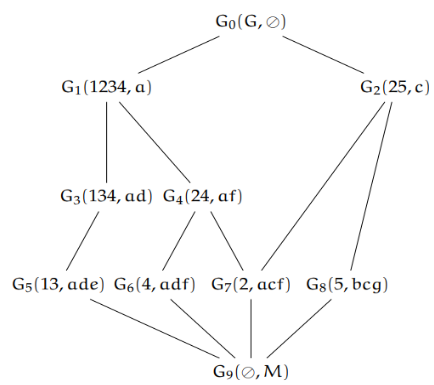

Figure 4. Hasse diagram from Table 6

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} M & a & ade & ad & \oslash \\ \oslash & M & a & af & c \\ \oslash & \oslash & M & ad & \oslash \\ \oslash & \oslash & \oslash & M & \oslash \\ \oslash & \oslash & \oslash & \oslash & M \end{pmatrix} \end{array}$$

Figure 5. Indiscernibility matrix

$$\Rightarrow IDS^{BC} = \{M\} \wedge \{a\} \wedge \{a \vee d \vee e\} \wedge \{a \vee f\} \wedge \{c\} \wedge \{a \vee d\} \Rightarrow \{a \wedge c \wedge d\} \vee \{a \wedge c \wedge f\}$$

From the above IDS^{BC}, the two-reduction decision are T_1={a ∧ c ∧ d} and T_2={a ∧ c ∧ f}.

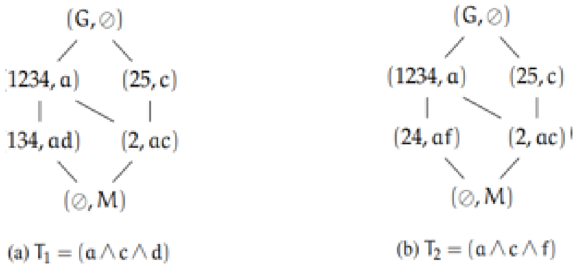

Figure 6. Reduct of breast cancer info

Finally, the reduction of the attribute was examined using the probabilistic value of the core. Hence the weighted attribute concept has been analysed by Hasse diagram (Figure 6). Thus, the optimal weight of each element in the core and noncore attributes are defined in the Tables 7 and 8.

Table 7. Weight for reduct T_1={a, c, d}

Reduct	$p(v_{ij})$	$\alpha(v_{ij})$	$Core[v_{ij}]$	w_i
a	0.8	0.2575	0.36	0.2096
c	0.4	0.5288	0.36	0.4304
d	0.6	0.4422	0.36	0.3599

Table 8. Weight for reduct T_2={*a, c, f*}

Reduct	$p(v_{ij})$	$\alpha(v_{ij})$	$Core[v_{ij}]$	w_i
a	0.8	0.2575	0.36	0.1958
c	0.4	0.5288	0.36	0.4021
f	0.4	0.5288	0.36	0.4021

From the above example, the optimistic reduction obtained using the weighted attribute concept that are analyzed from the non-core elements. Thus, we investigate the information table and determine the minimal attribute of breast cancer that classifies the mammogram images.

From the above systematic study, the weighted attribute concept was analyzed from the attribute reduction and consistency set. The indiscernibility matrix was formed from the granular ordered pair of objects and attributes. The reduct was examined through the probability of weighted attribute concept of core and non-core elements. The weight of each attribute was characterized by the probability values and defined the significance of the tuber affected in the mammogram images. Hence from the above example the attribute contract ranges from *a*={9.1-10.9} and IDM range from *c*={3.1-4} are the essential attribute. In addition, we analyze the attribute IDM that ranges from *d*={4.1-5} and dissimilarity that ranges from *f*={2.6-2.7} having a clear knowledge about the weight that emphasizes with the decision attributes. Thus, the weight of noncore element *d* is lesser than the *f*. Therefore, we come to the conclusion that reduct {*a, c, f*} gives an optimal result than {*a, c, d*}.

6. CONCLUSIONS

Weighted attribute reduction is effective in various fields, such as machine learning technology and knowledge management. A weighted attribute concept is presented in this paper that incorporates entropy information and characteristic knowledge of the core into the decision attribute. Using weighted attribute concepts, we have identified the degree of each attribute and determined its significance. By using an algebraic structure, we defined the join (meet) irreducible set and discussed its features with the consistency set. The weighted attribute concept is also based on the noncore elements behaviour. The weight of an attribute reduction has therefore been investigated. Further, this classification of core and noncore elements illustrates the extensive knowledge of the information system. In our study, we examined the information data obtained from mammographic images to demonstrate a different approach to weighted attributes.

It has been observed that a weighted concept using the probability of decision attribute is effective in the field of reduction. This work can be further examined by defining a structural operator and analysing the outcome of the features.

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