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Optimizing Expenditure Functions Through Economic Cybernetics: Analyzing Linear and Non-Linear Programming Approaches

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ABSTRACT

This study explores the analysis of expenditure functions within the framework of economic cybernetics, a discipline that applies automatic control theory principles to manage economic processes, a concept particularly pertinent to socialist economies. The investigation delves into total expenditures, average expenditures, and marginal expenditures, and presents a methodology for determining the elasticity coefficient for each expenditure type. There are several coefficients and particularly they serve as a key metrics for economists to asses importance of variables and their impact in each other having in mind the relationships they have in economy. The paper introduces a novel approach to associating these expenditures with linear and non-linear programming. It is known that for a given amount of production, the corresponding amount of production elements must be consumed. Therefore, this is predicted on the idea that consumption of resources (factors of production) dictates the amount of output produced because production necessitates the usage of resources. The examination of the expenditure function through the lens of economic cybernetics offers deeper insights into the evolving economic landscape of the 21st century.

1. INTRODUCTION

Microeconomics' expenditure function defines the minimum financial outlay required by an individual to reach a specific utility level, given the prices of accessible goods and the individual's utility function [1]. In the digital age, where internet integration into daily life becomes increasingly prevalent [2, 3], the concept of economic cybernetics has been gaining momentum [4]. Economic cybernetics, hybrid discipline merging economics and cybernetics, focuses on analyzing and enhancing the performance of complex economic systems using feedback control mechanisms and information theory. Within this interdisciplinary field, the innovative potential of enterprises assumes a critical role.

A significant determinant of an enterprise's innovative capabilities is the expenditure on Research and Development [2, 5]. Within the scope of economic cybernetics, the elasticity coefficient (ε) assumes a pivotal role [6]. This coefficient, a key metric for economists, measures the responsiveness of various economic variables to changes in others, offering valuable insights into the dynamic interrelationships within an economy.

Therefore, the analysis of the expenditure function from the perspective of economic cybernetics becomes critical to comprehend the evolving economic landscape of the 21st century. This research paper aims to contribute to this understanding by scrutinizing the expenditure function through the prism of cybernetic theory and investigating the impact of digital technologies on consumer spending patterns.

To enhance our understanding of these dynamics, the paper will delve deeper into the expenditure function's role within economic cybernetics, explore the factors influencing the elasticity coefficient, and examine how these insights can be applied to optimize expenditure in the digital age. The paper will also investigate the implications of these findings for policymakers and business leaders looking to leverage the principles of economic cybernetics to drive economic growth and innovation. By extending the understanding of the expenditure function within the context of economic cybernetics, this research aims to provide a foundation for future studies on economic optimization in the era of digital transformation.

2. DATA AND METHODS

2.1 General expenditure function

It is known that for a given amount of production, the corresponding amount of production elements must be consumed [7, 8]. This is predicted on the idea that consumption of resources (factors of production) dictates the amount of output produced because production necessitates the usage of resources [9-12]. For the given price of production factors, according to the production function, the general costs of the production process can be calculated. This means that the theory of consumption can also be defined as "the theory of production performed in the monetary values of the constituent elements". For the analysis of production, the Cobb-Douglas production function offers a straightforward but reliable paradigm.

The Cobb-Douglas production function stands as a fundamental model in the economic analysis of production, offering a lucid but robust framework. As stipulated by the Cobb-Douglas production function, key determinants in production encompass labor, capital, and specific parameters (α and β) that outline the production technology [6]. These parameters, known as output elasticities, gauge the responsiveness of output to a change in labor or capital inputs while holding other factors constant.

Important details about the production process are revealed by this mathematical depiction. The parameters α and β also signify the contribution of each input to the total output. For example, if α is greater than β, it would indicate that labor has a greater impact on output than capital. Understanding these factors enables a more thorough examination of production, assisting organizations and decision-makers with resource allocation and production optimization decisions [13].

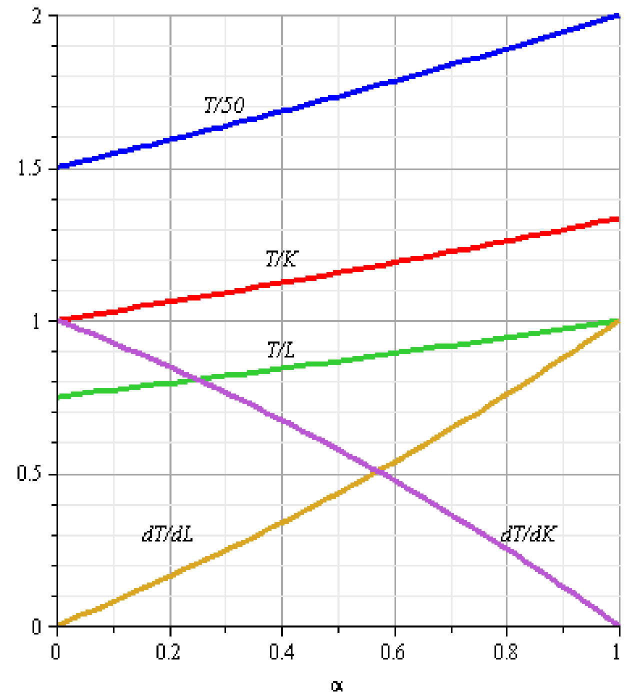

Figure 1. Graphic presentation of relevant expenses

According to the Cobb-Douglas production function, the factors that determine production include labor, capital, and the particular parameters (α and β) that define the production technology. These standards are essential for comprehending economic behavior and guiding choices in both corporate and policy situations. Let's start with the criterion dependence of production according to Cobb-Douglas [14]:

$$Q = AL^{\alpha}K^{1-\alpha}\ ; \quad 0 < \alpha < 1 \tag{1}$$

where:

Q – is the productivity of the production process,
L – labor consumed during production,
K – capital spent on production,
A – constant of proportionality,
α – production elasticity constant.

The total expenditure function is given by:

$$T = \frac{\partial Q}{\partial L}\cdot L + \frac{\partial Q}{\partial K}\cdot K \tag{2}$$

respectively:

$$T = A \cdot L^{\alpha} \cdot K^{1-\alpha} \tag{3}$$

The average costs (per units) are determined according to the criterion variation, T/L and T/K so that in this case it is obtained:

$$\frac{T}{L} = A\left(\frac{K}{L}\right)^{1-\alpha}\ ; \quad \frac{T}{K} = A\left(\frac{L}{K}\right)^{\alpha} \tag{4}$$

The marginal (marginal) costs are determined according to the principle criterion dependences, ∂T/∂L and ∂T/∂K obtaining the following:

$$\begin{aligned} \frac{\partial T}{\partial L} &= A\cdot\alpha\cdot\left(\frac{K}{L}\right)^{1-\alpha}; \\ \frac{\partial T}{\partial K} &= A\cdot(1-\alpha)\left(\frac{L}{K}\right)^{\alpha} \end{aligned} \tag{5}$$

In Figure 1, the corresponding costs for the design parameters are graphically presented: A=1, L=100, K=75.

3. RESULT AND DISCUSSION

3.1 Coefficient of elasticity in economic cybernetics

In the context of economic cybernetics, the coefficient of elasticity is also of particular importance (ε). It is known that for two economic variables X (independent variable) and Y (dependent variable), the elasticity coefficient is determined according to:

$$\varepsilon(Y,X) = \frac{\partial Y}{\partial X}\frac{X}{Y} \tag{6}$$

Next, the elasticity coefficients can be assigned for the corresponding expenses defined previously. Thus, for the coefficient of elasticity of total expenditure (T) to labor (L), ε (T, L), and to capital (K), ε (T, K), it is obtained:

$$\varepsilon(T,L) = \alpha\ ; \quad \varepsilon(T,K) = 1-\alpha \tag{7}$$

The coefficients of elasticity to the corresponding average expenditure are determined according to:

$$\begin{aligned} &\varepsilon\left(\frac{T}{L},L\right) = \alpha - 1; \quad \varepsilon\left(\frac{T}{L},K\right) = 1-\alpha \\ &\varepsilon\left(\frac{T}{K},L\right) = \alpha\ ; \quad \varepsilon\left(\frac{T}{K},K\right) = -\alpha \end{aligned} \tag{8}$$

The coefficients of elasticity to the respective marginal costs are given according to:

$$\begin{aligned}
&\varepsilon\left(\frac{\partial T}{\partial L}, L\right)=\alpha-1; \\
&\varepsilon\left(\frac{\partial T}{\partial L}, K\right)=1-\alpha \\
&\varepsilon\left(\frac{\partial T}{\partial K}, L\right)=\alpha; \\
&\varepsilon\left(\frac{\partial T}{\partial K}, K\right)=-\alpha
\end{aligned} \tag{9}$$

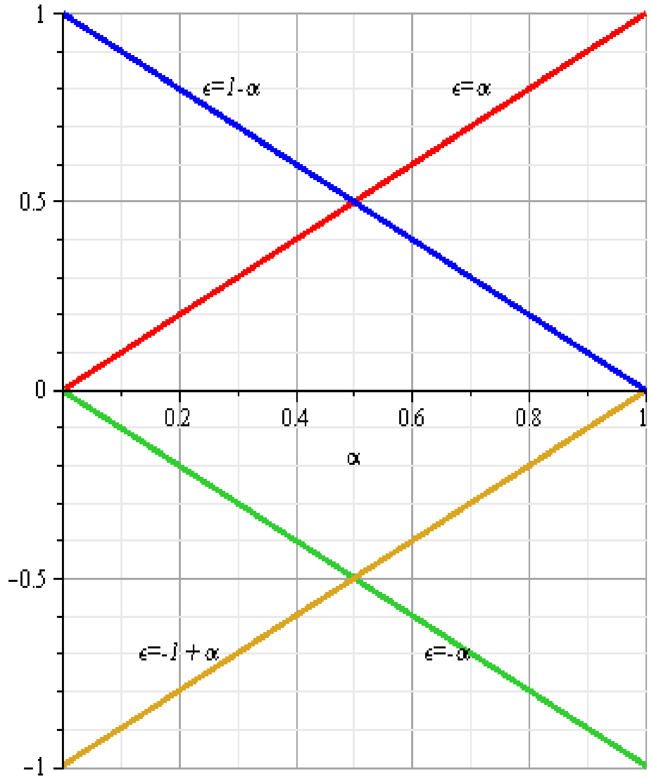

Figure 2. Graphic presentation of the characteristic coefficients of elasticity

The corresponding characteristic elasticity coefficients are graphically presented in Figure 2. For example, for α=0.2, when capital (K) changes by 1%, then total expenditure (T), the average expenditure on labor (T/L) and marginal expenditure on labor ($\partial T / \partial L$) increased (plus sign) by 0.8%, and so on. In Figure 2, the graphical representation shows how the mentioned coefficients respond to changes in the value of α.

Next, the total (overall) expenses presented in the form will be analyzed:

$$T = AQ^3 - BQ^2 + CQ \tag{10}$$

The average costs according to Eq. (10) are:

$$Q = \frac{T}{Q} = A\cdot Q^2 - B\cdot Q + C \tag{11}$$

Marginal costs are given according to the:

$$\frac{\partial T}{\partial Q} = 3\cdot A\cdot Q^2 - 2\cdot B\cdot Q + C \tag{12}$$

The possibility of determining the maximum value for the average consumption is noted:

$$\begin{aligned}
&\frac{\partial (T/Q)}{\partial Q} = 0; \\
&\left(\frac{T}{Q}\right)_{\max} = C - \frac{B^2}{4A}; \\
&Q = Q_e = \frac{B}{2A}
\end{aligned} \tag{13}$$

as well as for marginal costs:

$$\begin{aligned}
&\left(\frac{\partial T}{\partial Q}\right)_{\max} = C - \frac{B^2}{3A}; \\
&Q = Q_e = \frac{B}{3A}
\end{aligned} \tag{14}$$

In Figure 3 graphically presents the total, average and marginal costs for the design parameters: A=1, B=6, and C=20.

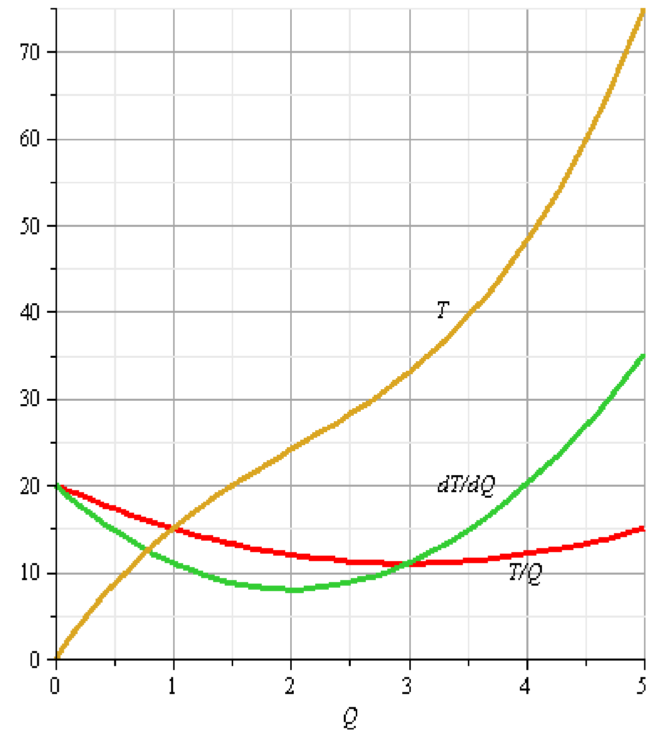

Figure 3. Graphic presentation of the relevant expenses according to Eq. (10)

The coefficient of elasticity ε (T, Q) is now given by:

$$\varepsilon(T,Q) = \frac{3AQ^2 - 2BQ + C}{AQ^2 - BQ + C} \tag{15}$$

The coefficient of elasticity ε (T/Q, Q) is determined according to the:

$$\varepsilon\left(\frac{T}{Q},Q\right)=\frac{Q\cdot(2\cdot AQ-B)}{AQ^2-BQ+C} \tag{16}$$

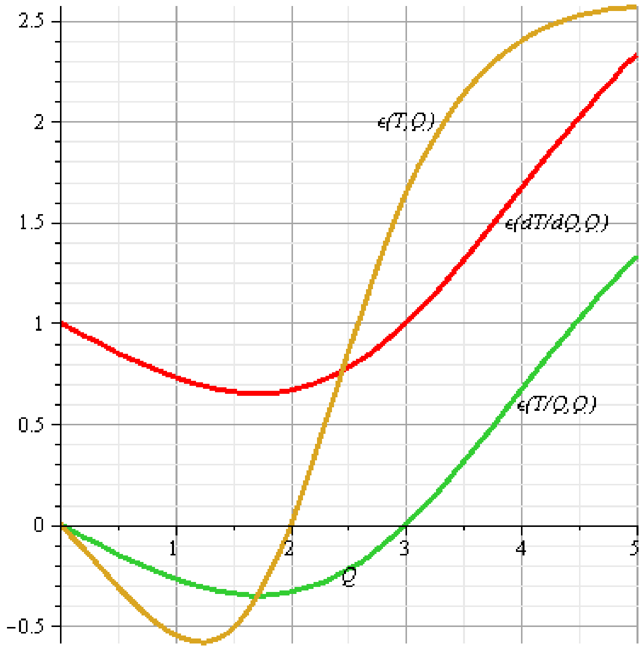

Figure 4. Elasticity coefficients for dependence according to Eq. (10)

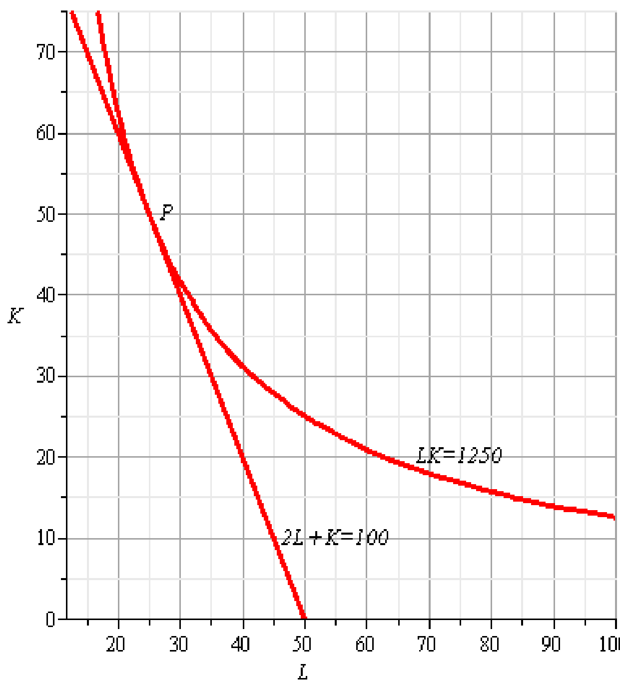

Figure 5. Regarding the optimal allocation of labor and capital resources

For marginal costs, the elasticity coefficient is required according to:

$$\varepsilon\left(\frac{\partial T}{\partial Q},Q\right)=\frac{Q(6AQ-2B)}{3AQ^2-2BQ+C} \tag{17}$$

The coefficients of elasticity are graphically presented according to Figure 4. For example, the minimum value of ε $(T, Q)_{\min}$=- 0.581 is reached for Q_e=1.225, which means that when the amount of production (Q) changes by 1%, total consumption decreases (minus sign) by 0.581%. The minimum value ε $(T/Q, Q)_{\min}$=- 0.348 is reached for Q_e=1.723, which means that when production (Q) changes by 1%, then average costs decrease by -0.348%. The minimum value ε ($\partial T / \partial Q$, $Q)_{\min}$=0.652 is reached for Q_e=1.723, which means that when productivity (Q) changes by 1%, marginal costs increase (plus sign) by 0.652%. The graphical representation in Figure 4 illustrates the variations in the elasticities of different expenditure measures concerning changes in the quantity of output (Q), providing valuable insights into the responsiveness of total, average, and marginal expenditures to variations in the level of production.

Furthermore, the case of the optimal combination of the allocation of labor resources (L) and capital (K) as production factors will be analyzed, so that the optimal value of total costs can also be sought, Figure 5.

Further, to the application of non-linear programming, it is convenient to apply the function (Φ) according to the Lagrange multipliers:

$$\Phi = A\cdot L^{\alpha}\cdot K^{1-\alpha}+\lambda\cdot(D-EL-FK) \tag{18}$$

According to the partial derivatives, $\partial\Phi/\partial L=0$; $\partial\Phi/\partial K=0$; $\partial\Phi/\partial\lambda=0$ it is obtained:

$$K=\frac{D(1-\alpha)}{F};\qquad L=\frac{\alpha D}{E};\qquad \lambda=\frac{A(1-\alpha)}{F}\exp\left[-\alpha\ln\left(\frac{E(1-\alpha)}{\alpha F}\right)\right] \tag{19}$$

The optimal value of the allocation of labor (L) and capital (K) resources is:

$$T=T_{opt}=A\left(\frac{\alpha D}{E}\right)^{\alpha}\left[\frac{D(1-\alpha)}{F}\right]^{1-\alpha} \tag{20}$$

The graphical presentation of the T_{opt} and the direction tangential to the optimization, EL+FK=D, is given in Figure 5. The point (economic state) P is the state where the direction of tangential optimization touches the isoquant T_{opt}. For design conditions, D=100, E=2, F=1, it is obtained, P(L=25; K=50). By optimizing this functional (Φ) with respect to labor (L), capital (K), and the Lagrange multipliers, the optimal values of labor and capital can be determined, along with the corresponding total expenditures. This approach enables the identification of the optimal resource allocation and expenditure pattern that maximizes the production output (Q) while satisfying the budget constraint (Y).

Optimal allocation involves determining the best combination of labor and capital that maximizes output, given a budget constraint. The Cobb-Douglas production function shows how variation in labor and capital affects output, and can thus help organizations allocate these resources more effectively. The marginal products of labor and capital, derived from the Cobb-Douglas production function, provide essential insights for this allocation.

The values as seen in Figure 5 indicate a potential optimal allocation of labor and capital, implying effective resource management to produce a particular level of output [13, 14].

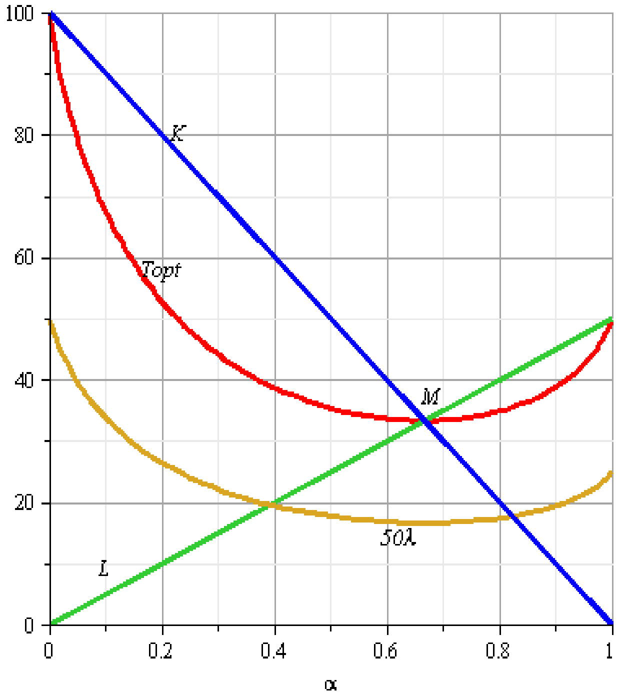

Figure 6. Graphic presentation of the characteristic sizes according to the Lagrange functional

Figure 6 graphically presents the characteristic sizes according to the Lagrange functional.

The Lagrange multiplier of 50 indicates how significant these restrictions were to the optimization process at point M, where the economy or firm has discovered an effective distribution of labor and capital that optimizes overall output while abiding by constraints. Constraints in the Cobb-Douglas production function can refer to a variety of limits, including financial restraints, restrictions on the availability of resources, or technological limitations. The trade-off between enhancing the production function and upholding these restrictions is quantified by the Lagrange multiplier of 50λ.

The minimum value of the functional T_{opt} is reached at the point (economic state) M so that the expressions are valid:

$$\alpha = \alpha_M = \frac{E}{E+F};$$
$$T_{opt(\min)} = \frac{AD}{E+F}; \tag{21}$$
$$M\left(\alpha_M; T_{opt(\min)}\right)$$

For design conditions, the following is obtained: $M(2/3; 100/3)$. Figure 7 graphically presents the coefficient of elasticity, ε (T_{opt}, α).

The minimum value of the coefficient in question is, $\varepsilon(T_{opt}, \alpha)_{\min}=-0.463$, which is reached for $\alpha=0.316$.

This implies that when the value of α, representing a parameter in the production function or resource allocation, changes by 1%, the coefficient of elasticity between total expenditures (T_{opt}) and a change by -0.463%. In other words, there is a negative relationship between the changes in α and the corresponding changes in total expenditures.

Figure 7 explores the coefficient of elasticity, this coefficient sheds light on how responsive total spending (T_{opt}) is to changes in the parameter, which describes some features of the resource allocation or production function.

Graphically representing this coefficient aids the visualization of the nature of this relationship, which is essential in understanding how changes in α affect the overall economic landscape.

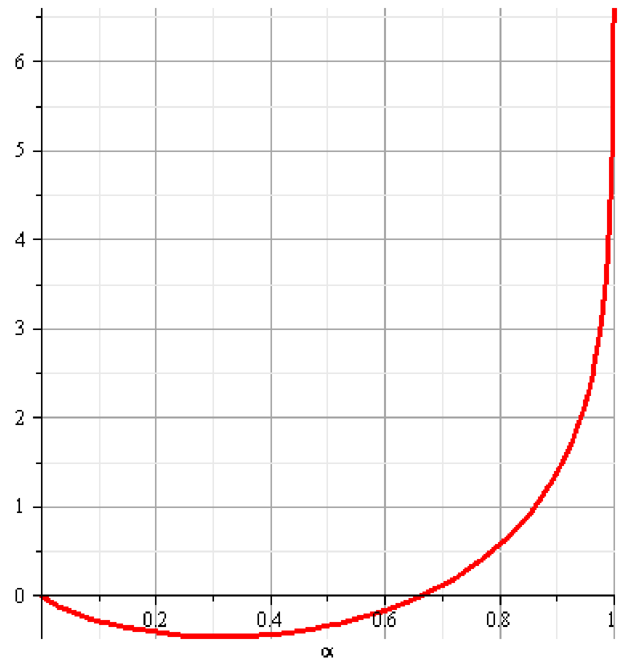

Figure 7. Graphic presentation of the elasticity coefficient ε (T_{opt}, α)

4. CONCLUSIONS

The study has illuminated key concepts integral to microeconomic expenditure analysis, such as total, average, and marginal expenditure. The research demonstrates how optimal labor and capital resource allocation -- the principal production factors -- can be achieved, and how the optimal total cost value can be determined. The determination of optimal total expenditure is linked with non-linear programming in the preliminary analysis. Within this context, a novel approach is proposed, enabling a more in-depth professional and scientific study and facilitating a more comprehensive understanding of the subject matter. Non-linear programming, a mathematical technique used to optimize problems where the objective function and constraints are non-linear, is commonly applied in economics, engineering, and science to tackle complex optimization issues. The novel approach to non-linear programming proposed herein holds potential implications for optimizing resource allocation across diverse fields, including manufacturing, finance, and healthcare. In summary, this paper ventures into the analysis of expenditure functions and their optimization in the context of labor and capital allocation. It also presents a fresh approach to non-linear programming that can be harnessed to optimize various intricate issues across different sectors.

The findings open up several potential research directions within economic cybernetics and expenditure analysis, such as the exploration of different production functions, advanced non-linear programming techniques, and dynamic expenditure analysis. These areas of investigation may offer further insights into the optimization of expenditure functions.

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NOMENCLATURE

Q	the productivity of the production process
L	labor consumed during production
K	capital spent on production
A	constant of proportionality
α	production elasticity constant
T	total expenditure function
X	independent variable
Y	dependent variable
ε	coefficient of elasticity
Φ	the function according to the Lagrange multipliers