



A Comprehensive Numerical Analysis of Natural Convection in Nanofluids within Various Enclosure Geometries: A Review

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ABSTRACT

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This comprehensive review elucidates recent numerical and empirical investigations of laminar-scale, free convection heat transport within two and three-dimensional enclosures filled with nanofluids and hybrid nanofluids. A particular emphasis is placed on previously explored enclosure geometries, ranging from simplistic to complex regular shapes. It is identified that a research gap exists within three-dimensional numerical investigations of both simple and complex enclosures. The study reveals that the local peak of the Nusselt number on the surfaces of the cylinder and the enclosure manifests at the locations where the shortest distance is observed between the cylinder surface and the enclosure wall. At lower Rayleigh numbers (Ra), the heat transfer demonstrates a decreasing trend with increasing values of volume fraction for oxide nanofluids. Upon increasing Ra, total heat generation is noted to decrease for all volume fractions and all positions of the conductive baffle. The Bejan and average Nusselt numbers display an inverted relationship to each other for the same range of Rayleigh numbers and solid volume fractions. Interestingly, the average Bejan numbers augment and the entropy generations remain approximately constant at high Rayleigh numbers. The analysis further unveils that the heat transfer rate experiences an increase with higher values of solid volume fraction and Rayleigh numbers. The average Nusselt number shows an increasing trend with the rise of the Hartmann number. Additionally, the heat transfer rate witnesses an enhancement with the increased concentration of Carbon Nanotube (CNT) particles and Rayleigh numbers. This review underscores the need for future research to address the present gaps and to further our understanding of heat transfer dynamics in nanofluid-filled enclosures.

1. INTRODUCTION

Large-scale natural convection, fluid flow inside enclosures, and buoyancy-driven heat transfer have garnered substantial attention in recent research [1-10]. Both numerical and experimental investigations have been conducted, allowing for a quantitative and qualitative examination of various complex phenomena. The advent of computer modeling and simulation techniques has facilitated this exploration, enhancing the importance of these studies [11-17]. Examinations of these phenomena have been conducted for a multitude of cavity geometries, including square and non-square enclosures. These studies have investigated a variety of conditions that enhance heat transfer, proposing methods to improve natural convection heat transmission within cavities. Such methods include altering the cavity's inclination or shape, as demonstrated by Bairi [9] and Majdi et al. [18] with their parallelogram-shaped cavities. Al-Rashed et al. [19-21] and Hussein et al. [22] introduced oblique cubical and trapezoidal enclosures, respectively. Including fins within enclosed cavities has also been proposed as a method for enhancing heat transfer [3]. Ma et al. [23] incorporated a square fin in his enclosed cavity, whereas the shape and location of the fins varied among other researchers. Charazed and Samir [24] employed a rectangular-shaped fin, and Saeid [25] experimented with multiple shapes, including two isosceles

triangles, one triangular shape, two opposite triangular shapes, and a rectangular shape. These fins or baffles were either solid or sporadically permeable, much like those used by Asl et al. [26]. The cavity described by Siavashi et al. [27], for instance, contained various porous fins.

Nanofluids, defined as suspensions of nanoparticles in liquids such as water, oil, ethylene glycol, and molten salts, have emerged as a new generation of advanced heat transfer fluids. These two-phase systems, comprised of metallic or metallic oxide nanoparticles, or nanotubes, exhibit high thermal conductivity due to the greater thermal conductivities of metals compared to fluids. However, the stability of nanofluids, a critical factor for their efficiency in engineering applications, is challenged by agglomeration among particles due to interactive forces, which influence dispersion, rheology, and performance. The characterization of nanofluids, therefore, plays an essential role in evaluating their stability. Various mechanical and chemical techniques have been introduced to improve the dispersion of suspended particles in liquids, thus reducing the effect of agglomeration on the stability of nanofluids. A comprehensive understanding of nanofluid stability can guide the preparation of stable nanofluids with enhanced properties for a variety of applications [28-31].

In response to the limitations of conventional nanofluids, a novel extension known as hybrid nanofluids has been introduced. These fluids are created by hybridizing two

distinct nanoparticles with disparate properties within a base fluid [32]. Hybrid nanofluids aim to improve thermal properties beyond those of base fluids and mono-nanofluids, capitalizing on the synergistic effect of the combined nanoparticles. The efficiency of heat transmission, pressure drop, and stability in heat transfer applications are expected to be enhanced by a well-balanced hybrid nanofluid [32]. However, further research is recommended to address issues such as long-term stability, increased pressure drops or pumping power, excessive viscosity, and the lack of a suitable thermal conductivity model [33]. Similar to mono-nanofluids, hybrid nanofluids undergo a comparable fabrication process and thermophysical characteristic investigation. According to the literature review by Idris et al. [32] and Sidik et al. [34], hybrid nanofluids can be prepared using one- and two-step methods. These methods are often chosen based on their cost-effectiveness for mass production and their efficacy for smaller productions involving multiple chemical stages [35].

Enhancing the properties of the working fluid via various Rayleigh numbers (Ra) has also been considered a viable technique [36-39]. In recent times, the introduction of fluid additives has been a common method for improving heat transfer performance [40-43]. Researchers have confirmed that adding metal nanoparticles or other materials in specific volume fractions can enhance thermal properties [44-46]. This fraction is typically not to exceed the range of $0 \leq \varphi \leq 0.06$, which has been found to enhance the properties of two-phase fluids while avoiding agglomeration or dissipation of solid particles [47]. Lastly, the use of an external magnetic field enhancement scheme is another technique [14, 48], which has significant applications in metal casting and other industrial applications [49-51]. On the other hand, some researchers have leveraged the advantages of porous media in designing their cavities [52-54]. The goal of this review is to provide a comprehensive overview of the latest advancements in heat transfer enhancement techniques. The review will explore the use of different geometries and modifications within an enclosed cavity, the application of nanofluids, the advent of hybrid nanofluids, and the various enhancement techniques like the introduction of additives and the use of magnetic fields.

2. NATURAL CONVECTION INSIDE VARIOUS GEOMETRY CAVITIES

For various working fluids, including nanofluids and hybrid nanofluids, the natural convection heat transfer in various three-dimensional cavities has been investigated computationally and experimentally. Table 1 below display Natural convection in a classical enclosure filled with nanofluid and Table 2 below display Natural convection in a complex enclosure filled with nanofluid; while Table 3 below display Natural convection in a classical enclosure filled with hybrid nanofluid and Table 4 below display Natural convection in a complex enclosure filled with hybrid nanofluid respectively [54-120].

2.1 Free convection in a classical cavity with nanofluid

Lee et al. [55] studied the temperatures of a warm inner circular cylinder and a chilly exterior square cylinder are different, which in mathematics leads to natural convection. The heat plume and the gap, which were impacted by conduction and convection, respectively, produced surface maxima of local Nusselt numbers along the cylinder and

enclosure surfaces. Numerical analysis was used to examine the impact of nanoparticles on the free convection of water-based nanofluids confined in an open, rectangular container [56]. It was anticipated that the left wall would be at temperature T_h . A fluid with a purported temperature of T_c was reportedly entering the cavity region from the right end. According to research, for low Rayleigh numbers, the volume fraction of oxide nanofluids demonstrates a decreasing propensity for heat transfer. However, with higher Rayleigh numbers, a tendency toward increasing heat transmission was seen because of a rise in the nanofluid volume percentage. Free convection and entropy production in Cu-water nano-fluid were numerically investigated in a container with a conductive baffle integrated into the bottom hot wall [57]. Where a conductive copper baffle was located, the enclosures bottom wall was consistently warm (T_h). The temperature (T_c) on the left and right-side walls was lower than the temperature on the lower wall, and the upper wall was isolated. At maximum Rayleigh numbers, more convection leads to a higher total Nusselt number by raising the volume percentage and moving the baffle closer to the cavity's center. In a three-dimensional cubical enclosure filled with water/ Al_2O_3 nanofluid, the influence of the Rayleigh number was investigated [58]. Solid volume percentage, on free convection heat transfer and fluid movement, and entropy production, were quantitatively studied. The right sidewall of the enclosure was a temperature that is isothermally cold, while the left sidewall was a temperature that is isothermally hot. The surrounding enclosure walls were thought to be isolated. According to the findings, The solid volume percentage of nanoparticles and the Rayleigh number both lead to a rise in the mean Nusselt number. Additionally, the Bejan and mean Nusselt numbers react differently for the same range of Rayleigh numbers and volume percent. In square, bottom-heated, inclined, and differentially heated enclosures with a nanofluid, laminar natural convection was investigated [59]. The mean Nusselt number can be improved with higher Rayleigh numbers due to increased convection by raising the volume percentage and moving the baffle closer to the cavity's center. According to the findings, there was a heated wall of the enclosure that had an angle that corresponded to a maximum Nu number that was dependent only on the Ra number and independent of the particle volume concentration. Inside a square container with a nanofluid, natural convection was numerically explored [60]. The local temperatures T_h (from the heat source) and T_c (from the local cold temperature) are kept constant on the enclosure's vertical walls to the sides, respectively. The assumption of adiabaticity for horizontal walls. The outcome showed that the mean Nusselt number rises as the volume percentage of nanoparticles rises. Additionally, demonstrate that the Cu_water nanofluid performs well in terms of heat transmission. As the heat source's length diminishes, the mean Nusselt rate rises. In an enclosure containing nanofluids and subjected to different thermal boundary conditions at the sidewalls, the heat transmission with steady MHD-free convection chilling in the presence of an angled magnetic field was studied [61]. The rate of heat transmission is significantly accelerated by the nanoparticle in the base fluid, as has been shown. As the heat source gets longer, the mean Nusselt number goes down. The free convection of water-based nanofluids in a square cavity filled with nanofluids and partially heated at the lower wall is investigated numerically using different types of nanoparticles [62]. The cavity's staying wall is kept at a low temperature. For the whole range

of Rayleigh numbers, a rise in the nanoparticle's percentage was found to speed up heat transmission. The type of nanofluids was shown to have a substantial impact on the enhancement of heat transmission. The thermal efficiency of a cubic enclosure containing a magnetic nanofluid was investigated in the context of the Lorentz force and an inductive electrical field based on a magnetic field [63]. The results show that the Lorentz force is imposed in a direction orthogonal to the applied magnetic field and inversely proportional to the speed of the nanofluid. The previously described force was significantly affected by the volume percentage of nanoparticles, the strength of the magnetic field, and the velocity of the nanofluid. The results demonstrate that the thermal behavior of the base fluid is unaffected by the Lorentz force. In order to highlight the impact of magnetic field orientation on the Rayleigh-Bénard convection of a nanofluid (Al_2O_3 -water) inside a square enclosure, a numerical analysis was conducted. It was believed that T_h and T_c represented the temperatures at the lower and upper walls, respectively, of the square container with vertical, adiabatic walls that is involved in the problem [64]. The mean Nusselt number was shown to grow when the Hartmann number increased. Only when a magnetic field is present does the base fluid's thermal transfer improve when nanoparticles are added. Joubert et al. [65] studied the effects of enclosure volume percentage, magnetic field design, and magnetic field intensity on a magnetic nanofluid's free convection heat transfer in an enclosure that was differentially heated with and without the addition of an external magnetic field. The Nusselt number and mean heat transfer coefficient for the nanofluids were determined using the information from the cavities. Al_2O_3 -water nanofluid heat transmission in a space with vertical hot and cool walls was numerically examined [66]. The findings support experimental work in that dispersing nanoparticles in the base fluid reduces natural convection heat transfer. Al-Rashed et al. [67] examined the inclination angle's effect of a magnetic field on the free convection taking place inside a cubic cavity containing a carbon nanotube (CNT)-water nanofluid. Despite the fact that the vertical walls were heated differently, it was thought that the horizontal walls were isolated. The Nusselt number, isosurfaces of temperature, particle trajectories, velocity vectors, and the findings of a fluid flow using a single-phase model were used to show the results. A higher Rayleigh number and a higher CNT particle percentage were shown to result in the heat transmission being more intense. In a small space with CNT-nanofluid that was heated differently in all three dimensions, it was examined how a partially active magnetic field affected heat transfer by free convection [68]. A magnetic field was provided to the upper and lower halves of two cases to observe this effect, with the remaining walls being insulated in both cases. The outcome revealed that, even at the same Hartmann number, the placement of the magnetic field is crucial. The result is may be a useful metric for regulating the movement of heat and fluid inside a closed space. A closed cavity saturated with a nanofluid was numerically investigated to examine the turbulent free convection and the creation of entropy [69]. While the others were isolated, the horizontal walls were termed isothermal at various temperatures. The findings demonstrate that, in contrast to entropy, the Nusselt number rises with rising Rayleigh number when nanofluids are used. The natural convection was quantitatively examined in a square hollow containing an electrically conductive nanofluid and driven by a recurring temperature profile along one of the

vertical walls [70]. The upper and lower horizontal walls were retained as isolated. The temperature of the right wall was kept low, while the opposite vertical wall's temperature fluctuated sinusoidally over time at an average temperature. In contrast to lower Rayleigh numbers, where convective flow predominates, which Nusselt numbers are independent of, the finding was that Nusselt numbers are independently of both Rayleigh and Hartman numbers. In a three-dimensional cavity filled with nanofluids, natural convection heat transport was numerically explored [71]. A barrier kept at a hot, steady, and uniform temperature heats up this arrangement. While the rest remained adiabatic, the left- and right-side vertical walls were kept at a cold temperature of T_c . The cavity's fluid flow and heat transfer were examined for different combinations of the governing factors, specifically the type of nanofluid. According to the findings, at large Rayleigh numbers, the nanofluid's influence on natural convection was particularly noticeable. Intensification of the flow and a rise in the heat exchange are because of a rise in the volume percentage. A numerical examination of the free convection of a Fe_3O_4 -water nanofluid packed inside a square chamber with a rounded diagonal corner is the study's main objective [72]. A single magnetic source and the cavity received applied temperatures (hot and cold walls). We evaluated the effects of the Rayleigh number, Hartmann number, and round corner radius on the flow of nanofluids (streamlines and magnitude of velocity) and heat transfer (isotherms and temperature distribution). When convection or conduction dominates, depending on the kind of flow, heat transfer is measured. In a square cavity filled with boron-water nanofluid, free convection and entropy production were measured when the left vertical wall was heated and the right vertical wall was cooled while the horizontal walls were kept apart [73]. The results were compared to those of copper (Cu) and aluminum oxide (Al_2O_3) nanoparticles, which are routinely used in complex enclosures with nanofluid for natural convection applications. The results show that entropy generation dominates fluid friction irreversibility as Rayleigh number rises for all nanoparticles and that entropy formation increases with Rayleigh number for all nanoparticles. At high Rayleigh numbers, fluid friction accounts for the majority of entropy generation. In a cubic cavity under the influence of alternating and time-unvarying magnetic fields, Cu-water nanofluid natural convection heat transport was investigated numerically and experimentally [74]. For a range of Grashof values, the impact of the nanofluid volume percent on free convection heat transfer is examined. Four sides of the cubic container were insulated. One of the vertical plates received a consistent heat flow, and another vertical plate's temperature remained chilly at all times. Different Grashof numbers, nanoparticle volume fractions, and Hartmann numbers were reported along with the results of the hot wall mean temperature and mean Nusselt number.

2.2 Free convection in a complex cavity filled with nanofluid

Saleh et al. [75] examined the use of nanofluids to enhance heat transmission in a trapezoidal enclosure. A stream-vorticity framework was used to describe transport equations. The temperature in the left enclosure is always hot (T_h), while the temperature in the right enclosure is always cool (T_c). The horizontal straight walls at the upper and lower are kept isolated. Testing was done on water-Cu and water- Al_2O_3 nanofluids. The findings reveal a novel link between the mean

Nusselt number, the Grashof number, effective thermal conductivity, viscosity, and sloping wall angle. Water and copper form a nanofluid in a parallelogrammic cage that is differentially heated for the study of laminar free convection [76]. According to the findings, the inclusion of copper-water nanofluid significantly enhances the rate of heat transmission, and Changes in skew angle have an impact on the convection vortex's form. The researchers explored the quantitatively unstable spontaneous convection of a water-based nanofluid in a right-angle trapezoidal cavity generated by a uniformly inclined magnetic field [77]. The temperatures inside the cavity are kept at varied but constant levels on the left vertical and right sloping walls. The upper and lower horizontal enclosure walls are adiabatically heated. Along with the aspect ratio, dimensionless time, and angle of inclination of the magnetic field with respect to the gravity vector, the Rayleigh, Lewis, and Hartmann numbers are calculated. This parameter's impact on the development of streamlines, concentrations, and isotherms is examined, as well as how it affects the usual Nusselt number along the hot wall. In a nanofluid-filled right-angle porous trapezoidal container, constant free convection has been numerically analyzed [78]. The temperature of the sloped wall is maintained T_c , whereas the vertical wall is kept at T_h , where T_h is greater than T_c . Additionally, the enclosures upper and lower walls are isolate. It was discovered that Lewis numbers and aspect ratio parameter are decreasing functions of Rayleigh number, whereas Rayleigh numbers are a rising function of flow strength and Nusselt and Sherwood numbers. In a trapezoidal hollow that was partially filled with a porous nanofluid layer and partially filled with a non-Newtonian fluid layer, natural convection was quantitatively examined [79]. Water-based nanofluids with titanium nanoparticles and silver, copper, or alumina are chosen for investigation. The left sloping wall is kept at a steady temperature of T_h , while the right sloping wall is kept at T_c . The horizontal walls are separated from one another in the cavity. This value tends to decrease lower than other values ($\phi=0, 0.05$, and 0.1) when the Darcy number concentration is between $(10^{-4}, 10^{-3})$ because of the sloping walls of the cavity. It was shown that a maximum nanoparticle volume percent ($\phi=0.2$) results in a maximum overall Nusselt number. Cu-water nanofluids' free convection heat transfer was numerically examined in a parallelogrammic cavity with porous media [80]. While the sidewalls are only vulnerable to a little temperature variation, the enclosure's bottom and top are insulated. We take into account the Darcy flow and the nanofluid models proposed by Tiwari and Das. The results for streamlines, Nusselt numbers, and isotherms were provided in accordance with the percentage of nanoparticle volume, porosity, different types of porous matrix, aspect ratio, inclination angle, and different Rayleigh numbers. The free convection heat transfer in a carbon nanotube-EG-water nanofluid-filled trapezoidal container has been quantitatively investigated using a variety of parameters [81]. The side walls of the trapezoidal cavity were thermally insulated from the bottom and top walls of the cavity, which were kept at constant hot and cold temperatures, respectively. The findings demonstrate for all solid volume fractions, at low Rayleigh numbers, the mean Nusselt number (Nu_{Avg}) falls with rising angle of inclination (aspect ratio) ($Ra \leq 104$). Job et al. [82] studied numerically symmetrical wavy trapezoidal enclosures, Nanofluids made of single-walled carbon nanotubes (SWCNT) and Al_2O_3 in water are unstable in MHD-free convection. At first, the fluid inside the container is at rest and is always cool.

The thermal isolation of the upper wall causes the temperature of the wave-like lower wall to rapidly rise to a steady state of heat. According to the results, both nanofluids' flow velocities increase as Rayleigh numbers rise but decrease when Hartmann numbers climb. In order to see natural convection by heat lines, discrete iso-flux heating from the left sidewall of three different kinds of wavy, inclined enclosures filled with Al_2O_3 -water and Ag-water nanofluids was investigated [83]. The remaining areas of the left sidewall and the right sidewall were both isolated. According to the results of this work, heat functions for base and nanofluids rise for all three types of horizontal wavy enclosures as wave amplitude and Rayleigh number climb. In a tilted T-shaped enclosure containing several kinds of nanofluids, the laminar stable magneto-hydrodynamic natural convection was numerically investigated [84]. A portion of the enclosure's upper wall included an implanted uniform heat source. The other cavity walls were believed to be isolated, while the left and right sides of the enclosing leg were kept at a constant cold temperature. According to the findings, the mean Nusselt number decreases with increasing Hartmann numbers and heat source length, whereas it increases with heat source location, inclination angle, Rayleigh numbers, aspect ratio, and nanoparticle volume percentage. A successful computer simulation of free convection in an oblique hollow filled with silver water nanofluid for varying Rayleigh numbers, volume fractions, and slope wall inclinations was achieved. A constant cold temperature (T_c) was maintained on the left- and right-side walls of the hypothetical oblique hollow [85]. The top wall of the enclosure was kept insulated, while the floor was kept at a constant, heated temperature (T_h). It was found that as the Rayleigh number and the proportion of solid volume in the nanofluid increased, the average Nusselt number increased for all inclination angle ranges. Alsabery et al. [86] analyzed numerically free convection with spatial side-wall temperature in a trapezoidal cavity. The experiment will use water-based nanofluids containing Ag/ Cu or Al_2O_3 nanoparticles. The left sloping wall is implicit to assume side-wall hot temperature, which is greater than the right sloping wall cold temperature. The horizontal walls are isolated. It has been demonstrated that the rate of heat transfer rises with non-uniform heating increments, with convection having a greater impact on low wave number values. The best side wall inclination angle leads to the best heat transfer increase. With the use of a nanofluid made of water/copper nanoparticles, the natural convection in concave and convex parabolic enclosures was numerically studied [87]. The sidewalls of the enclosures were isothermal at a cold temperature, whereas the upper and lower walls are isolated. According to the findings, both types of enclosures' water and nanofluid flow circulation decrease as the heat source's location rises. For the purpose of simulating free convection, a fully open parallelogrammic chamber that was filled with Cu/water nanofluid and locally heated from its lower wall was employed [88]. The cavity's upper wall is thought to be completely open to the outside, while a particular spot in the lower wall is exposed to a nearby hot heat source. The bottom wall's other areas were kept adiabatic. The cavity's right, left sidewalls were both kept at an isothermal cold temperature and were thought to be inclined. The nanofluid entered through the top wall and was thought to be chilly, but it emerged from the same wall in an adiabatic state. The end result was the mean Nusselt number rises as the nanoparticle volume percent and Rayleigh number rise. Selimefendigil [89] examined the numerical analysis of free convection in a

trapezoidal cavity with conductive barriers and nanoparticles in a variety of shapes (spherical, blade-shaped, and cylindrical). The trapezoidal hollow's side walls were kept constantly heated and cold, while its upper and lower walls were isolated. The mean Nusselt number develops linearly with nanoparticle volume percent and is unaffected by the type of blockage. The slope of the curves is greatest for cylinder-shaped nanoparticles. Carbon nanotube (CNT) nanofluid was used to simulate three-dimensional natural convection flow within an open trapezoidal chamber [90]. In her research, the other walls were taken to be ideal thermal insulators, while maintaining an isothermal hot temperature on the inclination wall, and cold nanofluid entered the cavity through its right open boundary. It was discovered that an increase in Ha with maximum values of the Rayleigh number is associated with a reduce in the rate of heat transfer, whereas a rise in the volume percent of solids leads to a rise in heat transmission independent of the magnetic field's inclination. In a partially heated trapezoidal chamber with an inner cylindrical barrier, the free convection of an Al_2O_3 -water nanofluid was calculated numerically [91]. The trapezoidal cavity's other sides are locked at a lower temperature of T_c ($T_c < T_h$), while the bottom wall is kept at a greater constant isothermal state at T_h . As the Hartmann number computation gets easier, The cavity's non-dimensional velocity decreases from its greatest value. The Hartmann number rises when the local Nusselt number's lowest value drops. The experimental work that has been studied enhances the overall efficiency of the assembly [92]. Depending on the design under consideration, the inclined fins implanted in the wall that create open enclosures with parallelogram sections alter the aerodynamics in the active enclosure and increase the mean Nusselt number by 7% to 23%. The natural convective heat transmission is not considerably affected by the distance between the enclosure's two active walls when the aspect ratio is greater than 0.2, according to measurements on a 1/5-size prototype. These results are applicable across the Rayleigh number's treatment range. Ullah et al. [93] simulated numerically the free convection in a trapezoidal container filled with SWCNT-water nanofluids and heated by a flame source. Rayleigh number, heat generation parameter, and flame size modifications were made in order to explore the significant behavior of the isotherm and steam lines. When $r=0.6$, The Rate of heat transfer was discovered to be at its highest. In buoyancy-driven flow, the local Nusselt number rises.

2.3 Free convection in a classical cavity filled with hybrid nanofluid

Numerical convective heat transfer was investigated using a nanofluid enclosed in a right-angled triangle container heated by a sinusoidal temperature maintained from the lower side and subjected to a continuous magnetic field [94]. The triangular cavity's hypotenuse side was kept at a constant, cool temperature, and the other side was isolated. According to the results, both the nanoparticles' Rayleigh number and Hartmann number have a decreasing relationship with the Nusselt number. Results were compared with data that had already been collected by other writers. In a cubic enclosure with a micropolar hybrid nanofluid $Cu_Al_2O_3$ /water, natural convection was numerically analyzed [95]. The results show that the thermal conductivity and dynamic viscosity coefficient in an Al_2O_3 /water micropolar nanofluid tend to increase with the addition of Cu . The results showed that when

hybrid nanofluid, as compared to standard nanofluid, was employed with increasing solid volume concentrations, both heat and mass transmission increased. Furthermore, as the micropolar vortex parameter is increased, the mean Nusselt and Sherwood values reduce. In a superposed container with layers of composite porous-hybrid nanofluid, the numerical buoyancy-driven flow was investigated [96]. The remaining portions of the wall were thermally isolated, and the lower wall of the cavity was partially heated to produce heat flux. The temperature in the upper and vertical walls of the cage was kept at a consistent, freezing level. The findings showed that the hybrid nanofluid ($Cu-Al_2O_3$ /water) has a maximum rate of heat transfer than the pure fluid. Additionally, at $Ra \leq 105$. Using a square enclosure, the free convection of Al_2O_3 -MWCNT/water nanofluids at different bi-nanoparticle percent weights (Al_2O_3 -MWCNT; 80:20, 60:40, 40:60, and 20:80) was studied. In a square enclosure, Al_2O_3 -MWCNT/water nanofluids have demonstrated better natural convection performance [97]. For the purpose of forecasting Nuav, a new correlation based on Ra and the ratio of bi-nanoparticles has been established. The outcomes of this investigation further support the benefit that hybrid nanofluids have over single-particle nanofluids. Numerical analysis was done to determine how the hybrid Al_2O_3 - Cu /water nanofluid ($water-Cu/Al_2O_3$) nanoparticles in a porous square cavity affected the production of entropy and MHD convection [98]. the numerical findings demonstrated by streamlines, isotherms, velocity profiles, and the Nusselt number. They show that as Ra rises from zero to one hundred, the isotherms that are sensitive to estimation differences under Ha grow. Conduction transfer was more apparent at higher Ha . Additionally, it was noticed that convective heat transfer weakens with a rise in Ra while strengthening with a rise in Ha . Ra rises, resulting in a stronger flow cell. Hybrid nanofluids were utilized in the experiment, which was centered on free convective heat transfer [99]. The free convection of alumina-multiwalled carbon nanotube/water hybrid nanofluids was studied in a square cavity with two isothermal ($AR=1$) vertical walls. Two steps were used to make these nanofluids at a certain weight ratio of 10:90 Al_2O_3 : MWCNT at different nanoparticle concentrations in volume of 0.00, 0.05, 0.10, 0.15, and 0.20 vol%. Al_2O_3 -MWCNT/water hybrid nanofluids were applied, and the results showed increased natural convection. that compared to single nanoparticle nanofluids, hybrid nanofluids offer an advantage. Finally, it can be said that compared to DI water, hybrid nanofluids have much better heat transfer capabilities. In a square cavity with three layers, the impact of the power law index on the convective heat transfer of hybrid nanofluids was examined [100]. The impact of the solid fluid layer was another factor taken into account. The mean Nu was found to increase as Ra increased, reaching its highest value when $Ra=106$. By increasing Da , there was little change in the isotherms at the solid layer. By amplifying the undulations, the mean Nu falls. Raising the thermal conductivity ratio increased the rate of heat transfer at the heated boundary and solid fluid contact in the enclosure. Free convection heat transfer in a square porous chamber filled with a hybrid nanofluid made of water [35% MWCNT and 65% Fe_3O_4] and having a solid, wavy, finite wall [101]. The right wall is kept at a low temperature, while the left wavy wall is heated to a constant temperature. The top and bottom walls are thermally insulated. According to the findings, the mean Nusselt number rises as Ra rises while falling when Darcy and amplitude wave numbers rise. The free convection brought on by the

MWCNT-Fe₃O₄/Water hybrid nanofluid is investigated using the Lattice Boltzmann Methodology [102]. A rectangular container with differential heating is filled with the test fluid. Aspect ratios between (0.5-2.0), Rayleigh numbers between 103 and 105, and nanocomposite volume fraction all have an impact on the properties of heat and fluid flow as well as the creation of entropy. According to the findings, the Nusselt number grows as the Rayleigh number rises while falling as the aspect ratio rises. However, as the volume percentage of MWCNT-Fe₃O₄ rises up to 0.001, the mean Nusselt number increases. The average Nusselt number for the MWCNT-Fe₃O₄ nanocomposite either does not significantly increase or deteriorates as the volume percentage is further increased. A rectangular enclosure containing hybrid nanofluids (50 percent CuO, 50 percent Al₂O₃) and water has been used in experimental studies to study free convective heat transfer [103]. Phase-change material (PCM) serves as the wall that connects this cavity. While maintaining adiabatic conditions on the horizontal walls, the vertical walls were heated to various temperatures. The findings have been verified and show strong agreement with earlier work. The primary findings also revealed that as the concentration of nanomaterials increased, so did the rate of free convection's ability to transport heat. Symmetrical vortices may develop close to the heated wall as a result of high temperature-induced increases in natural convection fluxes. In a porous annulus between a zigzag triangle and different cylinders, Cu-TiO₂/EG hybrid nanofluid heat transfer under the influence of an angled magnetic field was numerically evaluated [104]. The inclined wall is heated by a source and is set to T_h , while the right-angled wall is cool and has a preset temperature, T_c . Both the cylinder and the triangle's base are adiabatic. The obtained Rayleigh number and the volume percentage of the nanoparticles can be considered. Features that are critical in influencing convection. The existence of a magnetic field restricts heat transport and raises the Hartmann number. Thermal transmission can be enhanced by triangular obstacles. The efficiency of convective flow can be affected by the cylinder's angular velocity. An angled partial periodic magnetic field's impact on a hybrid nanofluid's natural convection flow in isosceles right triangle cavities was quantitatively explored [105]. Additionally, the entire magnetic field is contrasted with the presence of a periodic partial magnetic field. Reduced fluid velocity and convective heat transfer were the results of the increase in Lorentz force. In every instance, During the initial phase of the periodic magnetic field, it was clear that there was an inhibitory impact. A partial periodic magnetic field may be an effective controller wherever fluid velocity and temperature must be kept under control inside the enclosure. Numerical analysis was done to determine whether natural convection would occur in a square cavity that contained a Cu, Al₂O₃, and water hybrid nanofluid and was internally two blocks of heat production provide the heat [106]. The two identically sized blocks are positioned at the same height and produce the same amount of heat. The enclosure was perfectly isolated by the remaining walls and is cooled by the right wall at a constant temperature, T_c . The findings showed that as the Rayleigh number rises, the cavity's heat exchange improves, lowering the maximum temperature.

2.4 Free convection in a complex cavity filled with hybrid nanofluid

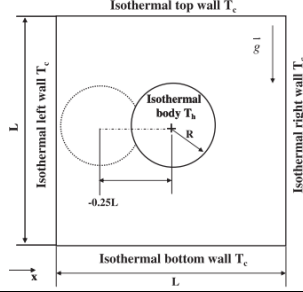
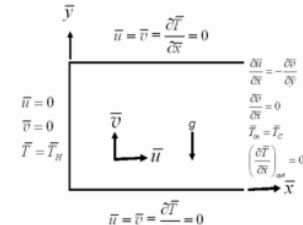
A hybrid nanofluid's free convection heat transfer into a

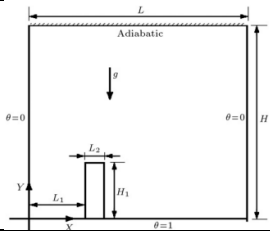
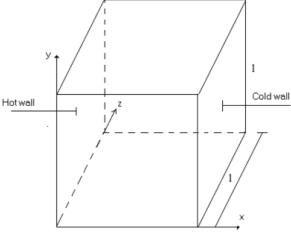
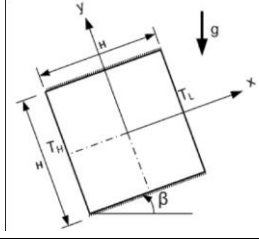
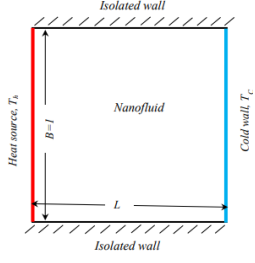
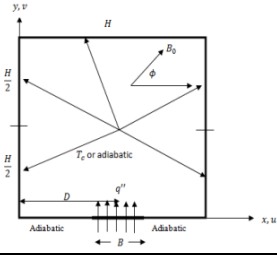
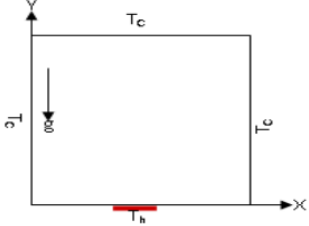
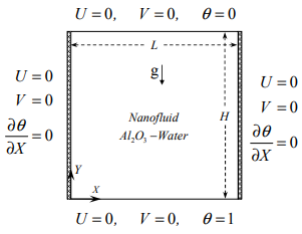
porous cavity subjected to a changing magnetic field was studied [107]. The upper and lower walls in this configuration had isothermal lines that extended from the left to the right side and vice versa, implying raised free convection heat transmission. Additionally, with larger Rayleigh numbers, a significant reduction in the Nusselt number is seen by raising the Hartmann number, this is due to the flow's reduced power. MgO-MWCNTs/EG hybrid nanofluid flow and heat transmission in a complex shape containing a porous medium were analyzed [108]. A constant, angled magnetic field and radiation effects were applied to the container. In addition to having an annular geometry, the cavity has an isothermal, wavelike exterior cold wall. Porous material employed in the porous space included glass balls and foam made of aluminum metal. The findings revealed that the overall improvement would decrease as the volume proportion of nanoparticles increased. The maximum enhancement value (Nusselt number ratio compared to the pure fluid) for a glass ball is roughly 1.17, and it is roughly 1.15 for aluminum metal foam when the volume percentage of hybrid nanoparticles is as low as 0.2 percent in a convective-dominant regime of natural convection flow with a Rayleigh number of 107. Entropy generation, fluid movement, and heat transport were all considered in connection to Cu-Al₂O₃-water hybrid nanofluids inside a complex-form container with a hot-half wall. the upper and lower surfaces of the enclosure are still isolated, but the sidewalls are formed of wavy walls with a cool isothermal temperature [109]. Heat transmission was shown to be larger in nanofluids than in pure water, and it was greatest in the instance of the Water-Cu-Al₂O₃ hybrid nanofluid. Increasing the volumetric flow rate of the nanoparticles improves heat transmission due to the increased thermal conductivity. Ra increases heat transmission because of the increased relevance of driving buoyant forces over resistive viscous forces. A hybrid nanofluid in a two-dimensional trapezoidal enclosure was examined for MHD-free convection via numerical modeling [110]. The enclosure was sinusoidally warmed from the bottom wall, while the sloped sides became cooler and the top wall remained insulated. As a working fluid, a hybrid nanofluid composed of MgO-Ag and water was utilized. It was found that as Ra increased, so did the strength of the stream functions and isotherms, whereas an increase in Hartmann number lowered flow circulation while increasing the strength of the isotherms. Isotherms, stream functions, and local and mean Nusselt numbers were used to present the results. The laminar flow of a nano-liquid in a trapezoidal cavity is explained, as are the convective exchanges that occur there [111]. The chamber in issue was trapezoidal in shape and contained a hybrid Cu-Al₂O₃/water nanofluid. An external magnetic field was acting on the enclosure with the zigzagged wall, which was consistently tilted at an inclination angle. γ . Therefore, raising the volume percentage of the hybrid nano-liquid becomes important in circumstances when free convection is weak. As the size of the heating element gets smaller, the Nu number rises. Parvin et al. [112] studied a numerical operation carried out in a curved cavity filled with a hybrid nanofluid that contained an inner cylinder with a wavelike shape and a magnetic field. By mixing copper and alumina nanoparticles with a normal water-based solution, the hybrid nanofluid was created. Natural convection flow occurred in the hollow as a result of the temperature differential between the hot, wavy cylinder inside and the cold, curve-shaped exterior. Heat transmission and fluid motion rates were shown to be substantially faster at higher values of

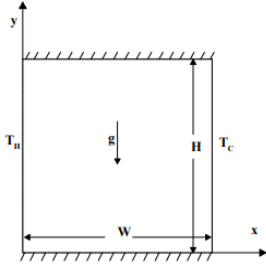
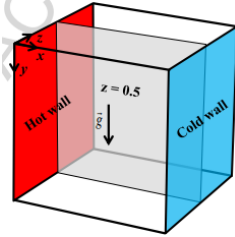
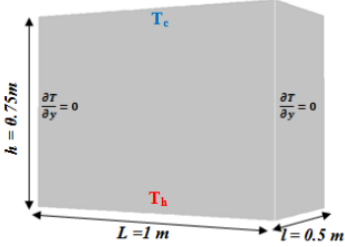
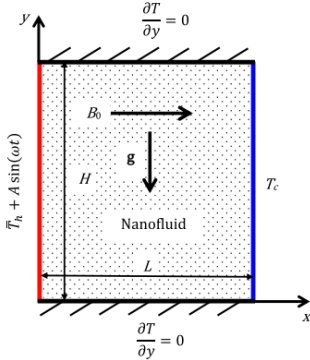
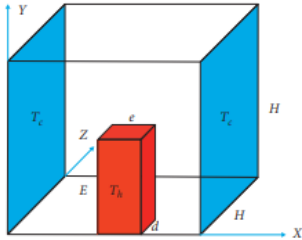
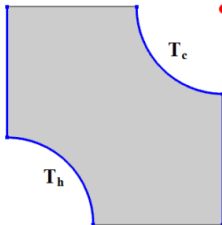
Ra. To improve the effectiveness of the heat transfer rate, the basic fluid was mixed with hybrid nanoparticles (Cu-Al₂O₃). When Ra is greater and the base fluid contains more hybrid nanoparticles, better heat transfer performance is attained. Al Using MHD nano liquid convective flow, a multi-walled carbon nanotube iron (II, III) oxide (MWCNT-Fe₃O₄) hybrid-nanofluid flow in an irregularly shaped cavity containing this fluid was investigated [113]. The cavity's interior and external borders were isothermally kept at temperatures of Th and Tc, respectively, while the cavity's side walls were adiabatically heated. It was discovered that increasing Ra results in an improvement in the thermal exchange rate within the strangely formed cavity. Higher Rayleigh and Darcy numbers were more pronounced in the decline in Nusselt numbers relative to Ha. Ghalambaz et al. [114] examined the thermal convective heat transfer and produced a water-Cu-Al₂O₃ hybrid nanosuspension that was irreversible in a peculiarly formed enclosure. The enclosure's interior and external borders are kept isothermally constant at temperatures of Th and Tc, respectively, which are high and low, and the side walls are adiabatic. In accordance with the findings, for all Rayleigh number magnitudes, raising the nanoparticle concentration accelerates the rate of entropy creation. Additionally, raising the width ratio (WR) speeds up heat transfer for high Rayleigh values. The efficiency of free convective heat transport within a rectotrapezoidal cavity containing an Al₂O₃-Cu and water hybrid fluid was investigated numerically [115]. The cavity's remaining walls are regarded as chilly, with the lower wall being evenly heated and the upper horizontal wall being insulated. The results of a unique thermophysical relationship that controls the thermal conductivity of the hybrid nanofluid are in agreement with those of the experiments. The research showed that adding nanoparticles reduced the friction factor, or pumping power, while increasing the thermal efficiency index and heat transfer rate. In the study of Asmadi et al. [116], a U-shaped hollow with isolated wavy sides is subjected to a numerical analysis of the free convection heat transfer of a copper-alumina hybrid nanofluid. The results show that the addition of hybrid nanofluid enhances thermal performance within a U-shaped cavity by 12% for two isolated wavy walls

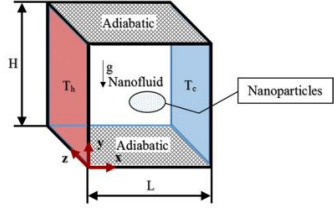
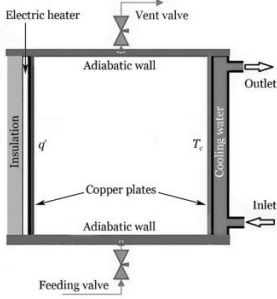
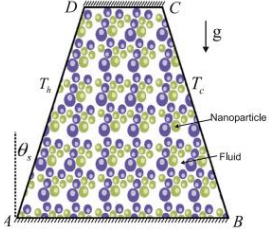
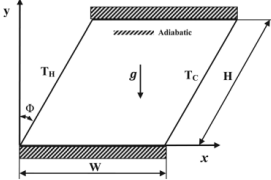
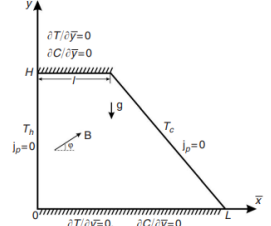
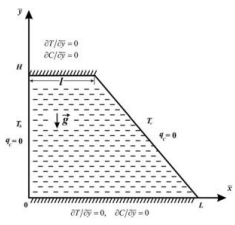
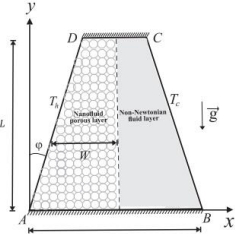
and by an average of 7% for one isolated wavy wall. The rate of heat transmission rises along with the Rayleigh number. In every case, the mean Nusselt number reduces as the amplitude and undulations increase. By adding a magnetic field and a semi-circle heater to the lower wall of the MWCNT-Fe₃O₄/water hybrid MPNF, the free convection and entropy of the system were studied [117]. In order to track changes in Nuavg and Sgen, the cavity's angle with regard to the horizontal was also changed. By rising the Ha and micro-rotation parameters, streamline intensity is reduced. When the micro-rotation parameter is increased from 0 to 2, Nuavg will decrease by 27%. The continuous thermo-gravitational convection of a Cu-Al₂O₃ (50-50%) water hybrid magneto-nanofluid was investigated using a novel cavity [118]. The enclosure was controlled using flush-mounted variable heaters and magnetic field-dependent (MFD) viscosity. Depending on where the heaters are located on the left and bottom limits, several cases (Cases I to IV) have been taken into consideration. Results showed that the location of the heaters and their geometry have a significant impact on all transport processes. In addition, compared to other physical factors, Cu-Al₂O₃ hybrid nanoparticles play a crucial role in elevating thermal transmission. The mean Nusselt number rises as a result of the incorporation of hybrid nanoparticles. In the presence of an applied magnetic field at an angle to the horizontal axis, the heat transfer of a hybrid nanofluid housed in a container with several heat sources on the bottom wall was examined [119]. The results show that changing the magnetic field parameter, the magnetic field's angle, the quantity and spread of heat sources, and the Rayleigh number significantly altered the flow pattern. There are more heat sources, a stronger magnetic field, a maximum Rayleigh number, and a maximum average Nusselt number. In a cavity with a trapezoidal shape and under the influence of partial magnetic fields, a hybrid silver-magnesium oxide-water nanofluid's natural convection flow is examined numerically [120]. As Rayleigh's number rises, buoyancy's influence over Lorentz's force becomes more evident. Convective heat transmission and fluid flow are both suppressed by the increase in Lorentz force.

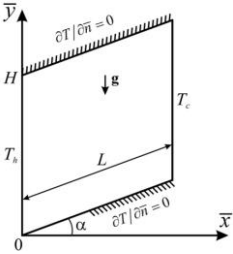
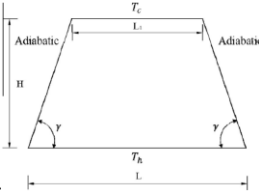
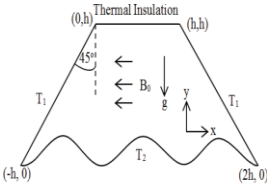
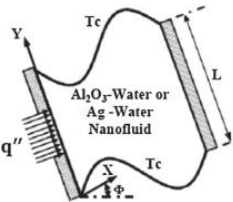
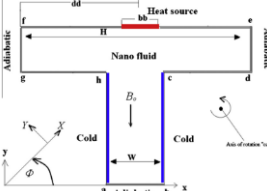
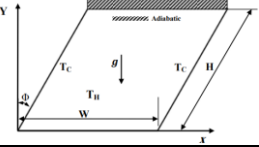
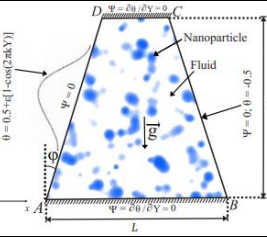
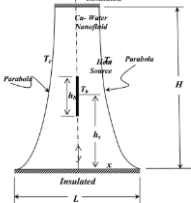
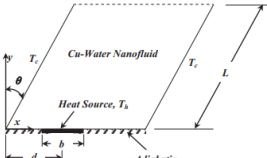
Table 1. Free convection in a classical cavity filled with nanofluid

References	Fluid Type Parameters	Model	Shape	Conclusion
Free Convection in a Classical Cavity Filled with Nanofluid.				
[55]	$10^3 \leq Ra \leq 10^6$	FVM		the local peak of the Nusselt number on the surfaces of the cylinder and the enclosure appear at the locations where the shortest distance between the cylinder surface and the enclosure wall.
[56]	$10^3 \leq Ra \leq 10^6$ $0.05 \leq \phi \leq 0.2$ water-Cu nanofluid	numerical simulations		low Rayleigh numbers, heat transfer exhibits a decreasing trend for increasing values of volume fraction of oxide nanofluids

[57]	$10^4 \leq Ra \leq 10^6$ $0 \leq \phi \leq 0.08$ Cu-water nanofluid	FVM		The total entropy generation decreases by increasing the Ra for all volume fractions and all positions of conductive baffle.
[58]	$10^3 \leq Ra \leq 10^6$ $0\% \leq \phi \leq 20\%$ water-Al ₂ O ₃ nanofluid	FDM		The Bejan and average Nusselt numbers have a reverse behavior to each other for the same range of Rayleigh number and solid volume fraction.
[59]	$10^4 \leq Ra \leq 10^6$ CuO-water nanofluid	two-phase lattice Boltzmann simulation		There is an angle associated with a maximum Nu number on the hot wall of the enclosure, which is a function of the Ra number and is independent of the particle volume concentration.
[60]	$10^3 \leq Ra \leq 10^6$ $0 \leq \phi \leq 0.10$ nanofluids (Cu, Ag, Al ₂ O ₃ and TiO ₂)	FVM		The average Nusselt number increases with an increasing volume fraction of nanoparticles.
[61]	$10^3 \leq Ra \leq 10^6$ $0 \leq Ha \leq 50$ $0 \leq \phi \leq 0.1$ (Cu), (Ag), (Al ₂ O ₃) and (TiO ₂)- water nanofluid	FVM		Increasing the heat source length leads to a decrease the average Nusselt number.
[62]	$10^3 \leq Ra \leq 10^6$ $0 \leq \phi \leq 0.2$ (Cu, Ag, Al ₂ O ₃ , TiO ₂)- water nanofluid	Finite element formulation using the Galerkin weighted residual technique		For higher values value of solid volume fraction and Rayleigh numbers heat transfer rate is increased.
[63]	$10^3 \leq Ra_E \leq 10^6$ Nano-fluid	FVM	-	Increasing the nanoparticle concentration to 6% tends to shift the free convection heat transfer to conduction heat transfer.
[64]	$\Phi = (0, 0.07)$ nanofluid (Al ₂ O ₃ -water)	FVM		The average Nusselt number is increased with the increasing of the Hartmann number
[65]	$1.7 \times 10^8 < Ra < 4.2 \times 10^8$ Fe ₂ O ₃ -ferrofluid	experimental	-	it was found that the best-performing magnetic field

				enhanced the heat transfer behavior by an additional 2.81% in Nu at $Ra=3.8 \times 10^8$
[66]	$10^3 \leq Ra \leq 10^6$ $0.5 \leq AR \leq 2$ $0\% \leq \phi \leq 7\%$ Al_2O_3 -water nanofluid	FEM		the average actual Nusselt number decreases with nanoparticles concentration increment.
[67]	$10^3 \leq Ra \leq 10^5$ $0^\circ \leq \gamma \leq 90^\circ$ $0 \leq \phi \leq 0.05$ Carbon nano-tube (CNT)-water	FDM		The heat transfer increases when the concentration of CNT particles and Rayleigh number are increased.
[68]	$10^3 \leq Ra \leq 10^6$ $0 \leq Ha \leq 100$ $0 \leq \phi \leq 0.05$ CNT-nanofluid	FVM	-	Heat transfer increases with increasing of nanoparticle volume fraction.
[69]	$Ra = 10^7, 10^8$ Al_2O_3 - water nanofluid	FVM		the average Nusselt number increases with the increase of Rayleigh number and the nanoparticles volume fraction.
[70]	$10^3 \leq Ra \leq 10^8$ Water-Cu nanofluid	spectral-element method		heat transfer for higher amplitudes where an increase with the peak Nusselt number is observed with increasing the forcing amplitude of the hot wall.
[71]	$10^3 \leq Ra \leq 10^6$ $0 \leq \phi \leq 0.1$ Water- Al_2O_3 , Cu, TiO_3 nanofluid	FVM		The effect of the nanofluid on natural convection is particularly apparent at high Rayleigh numbers.
[72]	$10^3 \leq Ra \leq 10^5$ $Ha = 50$ to 0 Fe_3O_4 -water nanofluid	FEM		the heat transfer becomes dominated by conduction instead of convection and the nanofluid temperature homogeneity was deteriorated.

[73]	$10^4 \leq Ra \leq 10^6$ Cu-water, Al_2O_3 -water nanofluids.	FVM		the average Bejan numbers increase and the entropy generations were found to be approximately the same at high Rayleigh numbers.
[74]	$10^4 \leq Ra \leq 10^6$ CuO-water Al_2O_3 -water	Experimental and numerical (FVM)		the heat transfer is improved, though the average Nusselt number is decreased.
Table 2. Free convection in a complex cavity filled with nanofluid				
References	Fluid Type Parameters	Model	Shape	Conclusion
Free Convection in a Complex Cavity Filled with Nanofluid.				
[75]	$10^5 \leq Ra \leq 10^6$ (Water-Cu, Al_2O_3) nanofluid	FVM		developed a new correlation for the average Nusselt number as a function of the angle of the sloping wall, effective thermal conductivity and viscosity as well as Grashof number
[76]	$10^4 \leq Ra \leq 10^6$ $0.5 \leq AR \leq 4$ $0 \leq \phi \leq 0.2$ Cu-water nanofluid	FVM		when solid volume fraction and Rayleigh number increase, the intensity of the circulation increases.
[77]	$10^4 \leq Ra \leq 10^6$ nanofluid	FEM		Nusselt and Sherwood numbers are increasing functions of the Rayleigh number.
[78]	$50 \leq Ra \leq 1000$ nanofluid	FDM		Nusselt and Sherwood numbers and flow strength are increasing functions of Rayleigh number.
[79]	$10^{-5} \leq Da \leq 10^{-1}$ $Ra = 10^5$ and 10^6 $0^\circ \leq \phi \leq 21.8^\circ$ ($0^\circ \leq \theta \leq 90^\circ$) $0 \leq \phi \leq 0.2$ (Water with Silver, Copper, Alumina and Titania) nanofluid	FVM		The overall Nusselt number has maximum value when the power-law index is less than one (pseudoplastic fluid).

[80]	$Ra \leq 13^7$ $0.1 \leq AR \leq 10$ $0 \leq \phi \leq 0.01$ Glass balls Aluminum foam Cu Base fluid (water)	FDM		the decrease of the porosity increases the porous matrix thermal conductivity while the decrease of the inclination angle and of the aspect ratio would boost the deterioration of heat transfer.
[81]	$10^3 \leq Ra \leq 10^6$ carbon nanotube– EG–water nanofluid	FVM		As Rayleigh number increases, buoyancy-induced convection heat transfer becomes dominant
[82]	$Ra = 50, 100, 150$ $Ha = 50, 100, 150$ Al_2O_3 -water and SWCNT-water nanofluids	FEM with polynomial pressure projection stabilization		In the case of the SWCNT-water nanofluid, the rate of heat transfer increases with increasing Ha.
[83]	$10^3 \leq Ra \leq 10^7$ $[\theta = 0^\circ, 30^\circ, 60^\circ \text{ and } 90^\circ]$ $[\phi = 0, 0.05, 0.1, 0.15 \text{ and } 0.2]$ Silver (Ag) Alumina (Al_2O_3)- water nanofluid	FVM		for all three types of horizontal wavy cavities, heat functions increase for both nano and base fluids when the wave amplitude and the Rayleigh number increase.
[84]	$10^4 \leq Ra \leq 10^6$ $0 \leq Ha \leq 100$ $0 \leq \phi \leq 0.2$ (Cu, Ag, Al_2O_3 , TiO_2)-water nanofluid	FDM		The mean Nusselt number increases with the solid volume fraction as the heat source length decreases and its location increases.
[85]	$Ra = 10^3 - 10^6$ $0 \leq \phi \leq 0.2$ Al_2O_3 -water	FDM		Average Nusselt number becomes constant for the smaller value of Rayleigh numbers due to domination of conductive regime.
[86]	$10^3 \leq Ra \leq 10^7$ $0^\circ \leq \phi \leq 21.8^\circ$ $0 \leq \phi \leq 0.2$ Ag, Cu, or Al_2O_3 nanoparticle- containing water- based nanofluids	FDM		enhancement increases the convection heat transfer and thereby the strength of the flow circulation and the values of average Nusselt number increases.
[87]	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 0.04$ Cu-water	FVM		For all values of the Ra, the use of the nanofluid instead of water inside the concave and convex parabolic enclosures leads to greater improvement of the values of average Nusselt number
[88]	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 0.2$ Cu–water nanofluid	FDM		When the Rayleigh number and nanoparticles volume fraction increase, the average Nusselt number increase

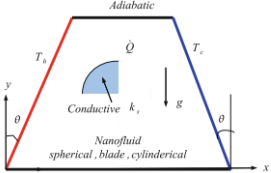
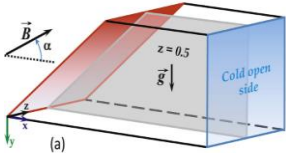
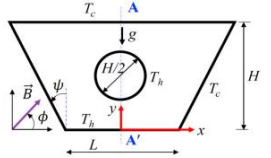
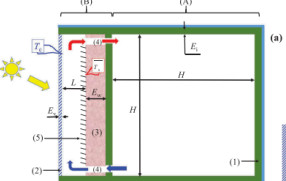
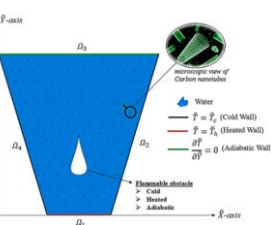
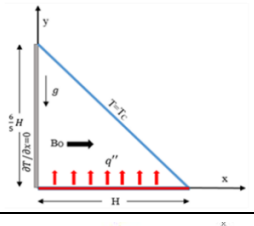
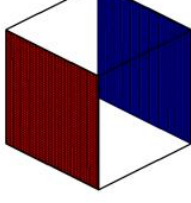
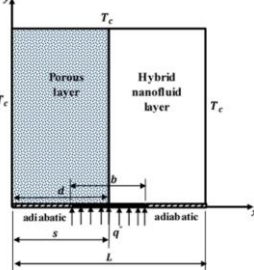
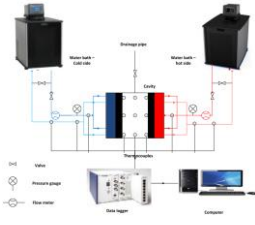
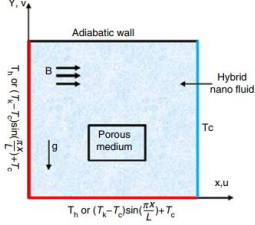
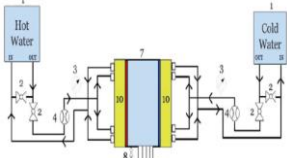
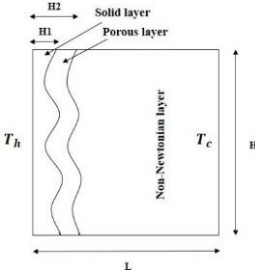
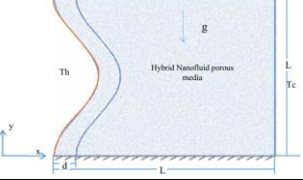
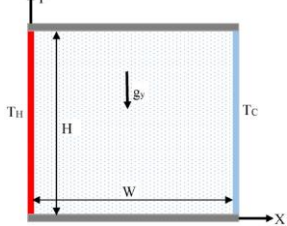
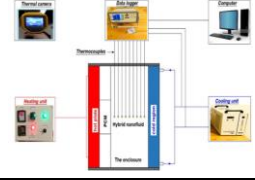
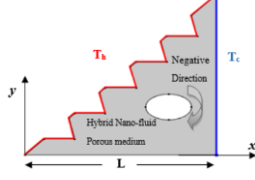
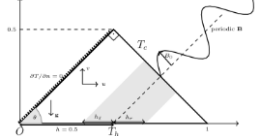
[89]	$10^4 \leq Ra \leq 10^6$ $0 \leq \phi \leq 0.4$ SiO ₂ - water nanofluid	FEM		For lower values of Rayleigh number, shape of the object is very effective for heat transfer enhancement
[90]	$10^3 \leq Ra \leq 10^5$ $0 \leq Ha \leq 100$ $0 \leq \phi \leq 0.05$ Carbon Nanotube (CNT)-nanofluid	FVM		at increase of Ha with high values of Rayleigh number is accompanied by a reduction in the heat transfer rate.
[91]	$10^3 \leq Ra \leq 10^7$ $0 \leq Ha \leq 60$ $0^\circ \leq \gamma \leq 90^\circ$ Al ₂ O ₃ -water nanofluid	FVM		At high Rayleigh numbers, values of the natural convection and buoyancy force become dominated, which leads to an increasing stream function
[92]	$Ra (2.81 \times 10^8 \text{ to } 4.14 \times 10^9)$	Experimental		The average Nusselt number is determined for all the tested configurations with a maximum uncertainty of 5%, taking into account the uncertainties of the measured physical parameters.
[93]	$Ra = 10^5, 10^7, 10^8$ Single wall carbon nanotubes (SWCNTs) mixed with water	FVM		The local Nusselt number predicts the increasing behavior for buoyancy driven flow.

Table 3. Free convection in a classical cavity filled with hybrid nanofluid

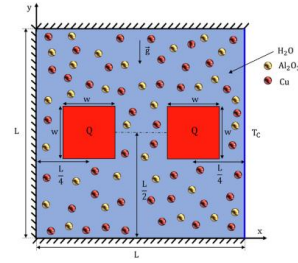
References	Fluid Type Parameters	Model	Shape	Conclusion
Free Convection in a Classical Cavity Filled with Hybrid Nanofluid				
[94]	$(10 \leq Ra \leq 10^6)$ Cu-Al ₂ O ₃ /water hybrid nanofluid	FEM		Nusselt number had been decreasing function of nanoparticles Rayleigh number and also it is a decreasing function of Hartmann number.
[95]	$(10^4 \leq Ra \leq 10^6)$ Cu-Al ₂ O ₃ /water hybrid nanofluid	FVM		the average Nusselt and Sherwood numbers decrease with the enhancement of micropolar vortex parameter.
[96]	$10^3 \leq Ra \leq 10^7$ $10^{-7} \leq Da \leq 1$ $0 \leq \phi \leq 0.2$ (Cu-Al ₂ O ₃ /water) hybrid nanofluid	FEM		Increasing the length of the heat source reduced the local Nusselt number

[97]	$Ra(1.65 \times 10^8 - 3.80 \times 10^8)$ Al_2O_3 -MWCNT/water hybrid nanofluids	Experiment		The results from this study further demonstrated the benefit offered by utilizing hybrid nanofluids over mono-particle nanofluids in heat transfer studies.
[98]	$(Ra=10^3-10^6)$, $(Da=10^{-5}-10^{-2})$, $(Ha=0-100)$ $(\phi=0-0.08)$ hybrid nanofluid Al_2O_3 - Cu/water	FEM		The entropy production boosts with growing Rayleigh number, while it declines with Ha
[99]	$Ra(2.81 \times 10^8 \text{ to } 8.58 \times 10^8)$ Al_2O_3 -MWCNT/ water hybrid nanofluids	Experiment		Nu_{av} augmented with an increase in Ra for Al_2O_3 -MWCNT hybrid nanofluids samples at different volume concentrations and the base fluid
[100]	$10^3 \leq Ra \leq 10^6$ (Ag-MgO Water) hybrid nanofluid	FEM		The average Nu decreases by increasing the undulations.
[101]	$(Ra=10^2, 10^3, 10^4, \text{ and } 10^6)$, Darcy number $(Da=10^{-2}, 10^{-4} \text{ and } 10^{-6})$ $(\phi=0.01, 0.03, \text{ and } 0.05)$, hybrid nanofluid of water and (35% MWCNT, 65% Fe_3O_4)	FEM		The temperature of the solid wave wall increases with the wave's amplitude. Consequently, with increased wave amplitude, the Nu_{hm} decreases.
[102]	$(Ra=10^4-10^5)$ MWCNT- Fe_3O_4 /Water hybrid nanofluid	LBM (Lattice Boltzmann Method)		the mean Nusselt number rises with the increase in Rayleigh number, while it falls as the aspect ratio increases
[103]	$(\Phi=0.03, 0.05)$, the cavity inclination angle $(\theta=0^\circ, 30^\circ, 45^\circ)$ 50% CuO- 50% Al_2O_3 water hybrid nanofluids	Experimental		The value of the Nusselt number increased by increasing the temperature difference,
[104]	$10^3 \leq Ra \leq 10^6$ $0 \leq Ha \leq 100$ $0.02 \leq \phi \leq 0.08$ Cu- TiO_2 /EG hybrid nano-fluid	FVM		Hartmann number reduced stream-function values and weakened the convective flow.
[105]	$10^3 \leq Ra \leq 10^6$ $0 \leq Ha \leq 100$ Al_2O_3 -Cu/water hybrid nano-fluid	Partition of the unity method based on radial basis functions (Rbf-Pum)		The rise in Ha number suppresses the fluid flow and heat transfer.

[106]

($Ra=10^3-10^7$)
Cu-Al₂O₃/water hybrid
nanofluid

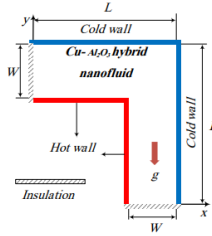
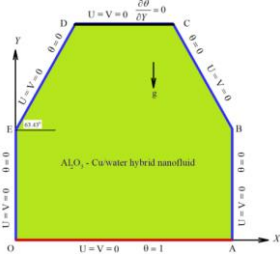
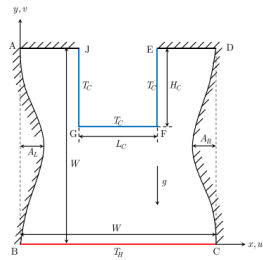
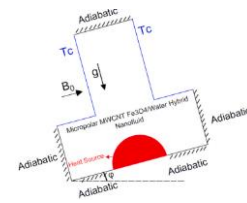
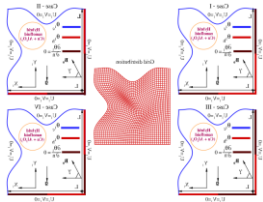
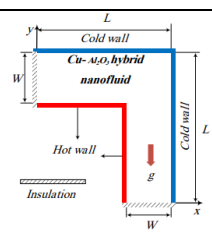
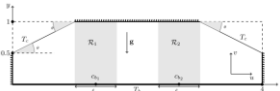
FVM



the increase of the Rayleigh number enhances the heat exchange in the cavity, resulting in a decrease in the maximum temperature.

Table 4. Free convection in a complex cavity filled with hybrid nanofluid

References	Fluid Type Parameters	Model	Shape	Conclusion
Free Convection in a Complex Cavity Filled with Hybrid Nanofluid				
[107]	$Ra=10^3-10^6$ $Ha=0-50$ Ag-MgO water hybrid nanofluid	FEM		at higher Rayleigh numbers, by increasing the Hartmann number, a significant decrease is observed in the Nusselt number.
[108]	$10^5 \leq Ra \leq 10^7$ MgO-MWCNTs/EG	FEM		The increase of Rayleigh number, Darcy number, and radiation parameter would increase the heat transfer rate in the cavity.
[109]	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 0.04$ Cu-Al ₂ O ₃ -water hybrid nanofluids	FEM		Increasing Ra improves heat transfer due to the developed importance regarding the driving buoyancy forces compared to the resistive viscous forces.
[110]	$Ra=10^3$ to 10^6 ($Ha=0$ to 60) ($\phi=0$ to 0.02) hybrid nanofluid (MgO-Ag/water)	FEM		the Nusselt number is increases with Ra and ϕ while it decreases with Ha.
[111]	$10^3 \leq Ra \leq 10^5$ $0 \leq \phi \leq 0.08$ hybrid Cu-Al ₂ O ₃ /Water nanofluid	Numerical		The Nu number rises with the reduction in the heating element size. Nu augments when the heating element is passed to the right, the inclined side of the enclosure.
[112]	$10^3 \leq Ra \leq 10^5$ Cu-Al ₂ O ₃ /water hybrid nanofluid)	FEM		Higher values of Ra contribute to boosting up the rate of heat transfer and fluid motion notably.
[113]	$10^3 \leq Ra \leq 10^6$ (MWCNT-Fe ₃ O ₄) hybrid nanofluid	FEM		The reduction in Nusselt numbers relative to Ha is more noticeable for higher Rayleigh numbers, as well as for higher Darcy numbers.

[114]	$10^3 \leq Ra \leq 10^5$ Cu-Al ₂ O ₃ Hybrid Nanofluid	FEM		for all magnitudes of Rayleigh numbers, increasing nanoparticles concentration intensifies the rate of entropy generation.
[115]	$10^4 \leq Ra \leq 10^5$ Al ₂ O ₃ -Cu/water hybrid fluid	FEM		the nanoparticle loading intensified the rate of heat transfer and thermal efficiency index at the expense of the higher friction factor or higher pumping power
[116]	$10^4 \leq Ra \leq 10^5$ Cu-Al ₂ O ₃ /water hybrid nanofluid	FEM		The heat transfer rate increases as the Rayleigh number increases. The value of the average Nusselt number decreases as the amplitude and undulations increase for all cases.
[117]	Angles of the cavity's inclination (0° - 90°): Ha (0-20), Ra (10^4 - 10^6) MWCNT-Fe ₃ O ₄ /Water Hybrid Nano-fluid	FEM		both micro-rotation parameter and Hartmann number reduce NC and Nuavg of hybrid nanofluid. Besides, Be number will increase by reducing Ra number.
[118]	$10^4 \leq Ra \leq 10^6$ $0 \leq Ha \leq 60$ orientation of magnetic field ($0^\circ \leq \gamma \leq 60^\circ$) and magnetic number ($0 \leq \delta_0 \leq 1$) $0 \leq \phi_{hnp} \leq 0.04$ Cu-Al ₂ O ₃ (50%-50%) water hybrid magneto-nanofluid	FEM		increase in the average Nusselt number up to 10.8% in Case-I, 12% in Case-II, 11.66% in Case-III and 11.65% in Case-IV.
[119]	$10^3 \leq Ra \leq 10^5$ Cu-Al ₂ O ₃ Hybrid Nanofluid	FDM		Due to the increase of the number of heat sources, angle of magnetic field and Rayleigh number, average Nusselt number is found to increase
[120]	$10^4 \leq Ra \leq 10^5$ Water Ag-MgO Hybrid Nanofluid	FDM		The large area of impact region of partial magnetic field results in inhibition of fluid flow and heat transfer.

3. THE GOVERNING EQUATIONS

The study hypotheses of the researcher state that under stable or unstable conditions, a set of equations can be employed to determine the numerical solutions in the area of 3D free convection inside various enclosure shapes. Equations involving energy, momentum, and continuity are employed to control the fluid inside the hollow. The governing equations

for the nanofluid and hybrid nanofluid, respectively, will be represented by an unstable model in this work.

3.1 The nanofluids governing equations

Due to the enhancements provided by the presence of solid nanoparticles, these equations for nanofluids underwent some changes. Density, viscosity, and thermal conductivity are

among the enhanced characteristics. the location where the governing equations start [71]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

The momentum equation in the x-axis:

$$\rho_{nf} \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial P}{\partial x} + \mu_{nf} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (2)$$

The momentum equation in the y-axis:

$$\rho_{nf} \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial P}{\partial y} + \mu_{nf} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \rho_{nf} g \quad (3)$$

The momentum equation in the z-axis:

$$\rho_{nf} \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial P}{\partial z} + \mu_{nf} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \quad (4)$$

The energy equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (5)$$

where, ρ_{nf} , μ_{nf} , α_{nf} are given by [103]:

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s \quad (6)$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \quad (7)$$

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (8)$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad (9)$$

The proposed equation of (k_{nf}) for suspensions is presented first by Maxwell [121] as:

$$\frac{k_{nf}}{k_f} = 1 + \frac{3 \left(\frac{k_s}{k_f} - 1 \right) \phi}{\left(\frac{k_s}{k_f} + 2 \right) - \left(\frac{k_s}{k_f} - 1 \right) \phi} \quad (10)$$

Later, Hamilton and Crosser [122] improved Maxwell's theory by adding the impact of particle shape. These models are merely weighted averages of conductivities for solids and liquids.

$$\frac{k_{nf}}{k_f} = \frac{k_s + (n-1)k_f - (n-1)\phi(k_f - k_s)}{k_s + (n-1)k_f + \phi(k_f - k_s)} \quad (11)$$

where, (n) is the shape factor, which is (3) in the case of a sphere and (6) in the case of a cylinder. The Maxwell-Garnetts [121] model can also approximate it for spherical nanoparticles as follows:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \quad (12)$$

There are a few higher-order models that attempt to

incorporate particle interactions by integrating higher-order terms such as those proposed by Jeffrey [106]:

$$\frac{k_{nf}}{k_f} = 1 + 3\beta\phi + \left(3\beta^2 + \frac{3\beta^2}{4} + \frac{9\beta^3}{16} \frac{\alpha+2}{2\alpha+3} + \dots \right) \phi^2 \quad (13)$$

where:

$$\beta = \frac{\left(\frac{k_s}{k_f} - 1 \right)}{\left(\frac{k_s}{k_f} + 2 \right)} \quad (14)$$

And the model of Davis [29] as:

$$\frac{k_{nf}}{k_f} = 1 + \frac{3 \left(\frac{k_s}{k_f} - 1 \right)}{\left(\frac{k_s}{k_f} + 2 \right) - \left(\frac{k_s}{k_f} - 1 \right) \phi} \left[\phi + f \left(\frac{k_s}{k_f} \right) \phi^2 + O(\phi^3) \right] \quad (15)$$

where: $f(10)=2.5$ and $f(8)=0.5$.

3.2 The hybrid nanofluid governing equations

For Hybrid Nanofluid (hnf) [123], the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (16)$$

The momentum equation in the x-axis:

$$\rho_{hnf} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu_{hnf} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (17)$$

The momentum equation in the y-axis:

$$\rho_{hnf} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu_{hnf} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \quad (18)$$

The momentum equation in the z-axis:

$$\rho_{hnf} \left(u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu_{hnf} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + (\rho\beta)_{hnf} g(T - T_{ref}) \quad (19)$$

The energy equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha_{hnf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (20)$$

where, ρ_{hnf} , μ_{hnf} , α_{hnf} are given by [123]:

$$\rho_{hnf} = (1 - \phi_{hnf})\rho_f + \phi_{Al_2O_3}\rho_{Al_2O_3} + \phi_{Cu}\rho_{Cu} \quad (21)$$

$$(\rho C_p)_{hnf} = (1 - \phi_{hnf})(\rho C_p)_f + \phi_{Al_2O_3}(\rho C_p)_{Al_2O_3} + \phi_{Cu}(\rho C_p)_{Cu} \quad (22)$$

$$(\rho\beta)_{hnf} = (1 - \phi_{hnf})(\rho\beta)_f + \phi_{Al_2O_3}(\rho\beta)_{Al_2O_3} + \phi_{Cu}(\rho\beta)_{Cu} \quad (23)$$

$$\mu_{hnf} = \frac{\mu_f}{(1-\phi_{hnf})^{2.5}} \quad (24)$$

4. CONCLUSIONS

The research on two- and three-dimensional laminar free convection inside various classical and complicated geometric enclosures is reviewed in detail in this publication. In numerical and experimental studies, the laminar (Ra) was shown to range between ($10^3 \leq Ra \leq 10^6$) for working fluids that were either pure nanofluids or hybrid nanofluids. The results from this study further demonstrated the benefit offered by utilizing hybrid nanofluids over mono-particle nanofluids in heat transfer studies. The following can be used to sum up the main conclusions:

1. The average Nusselt number increases with the increase of Rayleigh number and the nanoparticles volume fraction. Also, in the comparison with pure water, nanofluid presents higher average Nusselt number along the heating wall because the increased conductivity as a dominant parameter to increase heat transfer convection. The addition of nanoparticles into pure fluid decreases the entropy generation.
2. For higher values value of solid volume fraction and Rayleigh numbers heat transfer rate is increased.
3. Average temperature decreases for increasing the values solid volume fraction as well as Rayleigh numbers.
4. Nanoparticles with lowest value of thermal conductivity are regarded as the lowest heat transfer rate.
5. at the visualization of the heat lines on natural convection in a trapezoidal cavity partly filled with a nanofluid porous layer and partly with a non-Newtonian fluid layer. It is found that when the nanoparticle volume fraction is applied, the circulation intensity increases due to the increase in the thermal conductivity of nanofluid. The conduction heat transfer pushes the isotherms patterns within the nanofluid layer to take almost a diagonal shape, while the convection mode heat transfer forces the isotherms patterns within the fluid layer to appear with almost a horizontal line to the sloping walls.
6. Due to the effect of nonuniform heating, the increase in the strength of the flow circulation is observed with increasing values of the wave number, but at a high wave number, the strength of the flow circulation decreases.
7. Qualitatively, the enhanced-heat transfer situation is seen in all the three nanofluids employed in this study compared to that of the base fluid but the following general result holds: $Nu_{\text{water-Ag}} > Nu_{\text{water-Cu}} > Nu_{\text{water-Al}_2\text{O}_3}$
8. Regardless of the Rayleigh number, the local Nusselt number exhibits nonlinear
9. enhancement for the low value of Prandtl number ($Pr = 0.025$).
10. Heat transfer rate is greater at the top corner of hot wall while the local Nusselt number gradually reduces from bottom to top of the hot wall and is suddenly enhanced as we move close to the top corner of the wall.
11. Nu_{loc} and Nu_{avg} are enhanced for nanoparticle concentration and Rayleigh number, but the Hartmann number drops off the heat transmission.
12. Higher heat transference is anticipated for high fin length. Moreover, hybrid nanofluid explores high heat transmission rate compared to mono nanofluid.

13. The presence of hybrid nanoparticles enhances the heat transfer in the cavity. However, the increase of the concentration nanoparticles would reduce the magnitude of the enhancement. The maximum enhancement was observed for the very low volume fraction of nanoparticles, $\phi = 0.2\%$.
14. The geometry of the cavity induces minor effects on the flow and heat transfer patterns. A cavity with sharper edges results in a higher heat transfer rate.
15. at influence of multiple partial magnetic fields on natural convection of Ag-MgO/water hybrid nanofluid flow is investigated in a trapezoidal shaped cavity the Domination of buoyancy force on Lorentz force is pronounced with the increase in Rayleigh number.
16. The augmentation in Lorentz force suppresses fluid flow and convective heat transfer.
17. In a rectotrapezoidal enclosure whose bottom wall is uniformly heated. The lower Rayleigh number value leads to a higher thermal efficiency index. The hybrid nanoparticle loading accelerates the rate of heat transfer and the thermal efficiency index at the expense of the higher friction factor or higher pumping power.

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NOMENCLATURE

Be	Bejan number
C _p	at constant pressure, specific heat, J/Kg.K
c	The partition's location, m
g	the acceleration of gravity, m/s ²
Ha	Hartmann number
h	the partition's height, m
K	Thermal conductivity, W/m.K
Ma	Marangoni number
Nu	Nusselt number
P	Pressure, N/m ²
Pr	Prandtl number
Ra	Rayleigh number
T	Temperature, k
t	Time, s
u	The x-direction velocity component, m/s
v	The y-direction velocity component, m/s
w	The z-direction velocity component, m/s
x	the coordinate's horizontal component, m

y	the coordinate's vertical component, m
z	the axial direction coordinate, m

Greek symbols

α	Thermal diffusivity (m ² /s)
β	coefficient of thermal expansion (K ⁻¹)
γ	Angle of inclination of the magnetic field (°)
μ	Dynamic viscosity (Kg/m.s)
ρ	Density (Kg/m ³)
ϕ	percentage of solid volume for nanoparticles (-)
δ_0	magnetic number
θ	Cavity's inclination angle (°)
Φ	Volume concentration of the nanofluid
ϕ	Side wall Inclination angle (°)

Abbreviations

AR	Aspect Ratio
CFD	Computational Fluid Dynamic
CVM	Control Volume Method
DQM	Differential Quadrature Method
FDM	Finite Difference Method
FEM	Finite Element Method
FVM	Finite Volume Method
LBM	Lattice Boltzmann Method
MRT	Multiple Relaxation Time
PIV	Particle Image Velocimetry
Rd	Radiation number
TSM	Time Space Method
2D	Two-dimensional
3D	Three-dimensional

Subscripts

b	Block
c	Cold
D	Position of the hot source
E	External
eff	Effective
f	Fluid
I	Internal
L	Length
hnf	Hybrid Nano fluid
nf	Nano fluid
s	Solid particle