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A Binary Relation Fuzzy Soft Matrix-Theoretic Approach to Image Quality Measurement: Comparison with Statistical Similarity Metrics

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ABSTRACT

Image similarity assessment is a fundamental aspect of real-world applications and plays a crucial role in image processing. The structural similarity index (SSIM), which relies on statistical properties between two digital images, has been widely adopted. However, it struggles to detect or measure similarity at low peak signal-to-noise ratios (PSNR). In this study, a novel approach to image similarity for evaluating gray image quality is presented, termed Binary Relation Fuzzy Soft Matrix Image Similarity Measure (BR-FS-ISM). This method utilizes a fuzzy soft matrix-theoretic technique based on a new binary relation fuzzy soft matrix. The proposed BR-FS-ISM approach was tested under Gaussian noise conditions. Simulation results demonstrate that the novel BR-FS-ISM method outperforms the well-established SSIM metric, exhibiting the ability to detect and measure similarity at very low PSNR levels, with an average difference of approximately 10 dB. This paper suggests that the BR-FS-ISM approach offers a promising alternative to conventional statistical similarity measures for image quality assessment.

1. INTRODUCTION

Traditional mathematical methods have proven insufficient for addressing numerous real-world problems across various fields. In 1965, Zadeh [1] introduced the groundbreaking theory of fuzziness, which paved the way for other well-known set theories, such as intuitionistic fuzzy, vague, rough, and interval mathematics. Subsequently, in 1999, Molodtsov [2] proposed the soft set theory as a novel tool for dealing with uncertainty. Possible practical applications of soft sets in various problems have been explored by Molodtsov [2] and other researchers [3-5]. Aktaş and Çağman [6] demonstrated that every fuzzy set could be considered a soft set, suggesting that this theory is more general in nature.

In many instances, it is essential to compare two sets, which may be fuzzy, soft, vague, etc. Researchers often aim to determine whether two images or patterns are identical (similar), approximately identical (similar), or at least the degree to which they are identical (similar). Several researchers, including Chen et al. [7-9], Hong and Kim [10], Li and Xu [11], Pappis [12], Pappis and Karacapilidis [13], and others [14, 15], have investigated and resolved the problem of similarity measurement between fuzzy numbers, fuzzy sets, and vague sets. Recently, Majumdar and Samanta [16] and Williams and Steele [17] have introduced the study of similarity measures for soft sets and intuitionistic fuzzy soft sets. Similarity measures have extensive applications in various fields, such as pattern recognition, image processing, region extraction, coding theory, psychology, handwriting recognition, and decision-making.

Image similarity assessment is a critical aspect of practical applications. Image quality measurements play a significant role in image processing, as they can be employed to adjust or modify image quality and improve parameters in numerous image processing applications, such as image compression, image restoration, and image enhancement. Machine quality assessment aims to develop methods for objective quality assessment in comparison to subjective human image quality assessment [18].

A straightforward method to measure the similarity between two images (the original and the noisy version) is to calculate the mean squared error (MSE). Although easy to compute, MSE exhibits weak performance in pattern recognition [19]. The first notable objective measure was proposed by Wang and Bovik [20] in 2004, wherein image distortion was measured as a combination of three types of distortion: contrast, luminance, and correlation.

The above statistical measure called, (SSIM), used distance covariance to measure the similarity based on statistical properties or measurements such as mean and standard deviation as;

$$\rho(x, y)=\frac{(2-\mu_x-\mu_{y}+C_1)(2\sigma_{xy}+C_2)}{(\mu_x^2+\mu_y^2+C_1)(\sigma_x^2+\sigma_y^2+C_2)} \qquad (1)$$

where, $\rho(x,y)$ is the (SSIM) between two images x and y (original image and corrupted image), μ_x,μ_y are the statistical means, and σ_x^2, σ_y^2 statistical variances of pixel values in images x and y resp., σ_{xy} is the statistical covariance between two images, while the constants are given by $C_1=(K_1 L)^2$ and $C_2=(K_2 L)^2$, and where K_1 and K_2 are small constants, L is the maximum value of pixel (=255).

(SSIM) gives high image-similarity for noise free condition while it fails when noise begin increases.

The main motivation of this study is to associate and apply an important and well-known theory (fuzzy soft matrix theory) in mathematics with an important topic in image-processing (measurement of image- similarity). In addition to the need for a more efficient image- similarity measure.

There are many image- similarity measures in this manner, such as, information theoretic (HSSIM) [21], a hybrid (statistical and information theoretic ITSM [22], etc.

The above- similarity measures are depended on statistical or (and) information theoretical properties.

Now, we utilize a new scale measurement in this paper, (Fuzzy Soft Matrix-Theoretic Approach) to test similarity. Which we named it (Binary Relation Fuzzy Soft-Matrix-Theoretic), and show excellent results (i.e., its superior performance versus the classical statistical-similarity measure (SSIM) under additive Gaussian noise with several ratios of signal to noise. by ability to detect or measure similarity under very low PSNR. The average difference between the two image- similarity measurement is about 10dB.

In Section 2 of this article, we mention main definition and concept of Binary Relation-Fuzzy Soft Matrix and then we introduce our new measure in detail with algorithm. In Section 3, we explain a test environment of the proposed BR-FS-SM measure. Section 4 presents the main results and discussion. Section 5 shows the performance under Gaussian noise, and section 6 contains the conclusions of our new measure with illustrative figures.

1.1 Related work

Application of fuzzy soft (set) matrix-theory in image-processing is possible, when assuming that we can treat images as Binary Relation-Fuzzy Soft Set (Matrix).

There are some works in image-processing used definitions of fuzzy, and fuzzy soft sets or some types of them (i.e., treat images as fuzzy, and fuzzy soft sets or some types of them). In the following, some of them are mentioned;

Using the fuzzy set for edge detection is proposed in the study [23], an image is considered or taken as a fuzzy set and pixels are considered as elements of fuzzy set. That proposed measure converted the color images to a partially segmented images; finally, an edges detector is convolved over the partially segmented images to obtain an edged image. Also, an image segmentation measure using intuitionistic fuzzy and a new membership function is proposed in the study [24] by using restricted equivalence mapping, for finding the membership values of the image pixels.

In the study [25], in intuitionistic fuzzy sets, the information carried by the degree of membership and the degree of non-membership as a vector representation with the two elements, a cosine similarity approach between two intuitionistic fuzzy sets is proposed, it applied to pattern- recognition in image-processing. And in the study [26], A new Texture Image-Segmentation approach using Fuzzy Color Aura Matrices. Also, a new technique has been employed for the development of an optimum edge detection algorithm using fuzzy soft sets by using fuzzy soft relations [27].

Most of the previous work researchers used the definition of the fuzzy set (or fuzzy soft), and it does not fully correspond to the mathematical definition of a digital image (digital image is a binary function). Therefore, we have introduced a new definition that is more compatible with the definition of a digital image [28]. Which we will use as a basic idea in this research.

In this paper, we present Binary Relation-Fuzzy Soft Matrix theoretic image-similarity measure and show superior performance vs. the classical (statistical) similarity (SSIM) at Gaussian noise with deferent and various ratios of signal to noise.

2. BINARY RELATION-FUZZY SOFT MATRIX-THEORETIC MEASURE

First, we review the definitions given by us in the studies [2, 28], that was relied upon in designing the new Image-similarity measure.

2.1 Binary relation-fuzzy soft matrix

Let $\mathfrak{A}$ be an initial universe set and $\mathcal{E}_1, \mathcal{E}_2$ be two different sets of parameters. Let $\mathcal{P}(\mathfrak{A})$ denotes the power set of $\mathfrak{X}$. A binary relation fuzzy soft set over $\mathfrak{A}$, denoted by $\mathscr{bfs}$ is representing by a composite of membership function of fuzzy set w.r.t. power set $\#\mathscr{f}: \mathcal{P}(\mathfrak{A}) \to \mathrm{I}$; I=[0,1], with binary relation soft set $\mathscr{s}: \mathcal{E}_1 \times \mathcal{E}_2 \to \mathcal{P}(\mathfrak{X})$, Then $\#\mathscr{bfs}: \mathcal{E}_1 \times \mathcal{E}_2 \to \mathrm{I}$, is a membership function of binary relation fuzzy soft set where;

$$\#\mathscr{bfs}(\mathrm{e_i}, \mathrm{e_j}) = (\#\mathscr{f}\mathrm{o}\mathscr{s})(\beth_\mathrm{i}, \mathfrak{p}_\mathrm{j}), \forall (\beth_\mathrm{i}, \mathfrak{p}_\mathrm{j}) \in \mathcal{E}_1 \times \mathcal{E}_2, \text{where } \beth_\mathrm{i} \in \mathcal{E}_1 \text{ and } \mathfrak{p}_\mathrm{j} \in \mathcal{E}_2, \mathrm{i} = 1, \dots, \mathrm{m} ;\ \mathrm{j} = 1, \dots, \mathrm{h}$$

i.e., $\mathscr{bfs} = \left\{ \left((\beth_\mathrm{i}, \mathfrak{p}_\mathrm{j}), (\#\mathscr{f}\mathrm{o}\mathscr{s})(\beth_\mathrm{i}, \mathfrak{p}_\mathrm{j}) \right), \text{for all } (\beth_\mathrm{i}, \mathfrak{p}_\mathrm{j}) \in \mathcal{E}_1 \times \mathcal{E}_2,\ \text{s.t.} \beth_\mathrm{i} \in \mathcal{E}_1 \text{ and } \mathfrak{p}_\mathrm{j} \in \mathcal{E}_2, \mathrm{i} = 1, \dots, \mathrm{m};\ \mathrm{j} = 1, \dots, \mathrm{h} \right\}$

Then binary relation fuzzy soft matrix (also proposed by Abd Alhussain and Hassan [28]) denoted by $\mathcal{M}\mathscr{bfs}$ such that its membership function is $\#\,\mathcal{M}\mathscr{bfs} : \mathcal{E}_1 \times \mathcal{E}_2 \to \mathrm{I}$. (i.e., the Binary Relation-Fuzzy Soft Matrix $\mathcal{M}\mathscr{bfs}$ can be represented as the Table 1:

Table 1. The binary relation-fuzzy soft matrix $\mathcal{M}\mathscr{bfs}$

$\mathcal{M}\mathscr{bfs}$	$(\beth_\mathrm{i} \in \mathcal{E}_1)$ **Columns**
$(\mathfrak{p}_\mathrm{j} \in \mathcal{E}_2)$ Rows	$\#\mathcal{M}\mathscr{bfs}(\beth_\mathrm{i}, \mathfrak{p}_\mathrm{j}) = \mathscr{r}$, for all $(\beth_\mathrm{i}, \mathfrak{p}_\mathrm{j}) \in \mathcal{E}_1 \times \mathcal{E}_2$ s.t. $\beth_\mathrm{i} \in \mathcal{E}_1$ and $\mathfrak{p}_\mathrm{j} \in \mathcal{E}_2$, $\mathrm{i} = 1, \dots, \mathrm{m};\ \mathrm{j} = 1, \dots, \mathrm{h}$, where h and j are finite, and $\mathscr{r} \in \mathrm{I}$, I=[0,1]

The set of parameters $\mathcal{E}_1$ represent the columns of binary relation fuzzy soft matrix, i.e., $\beth_1$ represents the first column, $\beth_2$ represents the second column, …, and $\beth_\mathrm{m}$ represents the last column.

And the set of parameters $\mathcal{E}_2$ represent the rows of binary relation fuzzy soft matrix, i.e.

$\mathfrak{p}_1$ represents the first row, $\mathfrak{p}_2$ represents the second row, …, and $\mathfrak{p}_\mathrm{h}$ represents the last row.

Based on the above definition, we will design a novel image- similarity approach as follows.

2.2 Rationale

We noticed that (SSIM) which introduced in the study [2] gives good results of similarity; however, (SSIM) fails at low PSNR. So, we need an improver approach that can perform well at low (PSNR). We utilized the Binary-Relation fuzzy soft matrix to get the enhanced image-similarity measure (BR-FS-ISM); also, we tested the two measures (SSIM) and (BR-

FS-ISM) at disruptive conditions such as Gaussian noise.

2.3 The proposed binary relation-fuzzy soft- image-similarity measure

The design of the similarity approach (SSIM) was depended on statistical concepts on images. In this work we focus on Fuzzy Soft - theoretic concepts, specifically the Binary Relation-Fuzzy Soft Matrix, and propose the following Binary Relation-Fuzzy Soft Matrix - dependent approach.

Firstly, we propose an error estimate between a Binary Relation-Fuzzy Soft Matrix of original image x and a Binary Relation-Fuzzy Soft Matrix of noisy version y of it.

The Binary Relation-Fuzzy Soft Matrix - theoretic (BR-FS) measure can be designed as the following algorithm:

Algorithm

Step 1: Input the original Image x, convert it to the Binary Relation- fuzzy soft matrix ($\mathcal{M}bfs$(x)), then induce the noisy version ($\mathcal{M}bfs$(y)) from ($\mathcal{M}bfs$(x)).

Step 2: Compute the Binary Relation- Fuzzy Soft complement for $\mathcal{M}bfs$(x), $\mathcal{M}bfs$(y) (i.e., $[\mathcal{M}bfs(x)]^c$, $[\mathcal{M}bfs(y)]^c$) respectively.

Step 3: Compute $\mathcal{M}bfs(x) * \mathcal{M}bfs(y)$ and $[\mathcal{M}bfs(x)]^c * [\mathcal{M}bfs(y)]^c$, where * is the matrix multiply.

Step 4: Find the maximum of $(\mathcal{M}bfs(x) * \mathcal{M}bfs(y))$ and $([\mathcal{M}bfs(x)]^c * [\mathcal{M}bfs(y)]^c)$.

Step 5: Subtract the maximum value of 1(i.e., subtract 1).

Step 6: Divide the result above by the square of minimum value of $(\mathcal{M}bfs(x) + \mathcal{M}bfs(y) + c)$ and $([\mathcal{M}bfs(x)]^c + [\mathcal{M}bfs(y)]^c + c)$, where c is a very small positive constant. (The six above steps is to find the error E).

Step 6: B=mean (E), computes the mean of the values in E.

Step 7: (BR-FS-ISM)=(B -min(B))/(max(B)-min(B)).

Step 8: Compute Statistical Measure (SSIM) for Binary Relation- fuzzy soft matrix of original image $\mathcal{M}bfs$(X) and the noisy version $\mathcal{M}bfs$(Y).

Step 9: Compare (SSIM) with (BR-FS-ISM). (To deferent and same images).

Now, the Binary Relation-Fuzzy Soft Matrix - theoretic (BR-FS) error estimate can be designed as the following:

$$E(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) = \frac{[\max\left((\#\mathcal{M}bfs(x) * \#\mathcal{M}bfs(y)), ([\#\mathcal{M}bfs(x)]^c * [\#\mathcal{M}bfs(y)]^c)\right)] - 1}{[\min\left((\#\mathcal{M}bfs(x) + \#\mathcal{M}bfs(y) + c), ([\#\mathcal{M}bfs(x)]^c + [\#\mathcal{M}bfs(y)]^c + c)\right)]^2} \quad (2)$$

where, $\#\mathcal{M}bfs(x)$ and $\#\mathcal{M}bfs(y)$ are a membership function of Binary Relation- fuzzy soft matrix of original image and the noisy version resp., $[\#\mathcal{M}bfs(x)]^c$ and $[\#\mathcal{M}bfs(y)]^c$ are the complement of them. And c is a very small positive constant equal to 1e-6(c=1e-6=1×10^{-6}=0.000001=1u (micro)), inserted to avoid division by zero. Note that computes the mean of the values in E.

$$\Omega(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) = \text{mean}\left(E(\mathcal{M}bfs(x), \mathcal{M}bfs(y))\right) \quad (3)$$

Based on the above error estimate a Binary Relation-Fuzzy Soft -theoretic similarity measure $\lambda(\mathcal{M}bfs(x), \mathcal{M}bfs(y))$ (which we name as (BR-FS-ISM)) can be calculated as follows:

$$\lambda(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) = \frac{\Omega - \min(\Omega)}{\max(\Omega) - \min(\Omega)} \quad (4)$$

where:

$$0 \le \lambda(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) \le 1 \quad (5)$$

As in the measure (SSIM), (BR-FS-ISM) ranges between 0 (dissimilar case) and 1 (identical case). If we denote (SSIM) by $\rho(\mathcal{M}bfs(x), \mathcal{M}bfs(y))$, then:

$$0 \le \rho(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) \le 1 \quad (6)$$

3. THE TEST ENVIRONMENT

The proposed BR-FS-SM have been tested under effect of noise (Gaussian type), which is the most important source of noise in many image-similarity systems.

Different types of images have been used (tested) like; a (geometric shape), a (human face), and a (landscape).

4. RESULTS AND DISCUSSION

The above algorithm has been simulated using MATLAB. Figures 1 to 3 show performance of (SSIM) $\left(\rho(\mathcal{M}bfs(x), \mathcal{M}bfs(y))\right)$ and (BR-FS-ISM) $(\lambda(\mathcal{M}bfs(x), \mathcal{M}bfs(y)))$ using similar images at effect of Gaussian noise. Note; $0 \le \lambda(\mathcal{M}bfs(x), \mathcal{M}bfs(y)), \rho(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) \le 1$.

For completely similar (identical) images we have $\lambda(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) = \rho(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) = 1$; while for totally different (dissimilar) images we have $\lambda(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) = \rho(\mathcal{M}bfs(x), \mathcal{M}bfs(y)) = 0$.

We also implemented the Binary Relation-Fuzzy Soft-Theoretic based measure ((BR-FS-ISM)) as per equation (3). $\Omega(\mathcal{M}bfs(X), \mathcal{M}bfs(Y))$ is calculated under PSNR=-50 dB (total noise).

5. PERFORMANCE UNDER GAUSSIAN NOISE

After we implemented the Binary Relation-Fuzzy Soft-Theoretic based measure (BR-FS-ISM) as in the Eq. (4) and (SSIM) as in the Eq. (1), their performance of measure similarity is tested at noisy conditions, especially, when the second image is corrupted with the noise. Peak signal to noise ratio was used in this implementation as the following:

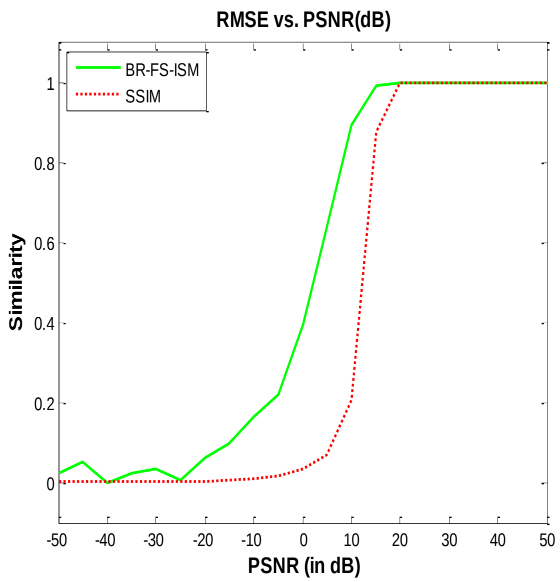
$$PSNR = \frac{L^2}{p_n}$$

where, L=1, and p_n is the Gaussian noise variance (power).

A maximal error can be taken or considered when noise power is very high (PSNR=-50 dB).

The result when using two similar images (an original image and a noisy version of it) is shown in Figures 1-3 for different and various types of images: Human face, landscape, and geometric shape. We used the different images "coins" and "cameraman" from MATLAB, and a human face from the famous face database AT&T [28].

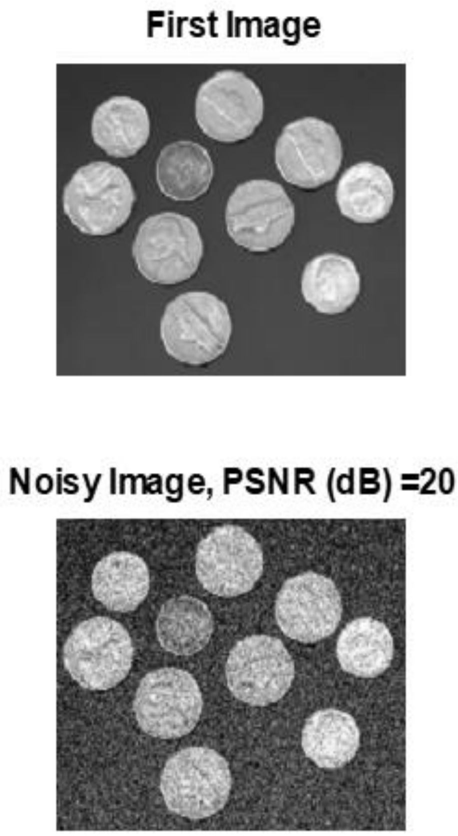
(a)



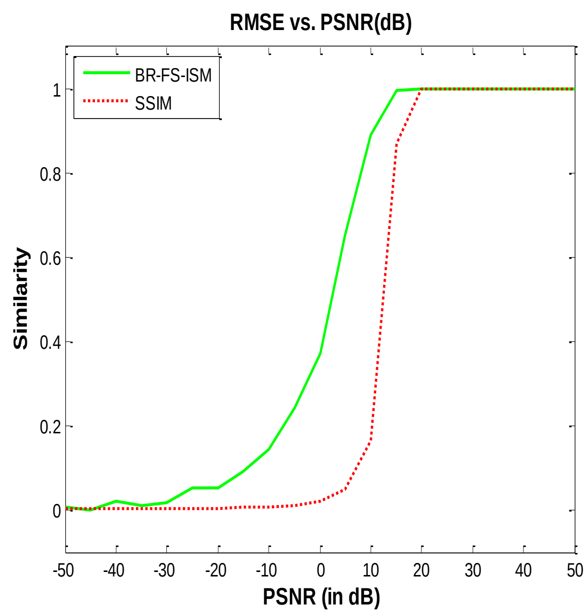
(b)

Figure 1. Performance of (BR-FS-ISM) and (SSIM) at landscape image with Gaussian noise: (a) The two test images (MATLAB Cameraman); (b) Similarity versus PSNR(dB)

Note: Clear that (BR-FS-ISM) outperforms (SSIM) by its ability to measure similarity at low PSNR (almost the difference is 10 dB).



(a)

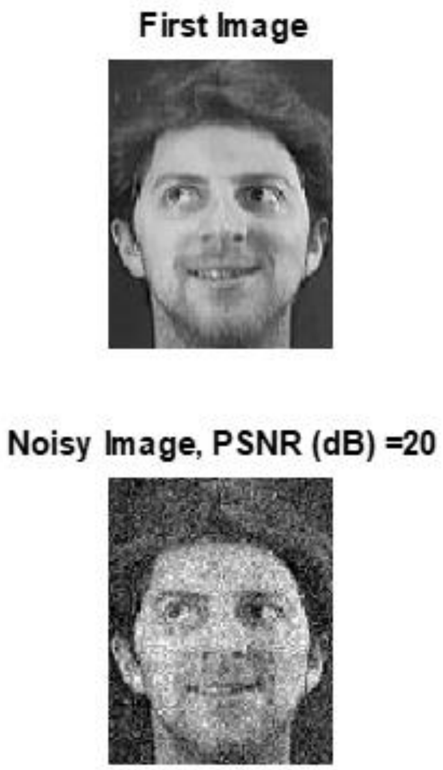


(b)

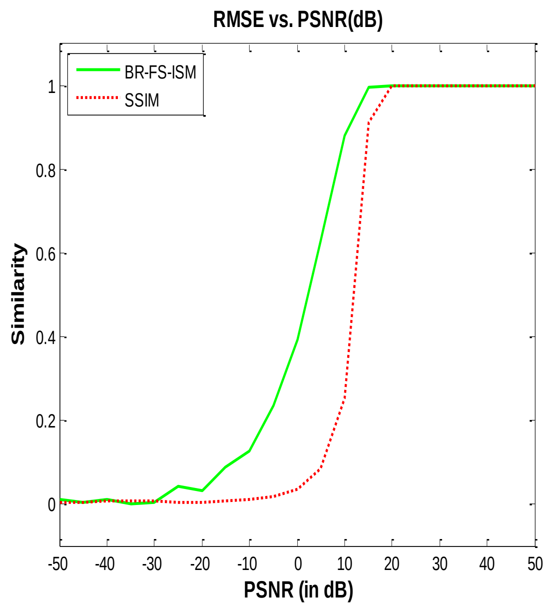
Figure 2. Performance of (BR-FS-ISM) and (SSIM) at geometric images (also from MATLAB)

Note: Clear that (BR-FS-ISM) still outperforms (SSIM) by its ability to measure similarity under low PSNR (almost the difference is 10 dB). Note and compare with Figure 1.

The performance of (BR-FS-ISM) as compared to the well-known (SSIM) shown in Figures 1-3, which consists of a test at Gaussian noise. These results show about 10 dB difference of ability for (BR-FS-ISM) over (SSIM) to detect and measure similarity at very low PSNR.



(a)



(b)

Figure 3. Performance of (BR-FS-ISM) and (SSIM) at a human face pose from (AT&T Database)

Note: Also Clear that (BR-FS-ISM) still outperforms (SSIM) by its ability to measure similarity under low PSNR (almost the difference is 10 dB). Note and compare with Figures 1 and 2.

6. CONCLUSIONS

We are managed and succeeded to associate and apply an important and well-known theory (fuzzy soft matrix theory) in mathematics with an important topic in image-processing (measurement of image- similarity). In addition, a novel Binary Relation- fuzzy soft matrix - theoretic, image-quality assessment approach has been proposed and tested vs. the most popular similarity measure (SSIM) at the effect of Gaussian noise. The approach is depended on Binary Relation-Fuzzy Soft-Matrix it is shown that the proposed approach (BR-FS-ISM) outperforms the (SSIM) by almost 10 dB of PSNR in Gaussian noisy environments for various and deferent types of images.

As a future work we can apply our new measure in face recognition, also we can propose another method for image-similarity with bipolar fuzzy soft environment or based on Binary-Relation-Multi-Fuzzy-Soft-Matrix which works for color images.

REFERENCES

[1] Zadeh, L.A. (1965). Fuzzy sets. Information and Control, 8(3): 338-353. https://doi.org/10.2307/2272014

[2] Molodtsov, D. (1999). Soft set theory—first results. Computers & Mathematics with Applications, 37(4-5): 19-31. https://doi.org/10.1016/S0898-1221(99)00056-5

[3] Chen, D.G., Tsang, E.C.C., Yeung, D.S. (2003). Some notes on the parameterization reduction of soft sets. In Proceedings of the 2003 International Conference on Machine Learning and Cybernetics (IEEE Cat. No. 03EX693), Xi'an, China, pp. 1442-1445. https://doi.org/10.1109/ICMLC.2003.1259720

[4] Maji, P.K., Roy, A.R. and Biswas, R. (2001) Fuzzy soft sets. Journal of Fuzzy Mathematics, 9: 589-602.

[5] Maji, P.K., Biswas, R., Roy, A.R. (2003). Soft set theory. Computers & Mathematics with Applications, 45(4-5): 555-562. https://doi.org/10.1016/S0898-1221(03)00016-6

[6] Aktaş, H., Çağman, N. (2007). Soft sets and soft groups. Information Sciences, 177(13): 2726-2735. http://dx.doi.org/10.1016/j.ins.2006.12.008

[7] Chen, S.M., Yeh, M.S., Hsiao, P.Y. (1995). A comparison of similarity measures of fuzzy values. Fuzzy Sets and Systems, 72(1): 79-89. https://doi.org/10.1016/0165-0114(94)00284-E

[8] Chen, S.M. (1995). Measures of similarity between vague sets. Fuzzy Sets and Systems, 74(2): 217-223. https://doi.org/10.1016/0165-0114(94)00339-9

[9] Chen, S.M. (1997). Similarity measures between vague sets and between elements. IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), 27(1): 153-158. https://doi.org/10.1109/3477.552198

[10] Hong, D.H., Kim, C. (1999). A note on similarity measures between vague sets and between elements. Information Sciences, 115(1-4): 83-96. https://doi.org/10.1016/S0020-0255(98)10083-X

[11] Li, F., Xu, Z.Y. (2001). Measures of similarity between vague sets. Journal of Software, 12(6): 922-927.

[12] Pappis, C.P. (1991). Value approximation of fuzzy systems variables. Fuzzy Sets and Systems, 39(1): 111-115. https://doi.org/10.1016/0165-0114(91)90070-7

[13] Pappis, C.P., Karacapilidis, N.I. (1993). A comparative assessment of measures of similarity of fuzzy values. Fuzzy Sets and Systems, 56(2): 171-174. https://doi.org/10.1016/0165-0114(93)90141-4

[14] Grzegorzewski, P. (2004). Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric. Fuzzy Sets and Systems, 148(2): 319-328. https://doi.org/10.1016/j.fss.2003.08.005

[15] Kacprzyk, J. (1998). Multistage fuzzy control with a soft aggregation of stage scores. Aggregation and Fusion of Imperfect Information, 118-137. https://doi.org/10.1007/978-3-7908-1889-5_8
[16] Majumdar, P., Samanta, S.K. (2008). Similarity measure of soft sets. New Mathematics and Natural Computation, 4(1): 1-12. https://doi.org/10.1142/S1793005708000908
[17] Williams, J., Steele, N. (2002). Difference, distance and similarity as a basis for fuzzy decision support based on prototypical decision classes. Fuzzy Sets and Systems, 131(1): 35-46. https://doi.org/10.1016/S0165-0114(01)00253-6
[18] Wang, Z., Bovik, A.C., Lu, L. (2002). Why is image quality assessment so difficult? In 2002 IEEE International Conference on Acoustics, Speech, and Signal Processing Orlando, FL, USA, 4: IV-3313. https://doi.org/10.1109/ICASSP.2002.5745362
[19] Wang, Z., Bovik, A.C., Sheikh, H.R., Simoncelli, E.P. (2004). Image quality assessment: From error visibility to structural similarity. IEEE Transactions on Image Processing, 13(4): 600-612. https://doi.org/10.1109/TIP.2003.819861
[20] Wang, Z., Bovik, A.C. (2002). A universal image quality index. IEEE Signal Processing Letters, 9(3): 81-84. https://doi.org/10.1109/97.995823
[21] Hassan, A.F., Cailin, D., Hussain, Z.M. (2014). An information-theoretic image quality measure: comparison with statistical similarity. Journal of Computer Science, 10(11): 2269-2283. https://doi.org/10.3844/jcssp.2014.2269.2283
[22] Abdulhassan, A.M.R., Hassan, A.F. (2020). Efficient Information-Theoretic-Statistical (ITSM) equation for face recognition technique: Comparison with statistical technique and information-theoretic technique. In IOP Conference Series: Materials Science and Engineering Thi-Qar, Iraq, 928(4): 042036. https://doi.org/10.1088/1757-899X/928/4/042036
[23] Khaire, P.A., Thakur, N.V. (2012). A fuzzy set approach for edge detection. International Journal of Image Processing (IJIP), 6(6): 403-412. https://doi.org/10.4236/jsip.2014.54015
[24] Chaira, T. (2010). Intuitionistic fuzzy segmentation of medical images. IEEE Transactions on Biomedical Engineering, 57(6): 1430-1436. https://doi.org/10.1109/TBME.2010.2041000
[25] Ye, J. (2011). Cosine similarity measures for intuitionistic fuzzy sets and their applications. Mathematical and Computer Modelling, 53(1-2): 91-97. https://doi.org/10.1016/j.mcm.2010.07.022
[26] Haliche, Z., Hammouche, K., Losson, O., Macaire, L. (2022). Fuzzy color aura matrices for texture image segmentation. Journal of Imaging, 8(9): 244. https://doi.org/10.3390/jimaging8090244
[27] Močkoř, J., Hurtík, P. (2021). Approximations of fuzzy soft sets by fuzzy soft relations with image processing application. Soft Computing, 25(10): 6915-6925. https://doi.org/10.1007/s00500-021-05769-3
[28] Abd Alhussain, Z.F., Hassan, A.F. (2022). N-Ary multi fuzzy soft set and N-Ary multi fuzzy soft matrix. International Journal of Computer Application, 184(15): 26-30. https://doi.org/10.5120/ijca2022922144