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Enhanced Single-Snapshot 1-D and 2-D DOA Estimation Using Particle Swarm Optimization

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ABSTRACT

In radar applications, target range, velocity (Doppler), and angle are the three primary measurements employed. Estimating the number of targets and their directions of arrival (DOAs) on the antenna array can be achieved through various methods. Conventional techniques such as the correlation and Multiple Signal Classification (MUSIC) algorithms offer a straightforward approach for DOA estimation. However, these methods necessitate an exhaustive search of the entire spectrum and require numerous temporal snapshots to accurately identify the spatial spectrum peaks. To address this challenge, Particle Swarm Optimization (PSO)-correlation and PSO-MUSIC methods have been proposed. These PSO-based techniques provide a systematic approach to locate the spatial spectrum peak in both one-dimensional (1-D) and two-dimensional (2-D) scenarios. In order to determine the exact target position, the global best particle location is iteratively updated by these methods. The statistical performance of the 1-D and 2-D PSO-correlation and PSO-MUSIC algorithms demonstrates that these techniques exhibit higher accuracy in comparison to existing single-snapshot DOA estimation methods. The estimation performance of the proposed algorithms is analyzed and justified by employing the Cramér-Rao bound (CRB).

1. INTRODUCTION

Sensors, widely employed in modern technology, generate outputs for various applications, including remote sensing [1], biomedical engineering [2], communications [3], and radar systems [4]. In applications such as radar, sonar, and cellular communication, direction of arrival (DOA) estimation algorithms are commonly used to determine the directions of incoming targets. However, these algorithms often demand a significant number of antenna array samples (snapshots) to function effectively, thereby increasing the computational burden on the receiver. In real-time scenarios, obtaining multiple time snapshots or prior knowledge of the number of incoming signals is often infeasible, limiting the available samples for impinging target estimation at the receiver.

Snapshots, referring to the number of time samples or measurements per sensor, are crucial for signal parameter estimation. In the context of DOA estimation, more snapshots provide increased information about signals arriving at the array, thus enhancing the accuracy and resolution of estimated directions [5]. However, practical limitations, such as signal duration and sensor sampling rate, often restrict the number of snapshots. Ideally, the number of snapshots should be at least equal to the number of sensors in the array for a well-posed DOA estimation. This paper proposes a novel approach to accurately estimate DOA even under single snapshot conditions, balancing snapshot numbers with computational complexity and signal-to-noise ratio considerations.

Recent advancements in DOA estimation [5-7] suggest potential solutions for single or few snapshot DOA estimation in practical applications, such as wireless location systems and compressive sensing (CS) based framework DOA estimation. By integrating nature-inspired optimization techniques, such as Particle Swarm Optimization (PSO), with traditional DOA estimation algorithms [8, 9], the performance of DOA estimation can be improved under single snapshot conditions.

Several studies have attempted to address the issue of degraded DOA estimation performance with single or few snapshots. In study [10], an improved Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm is proposed for determining the direction of incoming signals using a single temporal observation from a uniform linear array (ULA). For 2-dimensional DOA estimation [11], an algorithm is proposed that uses a single snapshot to construct three covariance matrices based on a dual parallel ULA. In study [12], a 2D-ZAP algorithm is proposed, which estimates DOA using l_1-norm with a single snapshot. Compressive sensing frameworks have also been employed for single snapshot DOA estimation [9]. In this paper, Particle Swarm Optimization (PSO) is combined with the MUSIC spectrum function for DOA estimation in low signal-to-noise ratio (SNR) environments [13, 14].

The present study addresses the degraded performance of conventional DOA estimation algorithms, such as correlation and MUSIC, under single snapshot conditions. The proposed PSO-Correlation and PSO-MUSIC algorithms combine these conventional methods with a PSO framework to improve spectral search and identify maximum values. As demonstrated in section 4, the statistical performance of the PSO-Correlation algorithm is significantly improved compared to existing DOA estimation algorithms. The Cramér-Rao bound (CRB) is employed to estimate unbiased parameter variances with a lower bound, i.e., DOA. The performance of the array is influenced by various parameters, including the number of sensors (N), array geometry, number of sources (D), number of snapshots, signal-to-noise ratio (SNR), and others.

2. PRELIMINARIES

2.1 Signal model

Signals emitted from the multiple sources in the far-field, will be detected by the sensor arrays as shown in Figure 1 illustrate a fundamental model of array signal processing.

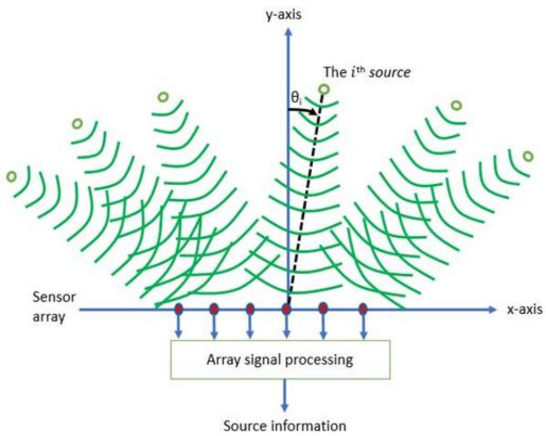

Figure 1. Basic model for array signal processing

The sensor array output is processed so that, we can infer source information such as the range, source directions, source powers, and velocity of sources. The sensors in Figure 1 are assumed to be on the x-axis, with the sources in the first or second quadrants of the xy-plane. The propagating wave-fronts are represented by green curves.

2.2 Signal model 1-D and 2-D

Let us considered an M-sensor Uniform Linear Array (ULA) and D-signal sources impinging. Sensors located at positions $[0, d, 2d, \dots (M-1)d]$ where d is the distance between tw antennas. Then the received signal can be modeled as [8] Eq. (1).

$$\boldsymbol{x(t)} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{D}} \boldsymbol{a_i}(\theta)\boldsymbol{S_i}(t) + \boldsymbol{n_i}(t) = \boldsymbol{AS}(t) + \boldsymbol{N}(t) \tag{1}$$

where, t is the time index, $\boldsymbol{S}(t) \in \mathbb{C}^{D \times 1}$ is the incoming signal wave, $\boldsymbol{n}(t) \in \mathbb{C}^{M \times 1}$ indicates the AWGN, and $\boldsymbol{A} \in \mathbb{C}^{M \times D}$ is the manifold matrix of the array as shown in Eq. (2).

$$\boldsymbol{A}(\theta) = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j\varphi_1} & e^{-j\varphi_2} & & e^{-j\varphi_D} \\ \vdots & & \ddots & \vdots \\ e^{-j(M-1)\varphi_1} & e^{-j(M-1)\varphi_2} & \cdots & e^{-j(M-1)\varphi_D} \end{bmatrix} \tag{2}$$

For i^{th} source with angle θ_i is defined as $\varphi_i = \frac{2\pi}{\lambda} d\sin(\theta_i)$. where, λ is wave length, θ_i is the incident angle of i^{th} source, and $1 \le i \le D$. The product of the array factor (AF) and the single element response yields the antenna array response. For a 1-D uniform linear array (ULA), the array factor is given by Eq. (3), while for a 2-D uniform rectangular array (URA), the array factor is given by Eq. (4) [15].

$$AF = \frac{1}{M}\left(\frac{\sin\left(\frac{M\varphi_1}{2}\right)}{\sin\left(\frac{\varphi_1}{2}\right)}\right) \tag{3}$$

$$AF = AF_x \times AF_y \tag{4}$$

where, AF_x is the array factor of the ULA in the x-axis direction and AF_y is the array factor of the ULA in the y-axis direction.

$$AF_x = \frac{1}{N}\left(\frac{\sin\left(\frac{N\varphi_x}{2}\right)}{\sin\left(\frac{\varphi_x}{2}\right)}\right) \;\&\; AF_y = \frac{1}{M}\left(\frac{\sin\left(\frac{M\varphi_y}{2}\right)}{\sin\left(\frac{\varphi_y}{2}\right)}\right) \tag{5}$$

where,
$\varphi_x = \frac{2\pi}{\lambda} d\sin(\theta)\cos(\phi)$ and $\varphi_y = \frac{2\pi}{\lambda} d\sin(\theta)\sin(\phi)$.

2.3 Cramér-Rao bound

The received observations from the antenna array $\mathbf{x}$ (of the Eq. (1)) is a real valued random vector, having probability density function $p(\mathbf{x};\, \boldsymbol{\alpha})$, $\boldsymbol{\alpha} = [\theta_i,\, Re\{A_i(k)\},\, Im\{A_i(k)\},\, \sigma_n]^T$ is real valued parameter vector where $1 \le i \le D$, $1 \le k \le K$, where K represents number of snapshots, and σ_n is the noise power. Assume that the pdf $p(\mathbf{x};\, \boldsymbol{\alpha})$ satisfy the regularity condition then the fisher information matrix (FIM) is defined as [15]:

$$[\mathrm{I}(\alpha)]_{\mathrm{ij}} = -E_x\left[\frac{\partial^2}{\partial[\alpha]_i \partial[\alpha]_j}\log\big(p(\mathbf{x};\, \boldsymbol{\alpha})\big)\right] \tag{6}$$

In the study [15], shown that the FIM is positive semi-definite, which means the FIM is invertible and the CRB is defined as

$$\mathrm{CRB}(\boldsymbol{\alpha}) = \mathrm{I}^{-1}(\boldsymbol{\alpha}) \tag{7}$$

CRB offers a lower bound on the variances of unbiased estimates of the parameters. CRB offer insights into the dependency of the array performance with respect to various parameters such as number of sensors M in the array, the array geometry, the number of sources D, the number of snapshots, and signal to noise ratio (SNR) more information on FIM and CRB can be found in study [16]. CRB for $\boldsymbol{\theta}$ [17] is defined as

$$CRB(\boldsymbol{\theta}) = \frac{\sigma_n}{2}\left\{\sum_{k=1}^{K} Re[(\boldsymbol{U}^H \boldsymbol{\Pi}_A^{\perp} \boldsymbol{U}) \odot \widehat{\boldsymbol{P}}^T]\right\}^{-1} \tag{8}$$

$$CRB_{Asympotic}(\boldsymbol{\theta}) = \frac{\sigma_n}{2}\left\{\sum_{k=1}^{K} Re[(\boldsymbol{U}^H \boldsymbol{\Pi}_A^{\perp} \boldsymbol{U}) \odot (\boldsymbol{P}\boldsymbol{A}^H \boldsymbol{\Sigma}^{-1} \boldsymbol{P})^T]\right\}^{-1} \tag{9}$$

where

$$\boldsymbol{A} = [\boldsymbol{a}(\theta_1)\, \boldsymbol{a}(\theta_2) \dots \boldsymbol{a}(\theta_D)], \tag{10}$$

$$\boldsymbol{U} = \left[\frac{\partial \boldsymbol{a}(\theta_1))}{\partial \theta_1} \frac{\partial \boldsymbol{a}(\theta_2))}{\partial \theta_2} \dots \frac{\partial \boldsymbol{a}(\theta_D))}{\partial \theta_D}\right], \quad (11)$$

$$\widehat{\boldsymbol{P}} = \frac{1}{K}\sum_{k=1}^{K} \boldsymbol{A}\boldsymbol{A}^H, \quad (12)$$

$$\boldsymbol{\Pi}_A^{\perp} = \boldsymbol{I} - \boldsymbol{A}(\boldsymbol{A}^H\boldsymbol{A})^{-1}\boldsymbol{A}^H, \quad (13)$$

and

$$\boldsymbol{\Sigma} = \boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^H + \sigma_n \boldsymbol{I} \quad (14)$$

The importance of the CRB is that, covariance of the unbiased estimator is lower bounded by the CRB.

2.4 Correlation algorithm

There are several methods available in the literature for DOA estimation, the easiest way to determine the DOA is through correlation. The model is of D signals incident on the array, corrupted by AWGN as shown in Eq. (1). We know that by the Cauchy-Schwarz inequality, as a function of θ, $\boldsymbol{A}^H(\theta)\boldsymbol{A}(\theta_i)$ has a maximum at $\theta = \theta_i$. The correlation method is given as Eq. (15).

$$P_{corr}(\boldsymbol{\theta}) = \boldsymbol{A}^H(\theta)\mathbf{x}(t) \quad (15)$$

$P_{corr}(\theta)$ is a non-adaptive estimate of the spectrum of the incoming data. The D largest peaks of the $P_{corr}(\theta)$ vs $\theta, \theta \in [-90^o\ 90^o]$ plot are the estimated DOA's.

2.5 MUSIC algorithm

The MUSIC estimates of θ_i are obtained by picking the D values of θ for which P_{MUSIC} is maximized. Maximization of P_{MUSIC} is usually done by evaluating it at the points of a fine grid using the Eq. (16).

$$P_{MUSIC}(\widehat{\boldsymbol{\theta}}) = \frac{1}{\boldsymbol{A}^H(\theta)\boldsymbol{U}_n\boldsymbol{U}_n^H\boldsymbol{A}(\theta)} \quad (16)$$

Procedure for the estimation of maximum using MUSIC algorithm is shown in the Figure 2.

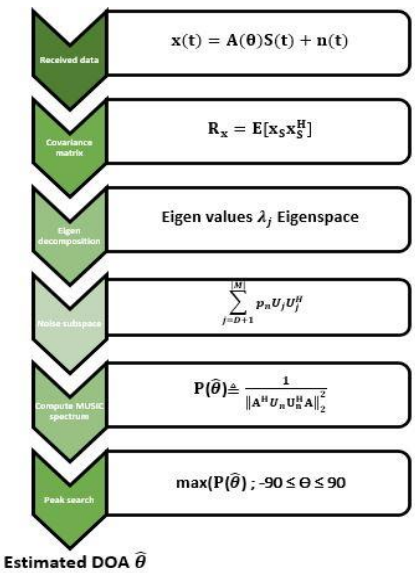

Figure 2. MUSIC algorithm methodology

2.6 Particle Swarm Optimization

PSO is optimization technique which is used to maximize or minimize a function in a feasible region, i.e., it is used to find global optimal solution in complex search space. It can also be used to solve multi-model function problems [18, 19]. PSO is a multi-agent parallel search technique which maintains a swarm of particles and each particle represents a potential solution in the swarm. All particles fly through a multi-dimensional search where each particle is adjusting its position according to its own experience and that of the neighbors.

The mathematical model of motion of the particle in the PSO is shown in the Figure 3 and can be described as Eq. (17) and Eq. (18):

$$x_i(t+1) = x_i + v_i(t+1) \quad (17)$$

where, $x_i(t)$ denote the position vector of particle i in the multi-dimensional search space $\mathbb{R}^n$ at time stamp t.

$$v_i(t+1) = wv_i(t) + r_1c_1[p_i(t) - x_i(t)] + r_2c_2[g(t) - x_i(t)] \quad (18)$$

where, $wv_i(t)$ is called as inertia term, $r_1, r_2 \in (0,1)$ are uniformly distributed random numbers, c_1, c_2 are called as acceleration coefficient.

Eq. (8) is used to update the position of the particle and Eq. (9) is used for updating the velocity of the particle $v_i(t+1)$, i.e., Eq. (9) is sum of three vectors parallel to previous velocity, parallel to vector connecting $\mathrm{x}_i(t)$ to $p_i(t)$ and vector parallel to vector connecting $\mathrm{x}_i(t)$ to $g_i(t)$. These two equations completely describe the mathematical model of PSO. Two equations are two simple rules which are obeyed by all the particles of the swarm.

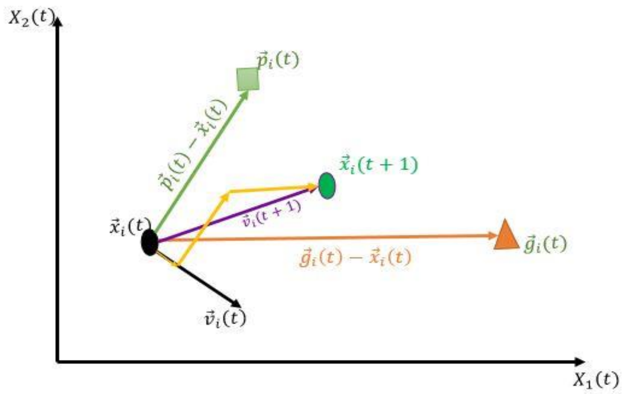

Figure 3. Velocity and position update for a particle in a two-dimensional search space

3. METHODOLOGY

With the latest advancements in DOA estimation, further solutions may be possible based on a single snapshot or a few snapshots. Using a single snapshot, it is possible to improve the performance of the DOA estimation by combining nature inspired optimization techniques like PSO with traditional DOA estimation algorithms as shown in the Figure 4 for the DOA estimation.

By using the P_{corr} and P_{MUSIC} Eqns. (15) and (16) respectively, as the cost function for PSO in DOA estimation, the goal is to find the set of DOA's that maximizes the amount of energy captured by the array. In the literature [1, 14] these approaches have been proved to be the most effective in finding the accurate DOA estimation in the presence of noise and other sources of uncertainty.

This can be formulated as an optimization problem as:

$$\underset{\theta\epsilon[-90^o,90^o]}{\operatorname{argmax}} \; P_{MUSIC}(\boldsymbol{\theta}) \tag{19}$$

Particle Swarm Optimization is a population-based optimization technique that searches for the optimal solution by iteratively updating the position Eq. (17) and velocity Eq. (18) of a swarm of particles. In the case of DOA estimation, the particles represent candidate solutions for the set of DOAs, and the objective function or cost function is the MUSIC spectrum function. For PSO-CORRELATION the objective function in Eq. (19) is replaced with correlation spectrum function which is in Eq. (15).

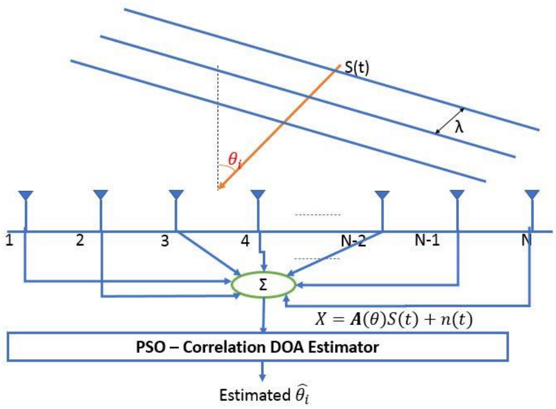

Figure 4. System model for PSO-Based estimation

Here we took the problem of degraded performance of the conventional DOA estimation algorithms when only single snapshot is considered. To be precise we have chosen fundamental beamforming method of DOA estimation correlation and high-resolution noise subspace-based method MUSIC. We have combined the correlation and MUSIC algorithm with PSO, that form an PSO-Correlation algorithm and PSO-MUSIC algorithm to improve the spectral search for maximum and it can also be seen in section 4 that statistical performance of the proposed techniques is much improved over the conventional DOA estimation algorithms.

4. SIMULATION AND RESULTS

This section covers the simulations for the 1-D and 2-D PSO-correlation and PSO-MUSIC algorithms in MATLAB. Statistical performance metrics mean square error and root-mean square error (RMSE) are used to calibrate these algorithms. Here the expressions for 1-D and 2-D RMSE are Eqns. (20) and (21) respectively.

$$RMSE = \sqrt{\frac{1}{MCRUN \times D} \sum_{j=1}^{MCRUN} \sum_{p=1}^{D} \left(\hat{\theta}_{j,p} - \theta_p\right)^2} \tag{20}$$

$$RMSE = \sqrt{\frac{1}{MCRUN \times D} \sum_{j=1}^{MCRUN} \sum_{p=1}^{D} \left\{ \begin{array}{c} \left(\hat{\theta}_{j,p} - \theta_p\right)^2 \\ +\left(\hat{\phi}_{j,p} - \phi_p\right)^2 \end{array} \right\}} \tag{21}$$

where, MCRUN is the number of independent runs, D is the number sources, $\hat{\theta}_{j,p}$ is the estimate value j^{th} MCRUN, θ_p is the true incoming bearing angle, and $\hat{\phi}_{j,p}$ is the estimate value j^{th} MCRUN, ϕ_p is the true elevation angle of the source.

Initially, we analyzed an incoming signal with a frequency of 2 GHz impinging with a bearing angle of 40° on a uniform linear array with antenna elements M=8, $d = \frac{\lambda}{2}$ is the distance between antenna elements, and K is the number of snapshots considered for the simulation. The simulated results were obtained by considering MCRUN=1000 independent Monte Carlo (MCRUN) runs, and the performance of 1-D correlation and 1-D MUSIC algorithms is evaluated at each SNR using the RMSE.

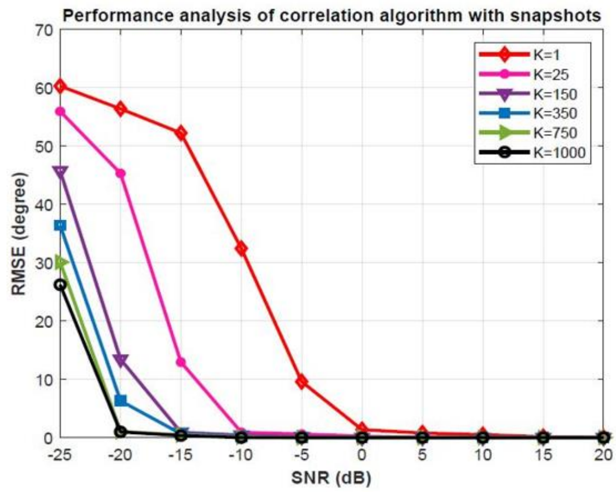

Figure 5(a). Performance analysis of 1-D correlation by varying snapshots

Figure 5(a) and 5(b) represents the performance of the 1-D correlation and 1-D MUSIC algorithms, the graphs demonstrate the performance of these algorithms by varying the number of snapshots K

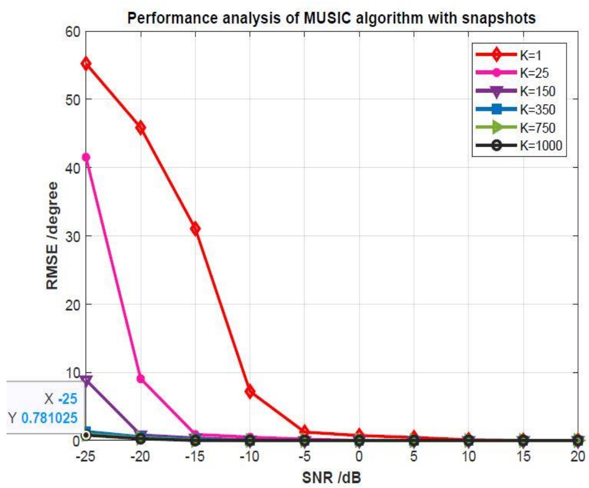

Figure 5(b). Performance analysis of 1-D MUSIC by varying snapshots

These methods exhibit high RMSE with a single or a smaller number of temporal snapshots, and as the number of temporal snapshots increases, RMSE decreases. It means that these algorithms require a greater number of time snapshots to work properly when signal to noise ratios is less, i.e., $\leq -5dB$. These algorithms are combined with Particle Swarm Optimization by considering the spectrum scan as the cost function.

Figure 6(a) and 6(b) represents the performance of the PSO-Correlation and PSO-MUSIC for 1-D and 2-D respectively. The performance of the proposed techniques along with traditional methods, is shown in the Figure 6(a) and 6(b). It can be observed that, the proposed methods outperform the standard methods like correlation, MUSIC, root-MUSIC, and MVDR in 1-D case and also in 2-D compared with traditional correlation and MUSIC, when only a single snapshot is available for DOA estimation and the SNR is very low.

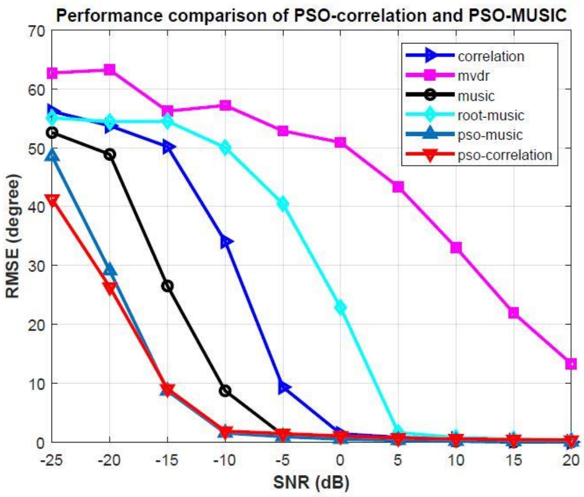

Figure 6(a). Performance analysis of 1-D PSO-Correlation and PSO-MUSIC with single snapshot K=1

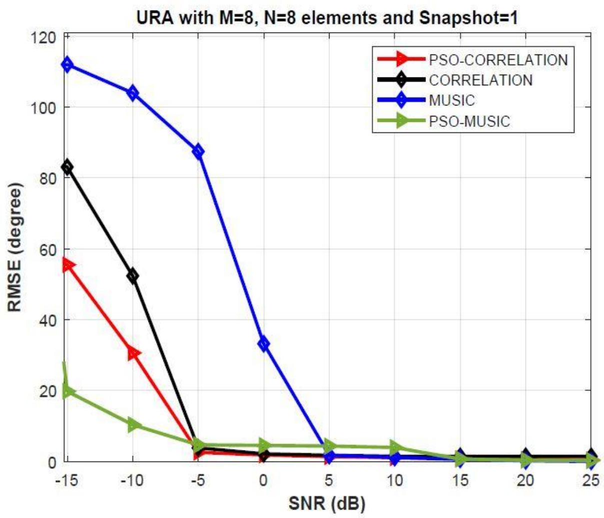

Figure 6(b). Performance analysis of 2-D PSO-Correlation and PSO-MUSIC with single snapshot K=1

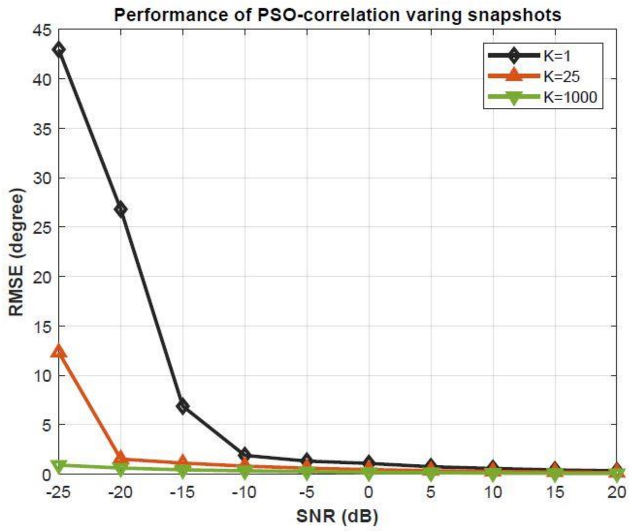

Figure 7(a). Performance analysis of the proposed methods 1-D PSO-correlation by varying snapshots

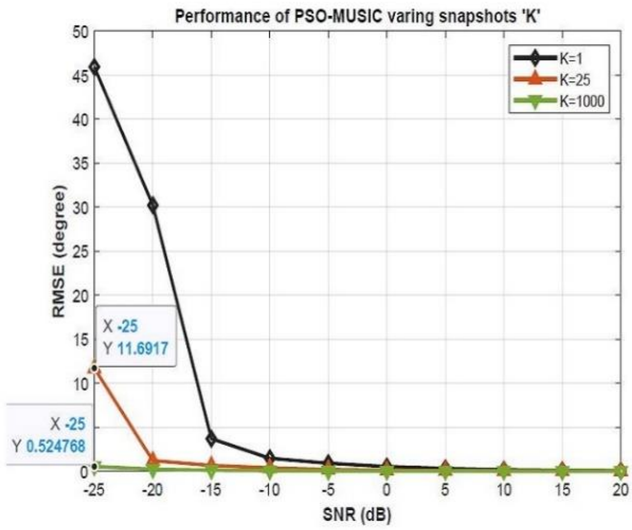

Figure 7(b). Performance analysis of the proposed methods 1-D PSO-MUSIC by varying snapshots

Then the efficiency of the suggested techniques PSO-correlation and PSO-MUSIC is determined with the snapshots K. The performance of these algorithms is evaluated by increasing the number of snapshots to K=1, K=25, and K=1000, as illustrated in the Figures 7(a) and 7(b). The RMSE of the proposed techniques decreases significantly as the number of snapshots increases in comparison to traditional methods correlation and MUSIC Figure 5(a) and 5(b).

Figure 8(a) and 8(b) compares the CRB for DOA estimation of correlation, PSO-correlation and MUSIC, PSO-MUSIC respectively. The CRB expression for θ_i as a function of SNR with single snapshot. It can be observed from the graph 8(a) and 8(b) that the expression is inversely proportional to the SNR and the proposed techniques exact more information of the parameter then the existing techniques with single snapshot. Figure 9 compares the performance of proposed techniques with 1000 snapshots.

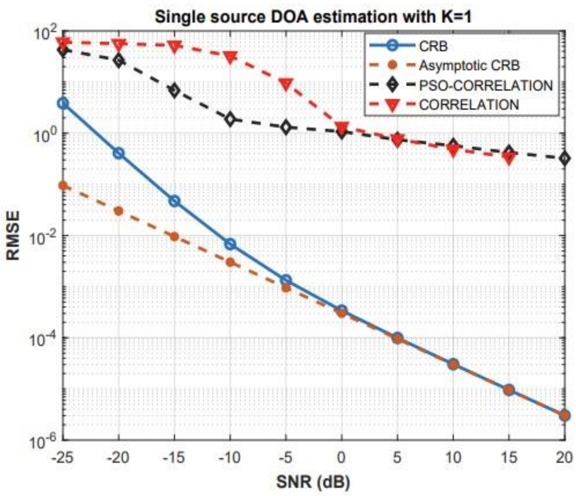

Figure 8(a). Performance of the proposed method PSO-CORRELATION with single snapshot for single source. The array configuration is ULA with M=8. Power source located at 40° with varying SNR

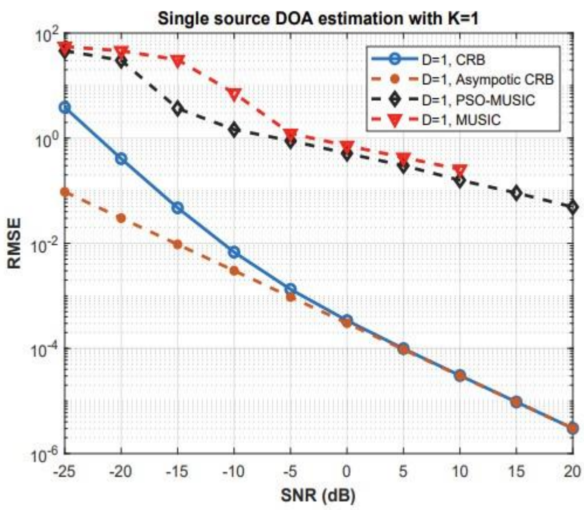

Figure 8(b). Performance of the proposed method PSO-MUSIC with single snapshot for single source. The array configuration is ULA with M=8. Power source located at 40° with varying SNR

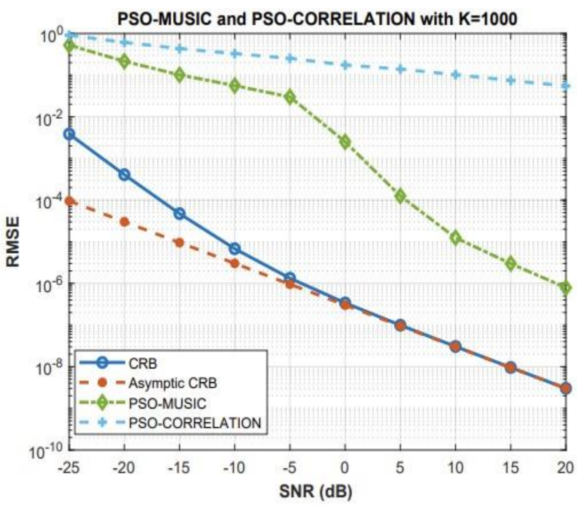

Figure 9. Performance analysis of the proposed methods 1-D PSO-MUSIC and PSO-Correlation with snapshots K=1000

5. CONCLUSION

In this paper, we have presented the efficiency of the proposed PSO-correlation and PSO-MUSIC methods for 1-D and 2-D DOA estimation using a single snapshot. Through extensive simulations and analysis, we have demonstrated that our proposed algorithms outperform traditional DOA estimation techniques in terms of accuracy, as indicated by the lower RMSE values obtained.

By leveraging the PSO technique, our methods provide a systematic approach to locate the spatial spectrum peak, leading to improved estimation precision. The iterative updating of the global best particle location in both 1-D and 2-D scenarios enables the determination of target positions with higher accuracy. Furthermore, we have employed the CRB to analyze and explain the estimation performance of our proposed algorithms, which further validates their effectiveness.

Although our proposed PSO-correlation algorithm has demonstrated superior performance in single-snapshot DOA estimation, there are several potential avenues for further research and exploration. Some future directions for extending this work are:

(i) Investigating the algorithm's ability to accurately estimate DOAs when faced with multiple simultaneously impinging signals

(ii) Extending the study to environments with significant multi-path propagation can provide insights into the robustness and reliability of our proposed methods.

In conclusion, the results presented in this paper demonstrate the effectiveness of the proposed PSO-correlation and PSO-MUSIC methods for single-snapshot DOA estimation in both 1-D and 2-D scenarios. The promising performance and the identified future research directions contribute to the advancement of DOA estimation techniques and open up possibilities for further improving the accuracy and applicability of such methods in practical radar systems.

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