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Boundary Layer Flow Of Silver and Titaniumoxide Nanofluids over Vertical Stretching Sheet

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ABSTRACT

The present paper focuses on the convective heat transfer of boundary layer flow of incompressible, viscous Nano fluids over a vertical linear stretching sheet. Two types of nanofluids such as silver - Water nanofluid and titanium oxide -Water nanofluid are considered for the present study. Various physical parameters for both types of Nanofluids for different volume fractions are calculated. The Boundary layer equations of motion and energy which are non-linear partial differential equations are reduced to non-linear ordinary differential equations by means of similarity transformations. The resulting non-linear ordinary differential equations are solved numerically by most efficient Nachtsheim-Swigert shooting iteration technique for satisfaction of asymptotic boundary conditions along with Fourth Order Runge-Kutta Integration Method. This study analyzes the effectiveness of heat transfer of Nano fluids in cooling plastic and rubber sheet.

Keywords: Nano fluids, Nanoparticles, Boundary layer equation, Stretching sheet.

1. INTRODUCTION

The cooling and heating effects are required by power manufacturing, transportation, electronics etc. These cooling and heating techniques are greatly needed for high energy device. It was clear that common fluids have limited heat transfer capabilities due to their low heat transfer capacity. However some metals are found to have very high thermal conductivity, may be three to four times higher than the common fluids. Therefore it is required to make a substance by combining these two, which will behave like a fluid and have thermal conductivity of a metal. Nanofluids are such substances, made by suspending the nano particles (with diameter 1 to 100 nanometer) in the common fluids, called base fluids. A very small amount of these nanoparticles in the base fluids increases the thermal conductivity by 15-40%.

Nanofluid, a name conceived by Choi at Argonne National laboratory, are fluids consisting of solid nanoparticles with size less than 100 nm suspended with solid volume fraction typically less than 4%. A nanofluid can enhance heat transfer performance compared to pure liquids. Nanofluids can be used to improve thermal management system in many engineering application such as transportation, micromechanics and instrument, HVAC system and cooling devices. Recently, many investigators studied nanofluid convective heat transfer in different geometry both numerically and experimentally.

Nanofluids are helpful in many areas of medical application like cancer therapy and laser based surgery which involves cooling of equipment.Nanofluid technology can be effectively used for cooling of powerful and small computers and other electronic devices. It can be also used for large scale cooling like aeroplanes, military system. Nanofluids can be widely used in large scale industries like chemical, food and beverages, oil, paper, textiles etc.

Das et al. [2] experimentally showed a two - to four - fold increase in thermal conductivity enhancement for water-based nanofluids containing Al_2O_3 or CuO nanoparticles over a small temperature range, 21^o – 51^oC. A comprehensive survey of convective transport in nanofluids has been made by Buongiorno [3], who gave a satisfactory explanation for the abnormal increase of the thermal conductivity. Buongiorno and Hu [4], studied on the nanofluid coolants in advanced nuclear systems. Ahmad and Pop [5] investigated mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids. Boundary layer flow of nanofluids over a moving surface in a flowing fluid was examined by Bachok et al. [6]. Khan and Pop [7] discussed the boundary-layer flow of a nanofluid past a stretching sheet. Makindeand Aziz [8] explained the boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. Daungthongsuk and Wongwises [9] studied analytically the effect of thermo physical properties models, for predicting the convective heat transfer rate for low concentration nanofluid.

Laminar boundary layer flow of nanofluid along a flat plate was studied by SP Anjali Devi and Juljia Andrews [16]. Vajravelu et al. [17] studied the effect of convective heat transfer in the flow of viscous Ag-water and Cu-water nanofluids over a stretching surface and it was seen that the role of nanoparticle volume fraction on the flow and heat transfer characteristics under the influence of thermal buoyancy and temperature dependent internal heat generation and absorption. Natural convection over vertical plate of nanofluid have been explained by Reddy et.al. [37]. Abdel Malik Bouchouchaand, Rachid Bassiah studied nanofluids in square cavity [36].

In this study, our objective is to investigate the effect of viscous nanofluid past over vertical stretching surface. Here the two nanofluids considered are Silver-water nanofluid and Titanium oxide-water nanofluid. Using an appropriate similarity transformation, the well-known governing partial differential equations are reduced to the ordinary differential equations. Numerical solution of the problem is obtained using Nachtsheim-swigert shooting iteration scheme for satisfaction of asymptotic boundary conditions along with Fourth order Runge-Kutta integration method. These numerical results for various physical parameters involved in the problem are demonstrated graphically. The skin friction coefficient and the non-dimensional rate of heat transfer are also presented numerically in tabular form for several values of the physical parameters.

2. FORMULATION OF PROBLEM

Consider a two - dimensional, steady, incompressible nanofluid past over vertical stretching sheet. The velocity of the stretching sheet is given by u = ax where a> 0, is constant acceleration parameter. X –axis is taken along the direction of stretching sheet. The flow is generated by the stretching sheet along X axis.

Using Prandtl boundary layer equations based on principle of conservation of mass, energy and momentum, equations of nanofluids is given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} = \vartheta_{nf}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Components of velocity along x and y directions are represented by u and v respectively, T is temperature, $\boldsymbol{\vartheta_{nf}}$is kinematic viscosity of nanofluids, $\boldsymbol{\alpha_{nf}}$ is thermal diffusibility of nanofluids,k $_{nf}$ thermal conductivity, (C p) $_{nf}$ is specific heat and ρ_{nf} density of nanofluids.

Apart from these equations, the boundary condition for velocity and temperature are

$$y = 0, u = u_w, v = 0, T = T_w + \text{a x},$$
$$y \to \infty, u \to 0, T \to T_\infty \tag{4}$$

The viscosity, heat capacity and thermal conductivity of the nanofluids depends upon volume fraction ϕ of nanoparticles used. The effective density of nanofluid is given by

$$\rho_{nf=(1-\emptyset)\rho_f} + \rho_s \tag{5}$$

And heat capacitance of nanofluid

$$(\rho C_p)_{nf\,=\,(1-\emptyset)(\rho C_p)_f} + \emptyset(\rho C_p)_s \tag{6}$$

Thermal expansion coefficient of nanofluid is given by

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \tag{7}$$

As given by Santraet. al. (Santra et. al. 2009), where ρ_f and ρ_s are density, (C p)$_f$ and (C p) $_s$ are specific heat capacitance of base fluid and solid particle respectively and β_f and β_s are thermal expansion coefficient of base fluid and solid particle respectively.

The dynamic viscosity of nanofluids as given by (Brinkman 1952) is as follows:

$$\mu_{nf} = \frac{\mu_f}{(1-\emptyset)^{2.5}} \tag{8}$$

$$\frac{k_{nf}}{K_f} = \frac{(k_s+2k_f)-2\emptyset\,(k_f-k_s)}{(k_s+2k_f)+\emptyset\,(k_f-k_s)} \tag{9}$$

Table 1. Density, capacitance and thermal expansion coefficient water, silver, titanium oxide

	(kg / m3)	Cp (J/ kg.K)	K (W / m.K)
Water	997.1	4179	0.613
Silver	10,500	234	429
Titanium oxide	4,2500	686.2	8.9532

To solve the equations following dimensionless variables are introduced:

$$\psi = (a\,\vartheta_{nf})^{1/2} x f(\eta) \tag{10}$$

$$\eta = (\frac{a}{\vartheta_{nf}})^{1/2} y \tag{11}$$

$$u = ax\, f'(\eta) \tag{12}$$

$$v = -\left(a\vartheta_{nf}\right)^{\frac{1}{2}} f\ (\eta) \tag{13}$$

where $\boldsymbol{\psi}$(x, y) is stream function and $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$, η is similarity variable. And,

$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \tag{14}$$

Continuity equation is satisfied and equations (2) and (3) along with boundary conditions. (4) are transformed and are written as:

$$f''' - (1-\emptyset)^{2.5}\left(1-\emptyset+\emptyset\frac{\rho_s}{\rho_f}\right)\left(f'^2 - ff''\right) +$$
$$\lambda\theta(1-\emptyset)^{2.5}\left(1-\emptyset+\emptyset\frac{(\rho\beta)_s}{(\rho\beta)_f}\right) = 0 \qquad (15)$$

$$\frac{1}{Pr_{nf}}\frac{k_{nf}}{k_f}\theta'' + \left(1-\emptyset+\emptyset\frac{\rho_s}{\rho_f}\right)\left(1-\emptyset+\emptyset\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)f\theta' = 0 \qquad (16)$$

Along with boundary conditions:

$\eta = 0, y = 0, f = 0, f' = 1, \theta = 1$
$$\eta \to \infty, y \to \infty, f' = 0, \theta = 0 \qquad (17)$$

The variables are defined as

$$Pr_{nf} = \frac{\vartheta_{nf}}{\alpha_{nf}} \qquad (18)$$

Kinematic viscosity and thermal diffusivity of nanofluids are respectively:

$$\vartheta_{nf} = \frac{\mu_{nf}}{\rho_{nf}} \qquad (19)$$

and

$$\alpha_{nf} = \frac{k_{nf}}{\rho_{nf}(C_p)_{nf}} \qquad (20)$$

And thus $(Pr)_{nf}$, the Prandtl number of nanofluid

$$Pr_{nf} = \mu_{nf}\frac{(C_p)_{nf}}{k_{nf}} \qquad (21)$$

where $\lambda = \frac{g(\rho\beta)_f}{\rho_f a^2}, Pr = \frac{\vartheta_f}{\alpha_f}$.

Skin friction coefficient:

The skin friction coefficient c_f is given by $C_f = \frac{\tau_w}{\rho_f U_w^2}$, where $\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}$. Using (11), we get $C_f Re_x^{1/2} = \frac{1}{(1-\emptyset)^{2.5}}f''(0)$.

Nusselt number:

Nusselt number is defined by: $Nu = \frac{xq_w}{k_f(T_w - T_f)}$ Where $q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}$. Using (11) we get $NuRe_x^{-1/2} = -\frac{k_{nf}}{K_f}\theta'(0)$.

3. SOLUTION OF PROBLEM

To solve equations (16) and (17) along with boundary conditions (18), shooting technique along with Runge – Kutta method of fourth order is used. Initial guesses were made for values of f'' to start shooting process as given by MA Satar and Maleque in 2011[37]. The value of η is found on each iteration loop by $\eta_{n+1} = \eta_n + \Delta\eta$. The step size $\Delta\eta = 0.001$ is used while obtaining the numerical solution with $\eta_{max} = 15$ and by considering the six decimal place as a criterion for convergence.

4. RESULTS AND DISCUSSIONS

In this section, we consider two types of water based nanofluids containing Silver(Ag) and Titanium oxide (Ti_2O_3). The Prandtl Number of the base fluid (Water) is kept constant at 7.2 and the effect of solid volume fraction φ is investigated in the range of $0 \leq \emptyset \leq 0.2$. In order to get the clear insight of the problem, numerical values of the solutions are obtained by fixing various values for the physical parameters involved having φ = 0.0 for base fluid and φ = 0.1 for the nanofluid when λ = 1.0. The results are exhibited through the graphs and tables. The heat transfer property of nanofluid over a vertical stretching sheet has been studied. Table 2 and 3 depicts density, capacitance, dynamic viscosity, thermal conductivity, Prandtl number of silver water and titanium oxide water nanofluids.

The study basically concentrates on flow of silver water and titanium oxide water nanofluids over stretching sheet. Results are shown for various physical parameters. Effect of heat transfer by changing volume fractions and Prandtl number have been analysed. The effect of change of volume fraction on various physical parameters has been depicted by graphs. Increase in volume fraction leds to increase in temperature profile of both types of nanofluids.

Figure 1 and 2 represents dimensionless temperature profile of silver water and titanium oxide water nanofluids respectively. It is noticed that increase in Prandtl number leds to decrease in temperature.

Figure 3 shows the effect of volume fraction on temperature profile of Titanium oxide nanofluids. It is observed that increase in volume fraction causes increase in temperature profile. There is very less effect of volume fraction on temperature.

Figure 4 depicts temperature profile of silver water and titanium oxide water nanofluids with Prandtl number 2.37 and 6.66 respectively. Increse in Prandtl number leads to decrease in temperature.

Figure 5 shows change in values of –θ'(0) with different Prandtl numbers.-θ'(0) denotes local heat transfer. It is noticed that Prandtl number increases as local heat transfer decreases.

Figure 6 shows change in volume fraction leds to volume fraction leds to change in Prandtl number. It is observed that with increase in volume fraction, Prandtl number decreases for both types of nnofluids. Decrease in Prandtl number of silver water nanofluid is more in comparison to titanium oxide nanofluid.

Figure 7 shows change in volume fraction causes change in density for both types of nanofluids. Increase in volume fraction increases density. Increse in density is more for silver water nanofluid than Titanium oxide nanofluids.

Hence following conclusions may be drawn:

- Nanofluids are better coolants than their base fluid as they have higher thermal conductivity.
- The increase in Prandtl number cause decrease in temperature for both silver water and titanium oxide water nanofluids.
- Both Prandtl number and heat capacity decreases with increase in volume fraction for both types of nanofluids
- Changing volume fraction also changes density and thermal conductivity for both types of nanofluids. Increase of volume fraction increases density and thermal conductivity and vice versa.

- Further, increase in volume fraction causes increase in heat transfer coefficient for both types of nanofluids.
- Viscosity remains unaltered for different types of nanofluids. However, increasing volume increase viscosity for both types of nanofluids.

Table 2. Thermophysical properties of silver Nanofluids

Φ	ρ_{nf}	$(Cp)_{nf}$	μ_{nf}	K_{nf}	$(Pr)_{nf}$
0.00	997.1	4179	0.001002	0.5970	7.02
0.01	1092.129	4139.55	0.001027	0.6556	6.05
0.02	1187.158	4100.1	0.001053	0.7161	5.01
0.03	1282.187	4060.65	0.001081	0.7858	4.21
0.04	1377.216	37550.82	0.01109	0.8488	4.02
0.05	1472.245	3981.75	0.001139	0.9432	3.66
0.06	1567.274	3942.3	0.001169	0.9991	3.17

Table 3. Thermophysical properties of Titanium oxide Nanofluids

ϕ	ρ_{nf}	$(Cp)_{nf}$	μ_{nf}	K_{nf}	$(Pr)_{nf}$
0.00	997.1	4179	0.001002	0.5970	7.02
0.01	1029.629	4144.072	0.001027	0.6153	6.92
0.02	1062.158	4109.144	0.001053	0.6428	6.84
0.03	1094.687	4074.216	0.001081	0.6596	6.76
0.04	1127.216	4039.288	0.01109	0.6701	6.69
0.05	1159.745	4004.36	0.001139	0.6902	6.56
0.06	1192.274	3969.432	0.001169	0.7013	6.43

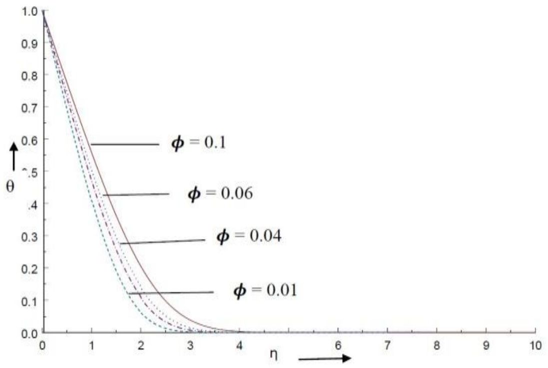

Figure 1. A comparative study of volume fraction against temperature profile for silver water nanofluid

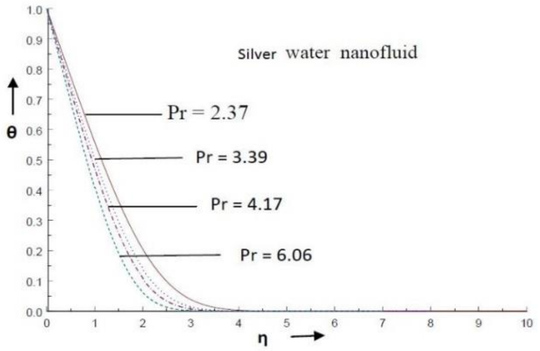

Figure 2. Temperature profile for various values of Pr no. of silver water nanofluid.

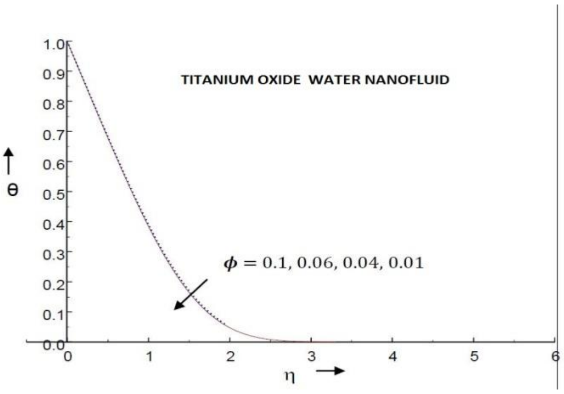

Figure 3. A comparative study of volume fraction against temperature profiles of Titanium oxide water nano fluid

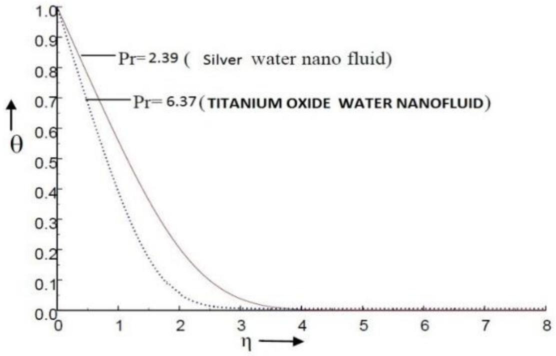

Figure 4. Temperature profiles of silver water and titanium water nanofluid

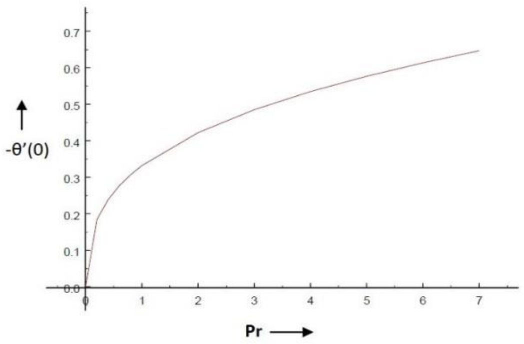

Figure 5. Effect of Prandtl number on local heat transfer

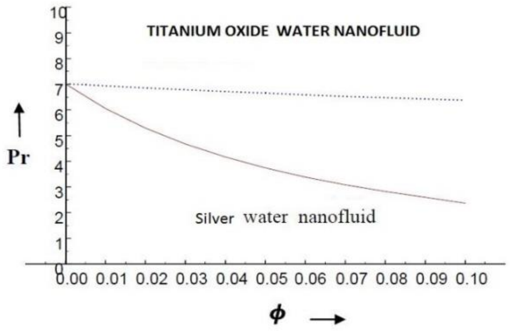

Figure 6. Prandtl number for different values of volume fraction for silver-water and titanium oxide water nanofluid

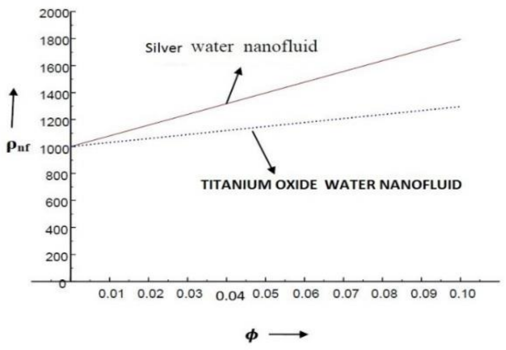

Figure 7. A comparative study of effect of volume fraction against density for silver-water and titanium oxide water nanofluid

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NOMENCLATURE

Cp	Specific heat
ρ	Density of fluid
B	Thermal expansion coefficient
T	temperature
G	Acceleration due to gravity
K	thermal conductivity
Nu	local Nusselt number
Re	Reynold’s number
A	thermal diffusiblity
B	thermal expansion coefficient
Φ	solid volume fraction
Θ	dimensionless temperature
η	similarity variable
μ	dynamic viscosity
f	fluids
nf	nanofluids
s	solid particles