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A Comparative Study of Synergetic and Sliding Mode Controllers for Pendulum Systems

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ABSTRACT

This study presents a performance comparison between a synergetic controller (SC) and a sliding mode controller (SMC) applied to a pendulum system. Initially, the mathematical model of the pendulum system is established. Subsequently, the design of both synergetic and sliding mode controls is elaborated, leading to the development of control laws for the proposed controllers. The effectiveness of the pendulum system controlled by SC and SMC has been validated through numerical simulations. These simulations revealed that the control signal exhibits chattering behaviour in the case of SMC, while this phenomenon is absent with the SC, demonstrating a distinct difference in performance between the two control systems.

1. INTRODUCTION

Most nonlinear systems have uncertain dynamic properties. As a result, controllers that are both high-performance and robust are needed. Recently, advanced algorithms have been used to design nonlinear controllers that are resilient and provide the required performance. One of them is the synergetic controller (SC) and sliding mode controller. It is a nonlinear and robust controller that may be used specifically with nonlinear systems whose parameters are uncertain. The sliding mode control (SMC) has been designed to ensure finite-time convergence of the sliding mode system dynamics. The sliding mode control utilizes nonlinear sliding surfaces instead of linear planes. SMC provides the benefit of finite time convergence and little steady-state error [1]. However traditional sliding mode control has singular points. The singularity can be avoided with non-singular sliding mode control, but the upper bounds of the disturbances must typically be known in order to calculate the switching gain [2, 3]. Synergetic control, which is similar to sliding mode control, is predicated on the idea that by using a continuous control law to guide a system to a wanted manifold with designer-selected dynamics, we can achieve results that are competitive with SMC while avoiding the latter's main drawback, chattering. The SC has the advantages of finite time convergence and low steady-state error. This control is crucial to the reliable operation of the pendulum system [4]. Recent control techniques and applied control to pendulum systems are discussed in the following literature: Tohma and Hamoudi [5] produced A study comparing the ASMC with the CSMC for simple pendulum found that the former was better at minimizing control action and chattering by setting the controller gain to an optimally small amount. Yakubu, Olejnik and Awrejcewic [4] proved the systemic impact of a variable pendulum length. The system's vertically stimulated parametric pendulum with variable length is constructed, resulting in quicker and longer oscillations than those of the constant-length pendulum. Hence, greater and more complex dynamics are realized. Ali and Naji [6] presented are designs for state feedback and state feedback with integrated controllers for the rotary inverted pendulum system. Using PSO, they were able to determine the ideal values for the state feedback gains. The suggested cost function incorporates the time response standards and restrictions to ensure that it is both resilient and able to fulfil the time response demands. To further track the system, state feedback plus integral was developed. Humaidi et al. [7] introduced an adaptive observer-based nonlinear backstepping control architecture. Both the reduced order adaptive observer and the backstepping design aim to predict the velocities of the cart and the pole, respectively. By means of simulated data in the MATLAB, the efficacy of an observer-based backstepping controller has been tested. Mokhtari et al. [8] developed adaptive neural network based on backstepping approach to a simple pendulum with in face of model uncertainties and perturbations. Berrahal et al. [9] developed observer-based adaptive backstepping that use the simple pendulum as an illustration. Using the observer technique, the adaptive backstepping control is examined.

In this study, two control design has been developed to control the angular position of simple inverted pendulum system. The first control approached is developed based on synergetic control theory, while the second control approach is established based on sliding mode control methodology.

The Contribution of this Work can be summarized:

- Design of synergetic control for pendulum system
- Design of sliding mode control for pendulum system
- Conducting comparison study between synergetic controlled system and sliding mode controlled system in terms of tracking performance, control efforts and robustness characteristic.

2. MATHEMATICAL MODEL

A free body diagram of the system is shown in Figure 1 [10]. It is instructive to work out this equation of motion also using

Lagrangian mechanics. The Lagrangian function is defined as

$$L = T - U \tag{1}$$

where, T is the total kinetic energy and U is the total potential energy of the mechanical system.

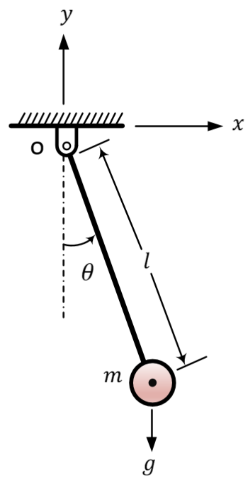

Figure 1. Free body of simple pendulum

To get the equations of motion, we use the Lagrangian formulation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) = F_i \tag{2}$$

where, q signifies generalized coordinates and F signifies non-conservative forces acting on the mechanical system. For the simplify pendulum, we assume no friction, so no non-conservative forces, so all F_i are 0. The aforementioned equation of motion is in terms of θ_p as a coordinate, not in terms of x and y. So we need to use kinematics to get our energy terms in terms of θ_p.

The linear velocity of the mass v_m is given by:

$$v_m = l.\dot{\theta}_p \tag{3}$$

The total kinetic energy T can be described by:

$$T = \frac{1}{2}\, m\, v_m^2 \tag{4}$$

Using Eq. (3), Eq. (4) can be rewritten as:

$$T = \frac{1}{2}\, m\, l^2\, \dot{\theta}_p^2 \tag{5}$$

The potential energy, U, depends only on the y-coordinate. Taking $\theta_p = 0$ as the position where $U = 0$,

$$y = l - l\cos\theta_p \tag{6}$$

Then, the potential energy U is given by:

$$U = m.g.y = m.g.l.\left(1 - cos\theta_p\right) \tag{7}$$

Now we have all the parts and pieces to complete the Lagran gian formulation. The Lagrangian function in terms of θ_p is

$$L = T - U = \frac{1}{2}\, m\, l^2\, \dot{\theta}_p^2 - m.g.l.\left(1 - cos\theta_p\right) \tag{8}$$

Using $q_1 = \theta_p$, the elements of Eq. (2) can be given by:

$$\frac{\partial L}{\partial \dot{\theta}_p} = m.l^2.\dot{\theta}_p \tag{9}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_p}\right) = m.l^2.\ddot{\theta}_p \tag{10}$$

$$\frac{\partial L}{\partial \theta_p} = -m.g.l.sin\theta_p \tag{11}$$

Now, putting these last two equations together

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_p}\right) - \frac{\partial L}{\partial \theta_p} = m.l^2.\ddot{\theta}_p + m.g.l.sin\theta_p = \tau_p \tag{12}$$

Let $x_1 = \theta$ and $x_2 = \dot{\theta}_p$, and $u = \tau_p$, the equation of state variables

$$\dot{x}_1 = x_2 \tag{13}$$

$$\dot{x}_2 = -\frac{g}{l}\sin(x_1) + \frac{1}{m\, l^2}\, u \tag{14}$$

where, l is the length of the pendulum in meters, g is the acceleration due to gravity, m is mass of the pendulum and u control action (τ torque of pendulum). For simplicity, Eq. (14) can be written in the following form:

$$\dot{x}_2 = f + b\, u \tag{15}$$

where, $f = -g\sin(x_1)/l$, $b = 1/(m\, L^2)$

3. CONTROL DESIGN OF PENDULUM SYSTEM

3.1 Control design based on synergetic control

Let ϵ the error between the actual angle position $x_1 = \theta$ and the desired $x_{1d} = \theta_d$ as follows:

$$\epsilon = x_1 - x_{1d} \tag{16}$$

Taking the first and second derivatives, one can have

$$\dot{\epsilon} = \dot{x}_1 - x_{1d} = x_2 - x_{1d} \tag{17}$$

$$\ddot{\epsilon} = \dot{x}_2 - \ddot{x}_{1d} = f + bu - \ddot{x}_{1d} \tag{18}$$

Let $\varphi(\epsilon)$ be defined as:

$$\varphi(\epsilon) = \alpha\, \epsilon + \dot{\epsilon} \tag{19}$$

Taking the derivative of Eq. (19), one can obtain

$$\dot{\varphi}(\epsilon) = \dot{\epsilon} + \alpha\ddot{\epsilon} \tag{20}$$

where, α is a scalar design for synergetic control. The definition of the $\varphi(\epsilon)$ with respect to the manifolds is given by

$$T\dot{\varphi}(\epsilon) + \varphi(\epsilon) = 0 \tag{21}$$

where, $T > 0$ is responsible for converging ratio of $\varphi(\epsilon)$ to manifold $\varphi(\epsilon) = 0$.

Using Eq. (20), Eq. (21) becomes

$$T(\dot{\epsilon} + \alpha\ddot{\epsilon}) + \varphi(\epsilon) = 0 \qquad (22)$$

$$T\dot{\epsilon} + T\alpha\,\ddot{\epsilon} + \varphi(\epsilon) = 0 \qquad (23)$$

Using Eq. (17) and Eq. (18)

$$T\dot{\epsilon} + T\alpha f + T\alpha\, b\, u - T\alpha\, \ddot{x}_{1d} + \varphi(\epsilon) = 0 \qquad (24)$$

According to Eq. (24), the control law can be deduced

$$u_{sc} = \frac{1}{T\alpha b}(-T\dot{\epsilon} - T\alpha f - \varphi(\epsilon) + T\alpha\, \ddot{x}_{1d}) \qquad (25)$$

where, u_{sc} represents the control action to pendulum system based on synergetic control.

3.2 Control design slide mode controller

In sliding mode control, the control law consists of two parts: equivalent part and switching part. The equivalent part of control signal is responsible for bringing the trajectory from initial states to the sliding surface, while the switching part tries to keep the trajectory on the sliding surface until reaches the origin [11, 12]. The sliding surface is given by

$$s = c\epsilon + \dot{\epsilon} \qquad (26)$$

where, c is scalar design parameter of positive value. The time derivative of Eq. (26) is given by

$$\dot{s} = c\dot{\epsilon} + \ddot{\epsilon} \qquad (27)$$

Using Eq. (18) to have

$$\dot{s} = c\dot{\epsilon} + f + bu_n - \ddot{x}_{1d} \qquad (28)$$

Setting $\dot{s} = 0$, the equivalent control law can be deduced

$$u_n = \frac{1}{b}\,(-c\dot{\epsilon} + f + \ddot{x}_{1d}) \qquad (29)$$

$$u_n = mL^2 \left(-c\dot{\epsilon} + \frac{g}{L}\sin(x_1) + \ddot{x}_{1d}\right) \qquad (30)$$

The total control law can be composed by adding the equivalent control part to switching part

$$u = u_n + u_s \qquad (31)$$

where, u_n represents the nominal (equivalent) control and u_s denotes the discontinuous control part, which is defined by

$$u_s = -\kappa\, sign(s) \qquad (32)$$

where, κ is a gain of positive value. As a result, the formula of control law can be given by

$$u_{smc} = mL^2\,(-c\dot{\epsilon} + \frac{g}{L}\sin\,(x_1) + \ddot{x}_{1d}) - \kappa sign(s) \qquad (33)$$

According to above analysis, two important remarks can be deduced [13, 14]:

Remarks 1: The control signal in synergetic control is continuous without interruptions, while there is chattering behavior.

Remark 2: According to Eq. (21), the synergetic control enforces the dynamic characteristics towards the manifold $(\epsilon) = 0$. The time constant T in Eq. (21) permits the change of the convergence rate of trajectory towards $\varphi(\epsilon) = 0$.

4. SIMULATION RESULTS

The effective of controllers have been verified via numerical simulation. The Simulink blocks within MATLAB environment has been used to represent the dynamic model of pendulum system and to synthesize synergetic and sliding mode controllers using Matlab functions. The codes used in developing the control algorithms reside inside these M-functions which are present within the library of MATLAB/Simulink. The block diagram of Figures 2 and 3 has been modelled by Simulink blocks for both controlled pendulum systems based on synergetic and slide mode controllers.

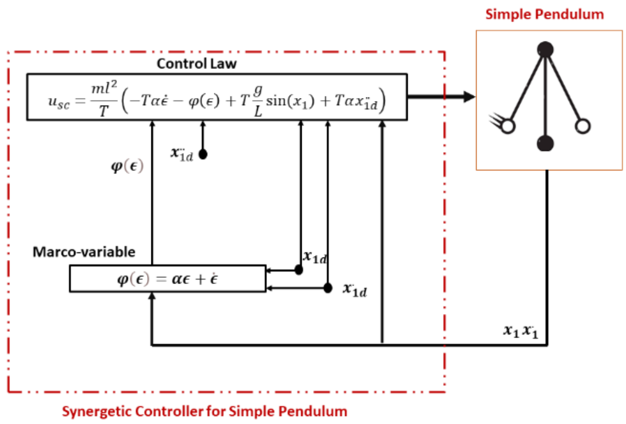

Figure 2. The schematic diagram of synergetic controlled pendulum system

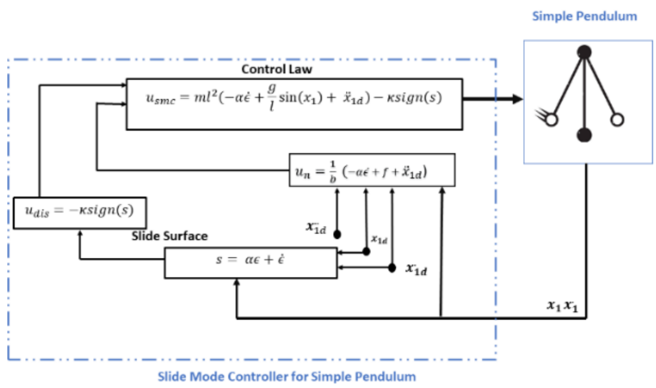

Figure 3. The schematic diagram of sliding mode controller for pendulum system

The parameters of pendulum system is selected as follows [8]:

$$m = 0.23 \text{ kg}, L = 0.5m, \text{ and } g = 9.81\ m/s^2\text{m}$$

The selected parameters of the synergetic and slide mode controller are as follows [7]:

$T = 0.1$, $\alpha = 2.3$, $c = 2.3$ and $\kappa = 2.4$

The external disturbance value $\delta_1 = 0.2$ and $\delta_2 = 0.3$. Both SC and SMC controllers have been applied to control the angular positions of pendulum system. Figures 4 and 5 show the behaviour of angular position under the control of proposed controllers. Figures 6 and 7 show the responses of tracking errors based on SC and SMC, respectively. It is clear that the controllers shows the can asymptotic stability of controlled system.

Synergetic control and sliding mode control are both used to control the angular velocity of a simple pendulum. The first and second controllers depicted in Figures 8 and 9 are extremely close to providing the pendulum's angular velocity.

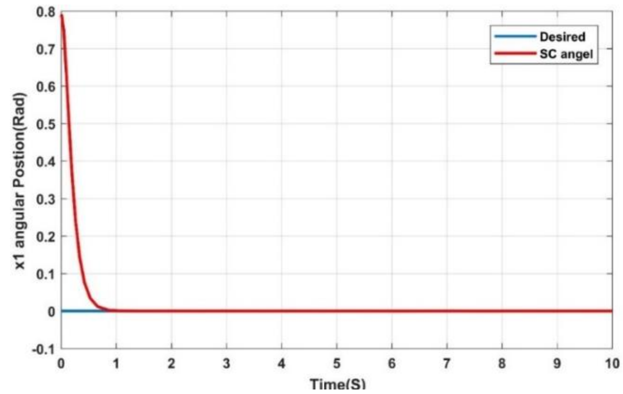

Figure 4. Tracking performance pendulum system using synergetic controller

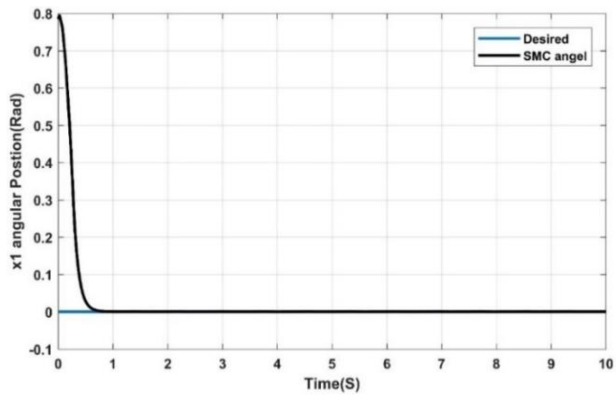

Figure 5. Tracking performance for pendulum system using sliding mode control

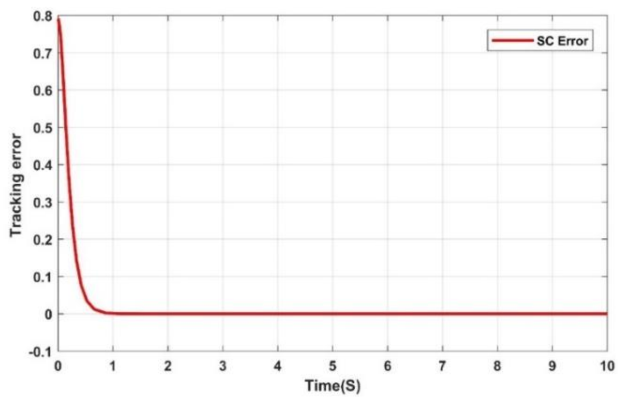

Figure 6. Tracking error for synergetic controller

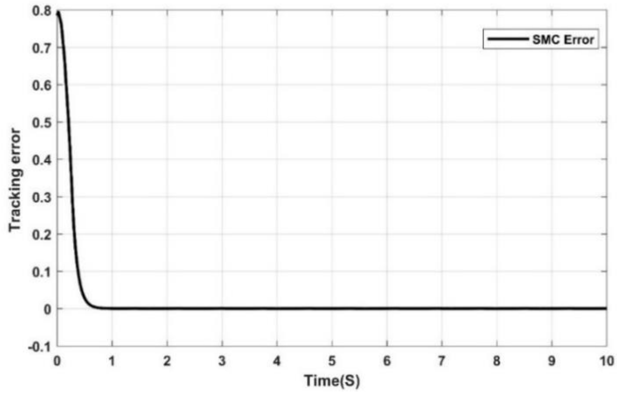

Figure 7. Tracking error for synergetic controller

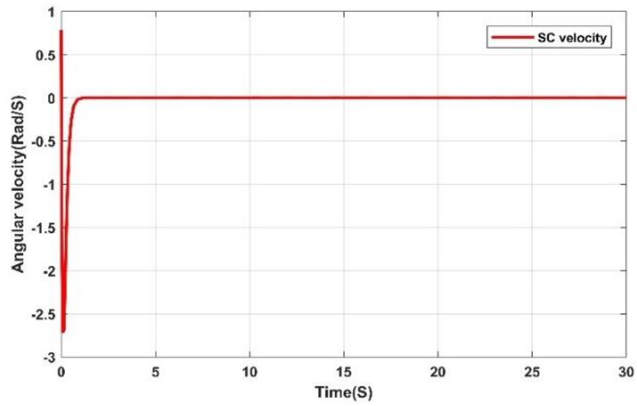

Figure 8. Angular velocity for simple pendulum using synergetic controller

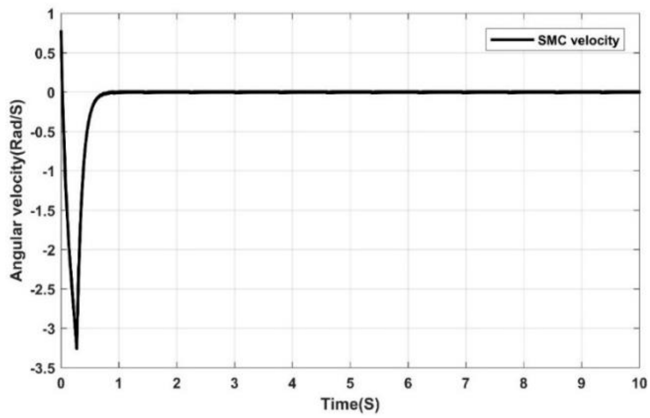

Figure 9. Angular velocity for simple pendulum using sliding mode controller

Figures 10 and 11 represent control action of pendulum, When the signum function is applied to the controller law, it causes the chattering phenomenon that affects SMC. In contrast, with SC controller, the control signal is modified, and there is no chattering phenomenon without the use of any chattering treatment tools.

To evaluate the system's robustness, the mass of the pendulum (m) and the length of the pendulum (L) are adjusted by 25%. Figures 12 and 13 depict the system response when the parameters are uncertain. It is demonstrated that the system is stable in spite of changes in system parameters. This indicates that proposed tow controllers may successfully compensate for system parameter changes.

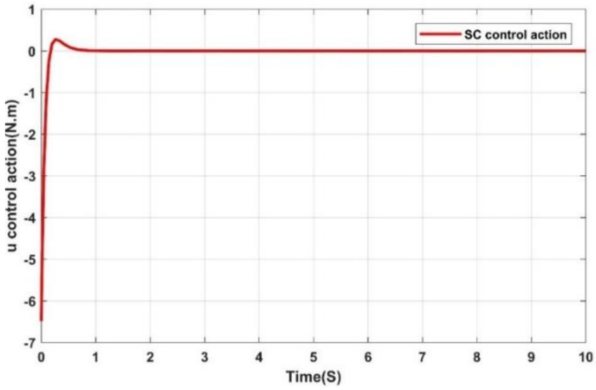

Figure 10. Control action for *s*ynergetic controller

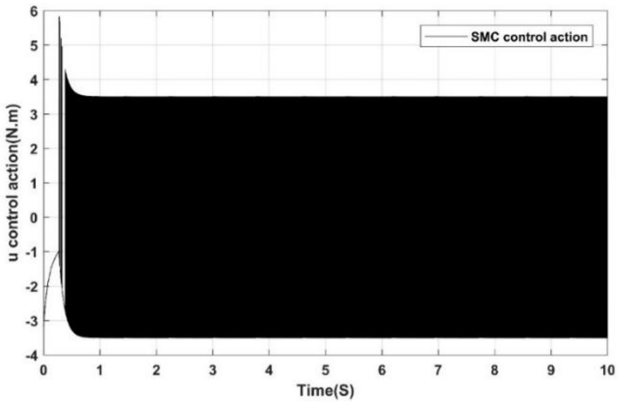

Figure 11. Control action for sliding mode controller

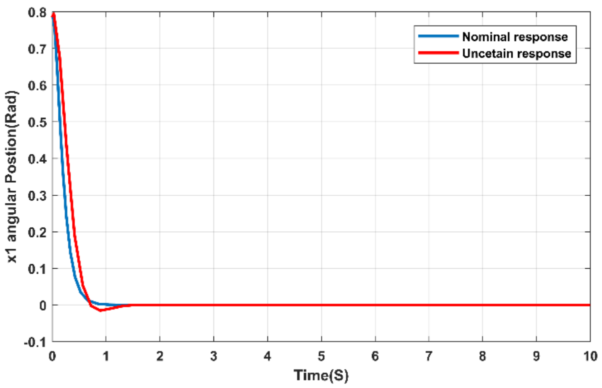

Figure 12. Uncertain system response using synergetic controller

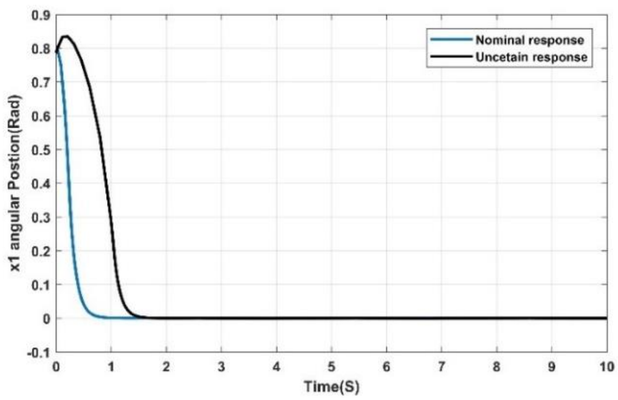

Figure 13. Uncertain system response using *s*liding mode controller

Table 1 shows the Performance between the SC and SMC controlled Pendulum systems. In a numerical sense, the RMSE value resulting from SC is equal to 0.1235 rad, while the RMSE given by SM is equal to 0.2756 rad. This indicates that the SC gives better performance and better error variance. This means that the synergetic controller gives greater resistance to changing parameters than the sliding mode controller, and therefore the SC controller is more Robust than the SMC controller.

Table 1. Performance between the SC and SMC controlled pendulum systems

Controller	Nominal Parameter (m, L)	% Variation	Uncertain Parameter (m, L)	ERMS
SC	0.25	25%	0.3125	0.1235
SMC	0.5	25%	0.625	0.2756

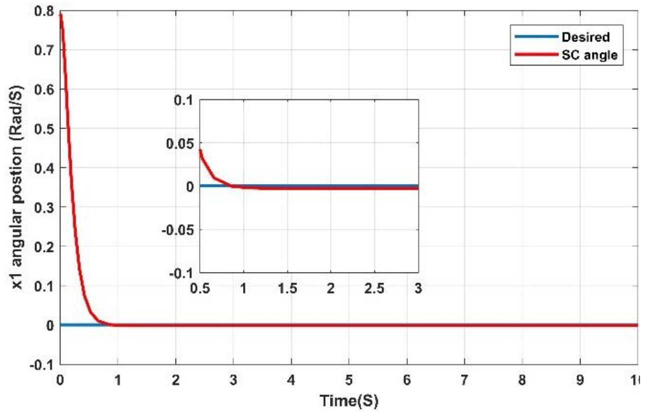

Figure 14. Angular response using synergetic controller under external disturbance

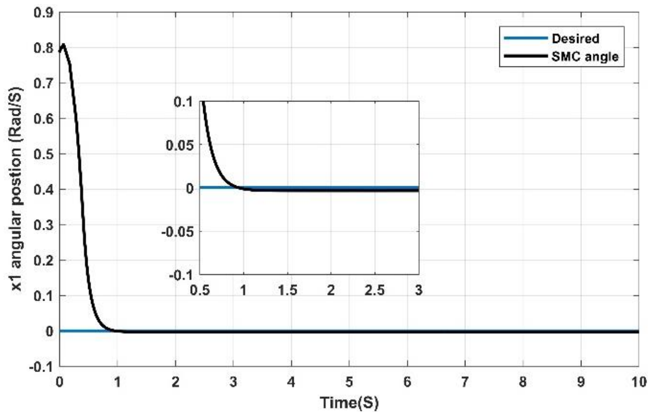

Figure 15. Angular response using sliding mode control under external disturbance

In the next scenario, the robustness characteristics of both controller have been evaluated by injecting disturbances on both channels of pendulum system. Actually, the basic pendulum dynamic equations, described by Eq. (3) and Eq. (4), can be rewritten as

$$\dot{x}_1 = x_2 + \delta_1 \tag{34}$$

$$\dot{x}_2 = -\frac{g}{L}\sin(x_1) + \frac{1}{mL^2}u + \delta_2 \tag{35}$$

where, δ_1 and δ_2 represents the external disturbance added to the system. Figures 14 and 15 show the effect of the external disturbance on the transient response of controlled system based on SC and SMC, respectively. The robustness of controllers are assessed by evaluating the deviation between the nominal response and the deviated response due to load application. The Root Mean Square of t Deviation (RMSD) has been calculated for both controllers. It turns out that the SC gives less RMSD (0.2634) than that based on SMC (0.2785). This leads to conclusion that the SC is more robust than SMC.

As future extension of this study, other control schemes can be used to control the pendulum system [15-20], or one can use modern optimization techniques to tune the design parameters of SC and SMC towards their improvement [21-25].

5. CONCLUSIONS

In this paper, the design and simulation of two controllers has been present for simple pendulum system. The first control design is based on synergetic theory and the second control design is based on SMC. The MATLAB programming software is used to simulate and verify the effectiveness of both controllers. The simulated results showed that the SC has better transient and robustness characteristics as compared to SMC. Moreover, the SC has no chattering behavior as compared to high chattering effect resulting from the SMC.

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NOMENCLATURE

L	Length of the pendulum m
m	Mass The of the pendulum kg
g	Acceleration due to gravity
τ	Torque of pendulum N.m
ϵ	The error between the actual angle position and the desired
T	The converging ratio of $\varphi(\epsilon)$ to manifold
u	Control action N.m
c	A scalar design for sliding mode control
u_{sc}	Control action of synergetic control
u_n	Nominal slide mode controller
u_{dis}	Discontinuous control action (sliding mode control)
s	Sild surface of slide mode control
κ	Gain design for sliding mode control (Discontinues)

Greek symbols

θ	Angular position
$\dot{\theta}$	Angular velocity
$\ddot{\theta}$	Angular acceleration
θ_d	Desired angle Rad
α	A scalar design for synergetic control
$\varphi(\epsilon)$	Manifold of synergetic control
δ_1	External disturbance 1
δ_2	External disturbance 2

Subscripts

SC	Synergetic control
SMC	Sliding mode control
RMSE	Root mean square error