Stability analysis of ferrothermohaline convection in a Darcy porous medium with Soret and MFD viscosity effects

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1. INTRODUCTION

The magnetization of ferromagnetic fluids depends on the magnetic field, the temperature and the density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. These leads to convection in ferrofluids in the presence of magnetic field gradient, known as ferroconvection, which is similar to buoyancy driven convection. Ferroconvection studies were initiated by Cowley and Rosensweig [1]. When a salty fluid gets heated up two types of inter-diffusive phenomena occur. They are known as (a) the Soret effect, (b) Dufour effect. Thermo diffusion, also called the Soret effect, is characterized by the Soret coefficient. Thermo diffusion in a ferrofluid in the presence of a magnetic field was investigated by Voelker and Odenbach [2]. The theory of convective instability of ferrofluid began with Finlayson [3] and extensively continued by Stiles and Kagan [4] and Siddheshwar [5]. Nanjundappa and Shivakumara [6] have investigated a variety of velocity and temperature boundary conditions on the onset of ferroconvection convective instability in an initially quiescent ferrofluid layer in the presence of a uniform magnetic field.

Ferroconvecitive instability in an anisotropic densely packed magnetic particle, treated as porous medium was studied by Vaidyanathan et al. [7]. Hurle and Jakeman [8] analyzed the Soret driven thermosolutal convection. Shevtsova et al. [9] carried out a study on the onset of convection in Soret driven instability. Soret driven ferrothermohaline convection in the presence and absence of a porous medium was investigated by Vaidyanathan et al. [10-11]. A linear stability analysis on the onset of Soret driven motion in nanoparticles suspension was made by Kim [12]. Hemalatha et al. [13-14] discussed the effect of rotation on Soret driven ferrothermohaline convection of dust particles with and without porous medium. The convection in ferromagnetic fluids is gaining much importance due to astounding physical properties. One such property is viscosity of the ferromagnetic fluid. Fluids with ferromagnetic properties may be formed by colloidal suspension of solid magnetic particles such as magnetite in a parent liquid. Viscosity of the fluid in a magnetic field is predicted by dimensional analysis to be a function of the ratio of hydrodynamic stress to magnetic stress in Rosenswig et al. [15]. The effect of uniform distribution of heat source on the onset of stationary ferroconvection was investigated by Rudraiah et al. [16]. The onset of convection in a sparsely packed porous layer with throughflow was studied by Shivakumara et al. [17]. The onset of surface tension driven convection in a superposed layers of fluid and saturated porous medium was studied by Shivakumara et al. [18]. The boundary and thermal non-equilibrium effects on the onset of Darcy-Brinkman convection in a porous layer was investigated by Shivakumara et al. [19].

The effect of rotation on double diffusive convection in a magnetized ferrofluid with internal angular momentum was studied by Sunil et al. [20]. The effect of a homogeneous magnetic field on the viscosity of a fluid with solid particles possessing intrinsic magnetic moments was investigated by Shliomis [21]. Kaloni and Lou [22] have investigated theoretically the convective instability problem in a thin horizontal layer of magnetic fluid heated from below under alternating magnetic field by considering the quasi stationary model with internal rotation and vortex viscosity. Thermal convection in a ferromagnetic fluid in the presence of a magnetic field dependent viscosity was investigated by Paras...
Ram et al. [23-24]. Paras Ram et al. [25-26] discussed the ferrofluid flow with magnetic field dependent viscosity due to rotating disk with and without porous medium. The effect of a magnetic field dependent viscosity on thermostosolutal convection ferromagnetic fluid in the presence and absence of porous medium was studied by Sunil et al. [27-28]. It is interesting to study the nature of variable viscosity on fluids. Viscosity may depend on temperature and the magnetic field also. The effect of temperature-dependent viscosity on the threshold of ferroconvection instability in a porous medium using the Brinkman model and Galerkin technique was investigated by Ramanathan and Muchikiel [29].

The effect of magnetic field dependent (MFD) viscosity on thermal convection in a ferromagnetic fluid with and without porous medium was investigated by Sunil et al. [30-31]. The effect of magnetic field dependent (MFD) viscosity on ferroconvection in a rotating with and without porous medium was studied by Vaidyanathan et al. [32-33]. The effect of a magnetic field dependent viscosity on the onset of convection in a ferromagnetic fluid layer heated from below and cooled from above in the presence of a vertical magnetic field with constant heat flux has been investigated by Nanjundappa et al. [34]. The effect of a magnetic field dependent (MFD) viscosity on ferroconvection in an anisotropic porous medium analyzed by Ramanathan and Suresh [35]. The comparison of theoretical and computational ferroconvection induced by magnetic field dependent viscosity in an anisotropic porous medium has been analysed by Suresh et al. [36]. Malashetty et al. [37-38] carried out an analytical study of linear and nonlinear double diffusive convection with Soret effect in couple-stress liquids with and without Dufour effect.

A nonlinear stability analysis for thermoconvective and double-diffusive magnetized ferrofluid with MFD viscosity has been investigated by Sunil et al. [39-40]. The effect of a magnetic field dependent viscosity on ferroconvection in an anisotropic porous medium in the presence of a horizontal thermal gradient was studied by Hemalatha and Sivaprabha [41]. Sunil et al. [42] have studied theoretically, the effect of magnetic field dependent viscosity on the thermal convection in a ferromagnetic fluid layer with or without dust particles. The effect of a magnetic field dependent viscosity on a Soret driven ferrothermolaline convection in a rotating porous medium was studied by Hemalatha [43]. Sekar and Raju [44-45] have studied the effect of magnetic field dependent viscosity on Soret-driven thermoconvection instability of ferromagnetic fluid in the presence of rotating and saturating an anisotropic porous medium of sparse particle suspension. More recently, the presence and absence of coriolis force on Soret driven ferrothermolaline convection saturating a densely packed anisotropic porous medium was studied by Sekar et al. [46-47].

In this paper, the effect of magnetic field dependent viscosity on Soret-driven ferrothermolaline convection in the presence of densely packed porous medium is studied. The overriding Soret effect on the salinity equation is also studied. Linear stability analysis is used. The conditions for the onset of stationary and oscillatory instabilities are obtained.

2. MATHEMATICAL FORMULATION

An infinitely spread layer of Boussinesq ferromagnetic fluid of thickness ‘d’ in the presence of densely packed porous medium heated from below and salted from above is considered. The temperature and salinity at the bottom and top surfaces z = ±d/2 are T0 ± ΔT/2 and S0 ± ΔS/2, respectively. Both the boundaries are taken to be free and perfect conductors of heat and solute. The Soret effect on the temperature gradient is considered.

The fluid is assumed to be incompressible fluid having a variable viscosity, given by \( \eta = \eta_1(1 + \delta B) \) where \( \eta_1 \) is taken as viscosity of the fluid when the applied magnetic field is absent and \( B = (B_1, B_2, B_3) \) is the magnetic induction. The variation in the coefficient of the magnetic field dependent viscosity \( \delta \) has been taken to be isotropic, that is, \( \delta = \delta_1 = \delta_2 = \delta_3 \) Hence the component wise \( n \) can be written as \( \eta_x = \eta_1(1 + \delta B_1) \eta_y = \eta_1(1 + \delta B_2) \) and \( \eta_z = \eta_1(1 + \delta B_3) \).

The mathematical equations governing the above investigation are as follows:

The continuity equation for an incompressible fluid is

\[ \nabla \cdot \mathbf{q} = 0 \]  

(1)

The momentum equation is

\[ \rho_0 \frac{Dq}{Dt} = -\nabla p + \rho\mathbf{g} + \nabla \cdot (\mathbf{H} \cdot \mathbf{B}) - \frac{\eta}{k} \mathbf{q} \]  

(2)

The temperature equation for an incompressible ferrofluid is

\[ \rho_c C_v \frac{dT}{dt} - \mu_0 \mathbf{H} \cdot \left( \frac{\partial \mathbf{M}}{\partial T} \right)_{\mathbf{v}, H} \frac{dT}{dt} + \mu_0 T \frac{\partial \mathbf{M}}{\partial T} \cdot \frac{d\mathbf{H}}{dt} = K_1 \nabla^2 T + \phi \]  

(3)

The mass flux equation is

\[ \frac{DS}{Dt} = K_1 \nabla^2 S + S_T \nabla^2 T \]  

(4)

where \( \rho_0, \mathbf{q} = (u, v, w), \mathbf{g} = (0, 0, -g) \), \( t, p, \mu_0, \eta, \mathbf{H}, \mathbf{B}, C_v, H, T, \mathbf{M}, K_1, S, S_T \) and \( \phi \) are the density, filter velocity, acceleration due to gravity, time, pressure, magnetic permeability, viscosity (variable), permeability of the porous medium, magnetic field, magnetic induction, heat capacity at constant volume and magnetic field, temperature, magnetization, thermal conductivity, salinity, mass diffusivity, Soret coefficient and viscous dissipation factor containing second-order terms in velocity, respectively.

Maxwell’s equations are

\[ \nabla \cdot \mathbf{B} = 0, \nabla \times \mathbf{H} = 0 \]  

(5)

Further \( B, M \) and \( H \) are related by

\[ \mathbf{B} = \mu_0 (\mathbf{M} + \mathbf{H}) \]  

(6)

Using Maxwell’s equation for non-conducting fluids one can assume that the magnetization is aligned with the magnetic field and depends on the magnitude of the magnetic field,
temperature and salinity, so that

\[ M = \frac{H}{H} M (H, T, S) \]  (7)

The magnetic equation of state is linearised about the magnetic field \( H_0 \), the average temperature \( T_0 \) and the average salinity \( S_0 \) to become

\[ M = M_0 + \chi (H - H_0) - K (T - T_0) + K_2 (S - S_0) \]  (8)

where \( \chi = (\partial M / \partial H)_{H_0,T_0} \) is the susceptibility, \( k = (\partial M / \partial T)_{H_0,T_0} \) is the pyromagnetic coefficient and \( k_2 = (\partial M / \partial S)_{H_0,T_0} \) is the salinity magnetic coefficient.

The density equation of state for a Boussinesq two-component fluid is

\[ \rho = \rho_0 \left[ 1 - \alpha (T - T_0) + \alpha_x (S - S_0) \right] \]  (9)

where \( \alpha \) is the thermal expansion coefficient and \( \alpha_x \) is the solute analog of \( \alpha \).

The basic state is assumed to be the quiescent state and taking the components of the magnetization and magnetic field in the basic state to be \([0,0,M_0(z)]\) and \([0,0,H_0(z)]\) the basic state quantities obtained are:

\[ q = q_0 = 0, \quad p = p_0(z), \quad \frac{\partial T}{\partial z} = -\beta_1 \Rightarrow T = T_0 - \beta_1 z, \]

\[ \frac{\partial S}{\partial z} = \beta_2 \Rightarrow S = S_0 + \beta_2 z, \]

\[ H_b(Z) = H_0 + \frac{K (T_b - T_0)}{1 + \chi} \frac{K_2 (S_b - S_0)}{1 + \chi} k, \]

\[ M_b(Z) = M_0 + \frac{K (T_b - T_0)}{1 + \chi} \frac{K_2 (S_b - S_0)}{1 + \chi} k. \]  (10)

where \( \beta_1 \) and \( \beta_2 \) are non-negative constants and \( k = (0,0,1) \) is the unit vector along vertical direction.

### 3. Linear Stability Analysis

The basic state is disturbed by a small thermal perturbation. Consider a perturbed state such that

\[ q = q_0 + \tilde{q}, \quad p = p_0 + \tilde{p}, \quad \tilde{w} = \tilde{w}_b(z) + \tilde{\eta}, \quad T = T_0 + \tilde{T}, \]

\[ H = H_0(z) + \tilde{H}, \quad M = M_0(z) + \tilde{M} \]

where \( \tilde{q}, \tilde{p}, \tilde{\eta}, \tilde{T}, \tilde{H} \) and \( \tilde{M} \) are perturbed variables and are assumed to be small. The perturbed state temperature and solute are \( T = T_0 - \beta_1 z + \tilde{T} \) and \( S = S_0 - \beta_2 z + \tilde{S} \). Let the components of the perturbed magnetization and the magnetic field be \( (M_1, M_2, M_0(z), M_3) \) and \( (H_1, H_2, H_0(z), H_3) \) respectively.

\[ H'_i + M'_i = \left( 1 + \frac{M_0}{H_0} \right) H_i \quad (i = 1, 2) \]  (11)

\[ H'_3 + M'_3 = (1 + \chi) H_3 - KT + K_2 S + S_T K T' \]  (12)

Let \( (B_1, B_2, B_3) \) denote the components of \( B \). Using Eq. (6), one gets the result \( B_i = \mu_0 (M_1 + H_i) \) and Eqs. (11) and (12) become

\[ B_i = \mu_0 \left( 1 + \frac{M_0}{H_0} \right) H_i \quad (i = 1, 2) \]  (13)

\[ B_3 = \mu_0 \left( 1 + \chi \right) H_3 - KT + K_2 S + S_T K T' + M_0 \]  (14)

When Eq. (5) is used in Eq. (1) and resulting equation is linearized with \( B_i \) \((i = 1, 2, 3)\) given by Eqs. (13) and (14), we obtain the following components

\[ \rho_0 \frac{\partial w}{\partial t} = \frac{\partial p}{\partial x} + \mu_0 (M_0 + H_0) \frac{\partial H_1}{\partial x} - \frac{\partial H_1}{\partial x} \frac{\eta}{k} \]  (15)

\[ \rho_0 \frac{\partial v}{\partial t} = \frac{\partial p}{\partial y} + \mu_0 (M_0 + H_0) \frac{\partial H_2}{\partial y} - \frac{\partial H_2}{\partial y} \frac{\eta}{k} \]  (16)

\[ \rho_0 \frac{\partial w}{\partial t} = \frac{\partial p}{\partial z} + \mu_0 (M_0 + H_0) \frac{\partial H_3}{\partial z} - \frac{\partial H_3}{\partial z} \frac{\eta}{k} \]  (17)

Differentiating Eqs. (15)-(17) with respect to \( x, y \) and \( z \) respectively and adding, the following equation is obtained upon using Eq. (1)

\[ \nabla^2 p = \mu_0 (M_0 + H_0) \left( \frac{\partial}{\partial z} (\nabla \cdot H') \right) + \mu_0 K_2 \beta_2 \frac{\partial H_1}{\partial z} \]  (18)

where \( H' \) has the components \( (H'_1, H'_2, H'_3) \).

From Eq. (5), \( H' = \nabla \phi \) where \( \phi \) is a scalar potential. Elimination of \( p \) from Eq. (15)-(17) and using Eq. (18) gives,

\[ \rho_0 \frac{\partial}{\partial t} \left( \nabla^2 \phi \right) = \mu_0 K_2 \beta_2 \frac{\partial^2 \phi}{\partial z^2} - \mu_0 K_2 \beta_2 \frac{\partial}{\partial z} \left( \nabla^2 \phi \right) \]  (19)
where \( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \) and \( \nabla^2 = \nabla^2 + \frac{\partial^2}{\partial z^2} \)

## 4. NORMAL MODE ANALYSIS

We now proceed to a normal mode analysis of the above stability problem. Let us take,

\[
f(x, y, z, t) = f(z, t) e^{i(k_1x + k_2y + k_3z)},
\]

\[
\phi = \phi(z, t) e^{i(k_1x + k_2y)}, \quad w = w(z, t) e^{i(k_1x + k_2y)},
\]

\[
T' = \theta(z, t) e^{i(k_1x + k_2y)}, \quad S' = S(z, t) e^{i(k_1x + k_2y)}
\]

(20)

with the wave number \( k_0^2 = k_1^2 + k_2^2 \)

Using Eq. (20) in Eq. (19), one gets

\[
\rho_0 \frac{\partial}{\partial t} \left( \frac{\partial^2 \phi}{\partial z^2} - k_0^2 \right) w = \mu_0 K_0 B_\phi \frac{1}{1 + \chi} \left[ 1 + \chi \right] \frac{\partial \phi}{\partial z} + k_0 S
\]

\[
+ \mu_0 K_0 B_\phi \frac{1}{1 + \chi} \left[ 1 + \chi \right] \frac{\partial \phi}{\partial z} - K \theta (1 - S_T) \left[ k_0^2 + \rho_0 g \alpha \theta k_0^2 S \right] \]

\[
- \rho_0 g \alpha \theta k_0^2 \theta + \frac{\mu_0 K_0}{1 + \chi} \left[ 1 + \chi \right] \theta \left[ k_0^2 + \rho_0 g \alpha \theta k_0^2 S \right] \]

(21)

\[
\eta \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) w + \eta k_0 \delta u_0 \left( M_0 + H_0 \right) w
\]

The temperature equation is

\[
\rho_0 C_{V,H} \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial \phi}{\partial t} = K_1 \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \theta
\]

\[
+ \left[ \rho_0 C_{V,H} \beta_\theta - \mu_0 K T_0 \beta_\phi - \mu_0 K_0 T_0 \beta_\phi \right] \frac{\partial \theta}{\partial t}
\]

(22)

where \( \rho_0 C_{V,H} = \rho_0 C_{V,H} + \mu_0 \rho_0 H_0 \)

The salinity equation is

\[
\frac{\partial S}{\partial t} + \beta_S w = K_s \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) S + S_T \left( \frac{\partial^2}{\partial z^2} - k_0^2 \right) \theta
\]

(23)

The magnetic potential equation is

\[
\left( 1 + \chi \right) \frac{\partial^2 \phi}{\partial z^2} - \left( 1 + \frac{M_0}{H_0} \right) k_0^2 \phi = -K \frac{\partial \theta}{\partial z} + K_2 \frac{\partial S}{\partial z} + S_T K \frac{\partial \theta}{\partial z} = 0
\]

(24)

The non-dimensional numbers can be written using

\[
t^* = \frac{vt}{d^2}, \quad w^* = \frac{wd}{v}, \quad T^* = \left( \frac{K_0 a R^{1/2}}{\rho_0 C_{V,H} \beta_\phi v d} \right) \theta, \quad k^* = \frac{k}{d^2},
\]

\[
\phi^* = \left( \frac{(1 + \chi) K_0 a R^{1/2}}{K_0 \rho_0 C_{V,H} \beta_\phi v d} \right) \phi, \quad z^* = \frac{z}{d}, \quad D = \frac{\partial}{\partial z}, \quad a = k_0 d,
\]

\[
S^* = \left( \frac{K_0 a R^{1/2}}{\rho_0 C_{V,H} \beta_\phi v d} \right) S, \quad v = \frac{\eta}{\rho_0}, \quad \delta^* = \mu_0 \delta H_0 (1 + \chi)
\]

Then Eqs. (21) – (24) become

\[
\left( \frac{\partial}{\partial t} + \frac{1}{k^*} \right) (D^2 - a^2) w^* = a R^{1/2} M_1 S_5 \phi^* + a R^{1/2} M_1 S_5 (1 - S_T) T^* + \frac{a^2}{k^*} M_1 \delta^* w^*,
\]

\[
-a R^{1/2} M_1 S_5 (1 - S_T) T^* + a R^{1/2} \left[ 1 + M_4 + M_5 \right] S^*,
\]

(25)

\[
P_r \left[ \frac{\partial T^*}{\partial t} + \frac{1}{k^*} (D^2 - a^2) T^* \right] = \left( D^2 - a^2 \right) T^* - a R^{1/2} (1 - M_2 - M_2 M_5) w^*,
\]

(26)

\[
P_r \frac{\partial S^*}{\partial t} = \tau (D^2 - a^2) S^* - a R^{1/2} M_4 w^*,
\]

\[
+ S_T \left( \frac{M_5}{M_6} \right) \left( \frac{R^2}{R^*} \right) (D^2 - a^2) T^*,
\]

(27)

\[
D^2 \phi^* - M_5 a^2 \phi^* - (1 - S_T) D T^* + \frac{M_5}{M_6} \left( \frac{R^2}{R^*} \right) \left( D^2 - a^2 \right) T^*,
\]

(28)

where the non-dimensional parameters used are

\[
M_1 = \frac{\mu_0 K^2 \beta_\phi}{(1 + \chi) \rho_0 g \alpha \theta}, \quad M_2 = \frac{\mu_0 K^2 T_0}{(1 + \chi) \rho_0 C_{V,H}}, \quad M_3 = \frac{K_2 \beta_\phi}{K_2 \beta_\phi},
\]

\[
M_4 = \frac{1}{1 + \chi} \rho_0 g \alpha \theta, \quad M_5 = \frac{\mu_0 K^2 \beta_\phi}{(1 + \chi) \rho_0 C_{V,H}}, \quad M_6 = \frac{K_5 \beta_\phi}{K_1},
\]

\[
P_r = \frac{\eta C_{V,H}}{K_1}, \quad \tau = \rho_0 C_{V,H} \frac{K_2}{K_1}, \quad R_s = \frac{\rho_0 C_{V,H} \beta_\phi g d^4}{v K_5},
\]

\[
R = \frac{\rho_0 C_{V,H} \beta_\phi g d^4}{v K_1},
\]

(29)

where \( R_s \) is the salinity Rayleigh number, \( R \) is the thermal Rayleigh number, \( P_r \) is the Prandtl number and other parameters describe non-dimensional parameters.

## 5. ANALYSIS OF SOLUTION AT FREE BOUNDARIES

The boundary conditions on velocity, temperature and salinity are

\[
w^* = D^2 w^* = T^* = D T^* = S^* = 0 \quad \text{at} \quad z^* = \pm 1/2.
\]

(30)
The exact solutions satisfying above equation (30) are

\[ w^* = Ae^{\alpha w} \cos \pi z^* \cos \pi z^* , \]
\[ S^* = Ce^{\alpha w} \cos \pi z^* , \]
\[ \phi^* = E e^{\alpha w} \sin \pi z^* \] 

where A, B, C and E are constants. These functions substituted in the set of Eqs. (25)-(28) give the following four linear homogeneous algebraic equations in the constant A, B, C and E:

\[
\begin{bmatrix}
(\sigma + \frac{1}{k})(\pi^2 + \pi' \pi'') - \pi' \pi'' \phi^* + \frac{1}{k} \pi' \pi'' M' a \delta & A \\
-aR^{(2)} [1 + M_4 (1 - S_T) + M_5 (1 - S_T)] & B \\
adR^{(2)} (1 + M_4 + M_5 M_6) A + aR^{(2)} M_1 (1 + M_4) E & 0
\end{bmatrix}
\]

(32)

For obtaining stationary instability, the time-independent term \( \mathbf{X} = 0 \). Eq. (36) helps one to obtain Eigen value \( \mathbf{R}_{SC} \) for which a solution exists:

\[ \mathbf{R}_{SC} = \frac{N^*}{D^*} \]

where

\[ N^* = \left( \pi^2 + \pi' \right) \left( \pi^2 + \pi' + a^2 M' \phi^* \right) - a^2 kR \pi^2 \]

\[ \left( 1 + M_4 + M_5 M_6 \right) \left( 1 - M_2 - M_2 M_5 \right) S_T \left( \frac{M_5}{M_6} \right) + M_6 \]

And

\[ D^* = a^2 k \left( 1 - M_2 - M_2 M_3 \right) \left( 1 + (1 - S_T) M_1 (1 + M_5) \right) \]

\[ - \frac{a^2 k M_4 (1 + M_5)}{\pi^2 + a^2 M_5} \]

\[ \left( 1 - M_2 - M_2 M_5 \right) S_T \left( \frac{M_5}{M_6} \right) \tau^{-1} + (1 - S_T) \tau^{-1} + M_5 \tau^{-1} \]

For \( M_1 \) very large, one gets the results for the magnetic mechanism, and the critical thermo magnetic Rayleigh number for stationary mode is obtained using

\[ N_{SC} = \mathbf{R}_{SC} M_1 = \frac{N^*}{D^*} \]

where

\[ N^* = \left( \pi^2 + \pi' \right) \left( \pi^2 + \pi' + a^2 M' \phi^* \right) - a^2 kR \pi^2 \]

\[ \left( 1 + M_4 + M_5 M_6 \right) \left( 1 - M_2 - M_2 M_5 \right) S_T \left( \frac{M_5}{M_6} \right) + M_6 \]

And

\[ D^* = a^2 k \left( 1 - M_2 - M_2 M_3 \right) \left( 1 + (1 - S_T) M_1 (1 + M_5) \right) \]

\[ - \frac{a^2 k M_4 (1 + M_5)}{\pi^2 + a^2 M_5} \]

\[ \left( 1 - M_2 - M_2 M_5 \right) S_T \left( \frac{M_5}{M_6} \right) \tau^{-1} + (1 - S_T) \tau^{-1} + M_5 \tau^{-1} \]
\[ D_r = a^2 k \left[ (1 - M_2 - M_2 M_5)(1 - S_T)(1 + M_5) \right] \]
\[ - \pi^2 \left[ \frac{a^2 k (1 + M_5)}{\pi^2 + a^3 M_3} \right] \]
\[ \left( 1 - M_2 - M_2 M_5 \right) \left[ S_T \left( \frac{M_5}{M_6} \right)^2 \pi^{-1} + (1 - S_T) + M_5 \pi^{-1} \right] \]

7. OSCILLATORY CONVECTION

The conditions for the onset of oscillatory stabilities are obtained as follows: Taking \( \sigma = \alpha + \alpha^2 > 0 \), in Eq. (36) and following the analysis and techniques of Vaidyanathan et al. [32], the critical Rayleigh number for oscillatory mode has been calculated using

\[ R_{OC} = \frac{U_1 U_3 + (U_1 U_2 + V_1 V_3) \sigma^2 + V_1 V_2 \sigma^4}{U_1^2 \sigma^2} \]

where

\[ U_1 = a^2 \left( \pi^2 + a^2 \right) \tau (1 - M_2 - M_2 M_5) \]
\[ \left( \pi^2 + a^2 M_3 \right) \left[ 1 + (1 - S_T) M_1 (1 + M_5) \right] \]
\[ + a^2 \left( \pi^2 + a^2 \right) M_1 (1 + M_5) \pi^2 \]
\[ \left( 1 - M_2 - M_2 M_5 \right) \left[ S_T \left( \frac{M_5}{M_6} \pi + (1 - S_T) + M_5 \right) \right] \]
\[ U_2 = \frac{1}{k} \left( \pi^2 + a^2 + a^2 M_5 \delta \right) \left[ \left( \pi^2 + a^2 M_3 \right) - \pi^2 M_2 (1 - S_T) \right] P_r^2 \]
\[ + \left( \pi^2 + a^2 \right)^2 \left[ \pi^2 M_2 \left( \frac{M_5}{M_6} \pi + (1 - S_T) \right) \right] P_r \]
\[ - \left( \pi^2 + a^2 M_3 \right) \pi (1 + \tau) \]
\[ \left[ a^2 R_i \left( 1 + M_4 + \frac{M_5}{M_6} \right) \right] \]
\[ V_1 = a^2 P_r \left[ 1 + M_1 (1 + M_3) (1 - S_T) \right] \]
\[ \left( 1 - M_2 - M_2 M_5 \right) \left( \pi^2 + a^2 M_3 \right) + \pi^2 M_2 M_5 \]
\[ + a^2 \pi^2 P_r M_1 (1 + M_5) \left[ (1 - M_2 - M_2 M_5) (1 - S_T) + M_5 \right] \]
\[ V_2 = \left( \pi^2 + a^2 \right) \left[ \pi^2 M_2 (1 - S_T) - \left( \pi^2 + a^2 M_3 \right) \right] P_r^2 \]

\[ V_3 = \frac{1}{k} \left( \pi^2 + a^2 \right) \left( \pi^2 + a^2 + a^2 M_5 \delta \right) \]
\[ \left[ \pi^2 M_2 \left( \frac{M_5}{M_6} \pi + (1 - S_T) \right) - \left( \pi^2 + a^2 M_3 \right) (1 + \tau) \right] P_r \]
\[ + a^2 R_i M_6 \pi \left[ 1 + M_1 + \frac{M_5}{M_6} \right] \left[ \left( \pi^2 + a^2 M_3 \right) - \pi^2 M_2 (1 - S_T) \right] \]
\[ - \tau \left( \pi^2 + a^2 \right) \left( \pi^2 + a^2 M_3 \right) \]

\[ \sigma^2 = \frac{U_1 V_1 - U_3 V_3}{U_1 V_2 - U_2 V_1} \]

\( R_{OC} \) and \( R_{SC} \) are critical Rayleigh numbers for oscillatory and stationary convection system. If \( R_{OC} > R_{SC} \), the system stabilizes through stationary mode. If \( R_{OC} < R_{SC} \), the system stabilizes through oscillatory mode.

8. METHODS OF SOLUTIONS

The Soret-driven thermoconvective instability of a ferromagnetic fluid layer heated from below and salted from above densely packed porous medium with MFD viscosity is considered using Darcy model. Perturbation method is applied and Normal mode analysis is adopted. In the perturbation method, due to the application of magnetic field, the system is perturbed from the basic state (quiescent state). Accordingly the governing and other equations are modified (as seen from Eqs. 11 to 17). Linear stability analysis is considered. Then Normal mode analysis is taken (as seen from Eq. 20). Non-dimensional analysis is carried out and the exact solutions satisfying the appropriate boundary conditions are taken yielding to algebraic equations. For getting non-trivial solution for the system of linear homogeneous equations, the coefficients of the dynamic variables are equated to zero and on simplification, the expression for \( R_{SC} \) is obtained. Varying the values of the parameters in the allowable range and getting the corresponding \( R_{SC} \) values, the stability pattern is discussed.

9. RESULTS AND DISCUSSIONS

Before discussing the significant results of the convective system, we turn our attention to the possible range of values of various parameters arising in the study. The Soret parameter \( S_T \) is assumed to take values from -0.002 to 0.002, the salinity Rayleigh number \( R_S \) is varied from -500 to 500 and the magnetization parameter \( M_1 \) is allowed to take values from 5 to 25. The Prandtl number \( Pr \) is assumed to be 0.01. The ratio of mass transport to heat transport \( \tau \) is assumed to be 0.03, 0.05, 0.07, 0.09 and 0.11. The coefficient of MFD viscosity \( \delta \) is assumed from 0.01 to 0.09. The magnetization parameter \( M_1 \) is assumed to be 1000. For these fluids, \( M_2 \) will have a negligible value and hence is taken as zero and permeability \( \kappa \) is assumed to take the values from 0.001, 0.003, 0.005, 0.007 and 0.009.

Figures 1, 2 and 3 represent the variation of \( R_{SC} \) versus \( R_S \) for different values of \( M_1 \), \( S_T \) and \( \delta \) When the salinity Rayleigh number \( R_S \) increases from -500 to 500, the critical magnetic Rayleigh number \( R_{SC} \) decreases. Therefore the system gets a destabilizing behaviour. It is observed from
figure 1 that the magnetization parameter $M_3$ is found to destabilize the system. Also, the stabilizing trend of Soret parameter $S_T$ and MFD viscosity $\delta$ are also seen from figures 2 and 3.

**Figure 1.** Variation of $R_{SC}$ versus $R_S$ for various $M_3$, $\tau = 0.003$, $k = 0.001$, $\delta = 0.01$ and $S_T = -0.002$

**Figure 2.** Variation of $R_{SC}$ versus $R_S$ for various $S_T$, $\tau = 0.003$, $k = 0.001$, $\delta = 0.01$ and $M_3 = 5$

**Figure 3.** Variation of $R_{SC}$ versus $R_S$ for various $\delta$, $\tau = 0.003$, $k = 0.001$, $\delta = 0.01$ and $M_3 = 5$

Figures 4 and 5 indicate the variation of the critical magnetic Rayleigh number $R_{SC}$ with respect to the Soret parameter $S_T$ for various $R_S$ and $\delta$. It is found that the increase in Soret effect stabilizes the system, thereby delaying the onset of convection. Both figures exhibit a stabilizing trend. This is due to the fact that the modulation of the salinity gradient by temperature gradient promotes stabilization. Positive values of $S_T$ stabilize the system which is more pronounced. The destabilizing trend of $R_S$ is seen from Fig. 4 and stabilizing behavior of $\delta$ is seen from Fig. 5, as would mean adding salt from the top.

**Figure 4.** Variation of $R_{SC}$ versus $S_T$ for various $R_S$, $\tau = 0.003$, $k = 0.001$, $\delta = 0.01$ and $M_3 = 5$

**Figure 5.** Variation of $R_{SC}$ versus $S_T$ for various $\delta$, $\tau = 0.003$, $k = 0.001$, $R_S = -500$ and $M_3 = 5$

**Figure 6.** Variation of $R_{SC}$ versus $M_3$ for various $\delta$, $\tau = 0.003$, $k = 0.001$, $R_S = -500$ and $S_T = -0.002$

Figure 6 gives the variation of the critical Rayleigh number $R_{SC}$ versus the non-buoyancy magnetization parameter $M_3$ for different MFD viscosity parameter $\delta$. It is seen from the figure that as the value of $M_3$ increases from 5 to 25, the value of $R_{SC}$ decreases for small value $\delta = 0.01$, thus the convective system has a destabilizing effect for $\delta = 0.01$ whereas for higher values of $\delta$ (0.05, 0.07 and 0.09), $R_{SC}$ gets increasing values. In this situation, the system has a stabilizing behavior which is increasing slowly.
Figure 7. Variation of $R_{SC}$ versus $\tau$ for various $\delta$, $R_S = -500, k = 0.001$, $S_T = -0.002$ and $M_3 = 5$

Figure 7 shows the variation of critical magnetic Rayleigh number $R_{SC}$ versus the mass transport to heat transport $\tau$ for different $\delta$. It is seen from this figure that the system destabilizes as the mass transport to heat transport $\tau$ increases. This is shown by a fall in $R_{SC}$ values. It is observed from the figure that the magnetic field dependent viscosity $\delta$ is found to stabilize the system.

Figure 8. Variation of $R_{SC}$ versus $k$ for various $\delta$, $R_S = -500, \tau = 0.003$, $S_T = -0.002$ and $M_3 = 5$

Figure 8 represents the variation of critical magnetic Rayleigh number $R_{SC}$ versus permeability of the porous medium $k$ for different $\delta$. It is clear that the system destabilizes as the permeability of the porous medium $k$ increases. This is indicated by a decrease in $R_{SC}$ values. The reason is that as the pore size increases, it becomes easier for the flow to destabilize the system. It is observed from the figure that the magnetic field dependent viscosity $\delta$ is found to stabilize the system.

Figure 9. Variation of $R_{SC}$ versus $\delta$ for various $M_3$, $R_S = -500, \tau = 0.003$, $S_T = -0.002$ and $k = 0.001$

From Fig. 9, illustrates that as $M_3$ increases, the values of $R_{SC}$ decreases for small values of $\delta$ whereas for higher values of $\delta$, $R_{SC}$ decreases for lowest values of $M_3$, and then increases for higher values of $M_3$. The same trend is seen from figure 6. The destabilizing trend of $R_S, k$ and $\tau$ is also seen from Figs. 11, 12 and 13. But stabilizing behavior of $S_T$ is seen from figure 10.

Figure 10. Variation of $R_{SC}$ versus $\delta$ for various $S_T$, $R_S = -500, \tau = 0.003$, $M_3 = 5$ and $k = 0.001$

Figure 11. Variation of $R_{SC}$ versus $\delta$ for various $R_S, S_T = -0.002, \tau = 0.003$, $M_3 = 5$ and $k = 0.001$

Figure 12. Variation of $R_{SC}$ versus $\delta$ for various $k$, $R_S = -500, S_T = -0.002, \tau = 0.003$, and $M_3 = 5$

Figure 13. Variation of $R_{SC}$ versus $\delta$ for various $\tau$, $R_S = -500, S_T = -0.002, M_3 = 5$ and $k = 0.001$

Figures 9-13 investigate the variation of $R_{SC}$ versus $\delta$ for different values of $M_3, S_T, k$ and $\tau$. From Figs. 9 - 13, one can
find that as the coefficient of MFD viscosity is increased from 0.01 to 0.09, the critical magnetic Rayleigh number increases. This means that the system is stabilized through viscosity variation with respect to magnetic field. This leads to the conclusion that the MFD viscosity delays the onset of convection for ferrofluid in a densely distributed porous medium.

10. CONCLUSIONS

The linear stability of thermohaline convection in a ferrofluid layer heated from below and salted from above saturating a porous medium subjected to a transverse uniform magnetic field has been considered with effect of Soret and magnetic field dependent (MFD) viscosity using Darcy model. In this investigation, the effect of various parameters like permeability of the porous medium, Soret parameter, MFD viscosity parameter, non – buoyancy magnetization, buoyancy magnetization, Prandtl number, ratio of mass transport to heat transport, Rayleigh number and salinity Rayleigh number on the onset of convection have been calculated. The thermal critical magnetic Rayleigh numbers for the onset of instability are also determined for both stationary and oscillatory modes, for sufficient large values of buoyancy magnetization parameter M1 and results are depicted graphically. Furthermore, the principle of exchange of stability is applied to find out the mode of attaining instability.

In conclusion, we see that convection can encourage in a ferromagnetic fluid by means of spatial variation in magnetization, which is induced when the magnetization of the ferrofluid depends on temperature and salinity. For the stationary convection, MFD viscosity has always a stabilizing effect, whereas the permeability of the porous medium has always a destabilizing effect on the onset of convection. In the absence of MFD viscosity (δ = 0) magnetization has always a destabilizing effect. In the presence of MFD viscosity, nothing specific can be said, since there is competition between the destabilizing role of the magnetization M3 and the stabilizing role of the MFD viscosity δ.

The coefficient of MFD viscosity δ effect has a destabilizing behavior for various values of Rs, τM3 and k which are observed in Figures 1 - 3, 6 - 13. But, the convective system has a stabilizing effect which is analyzed in Figures 4 and 5 for Soret parameter Sc. It is evident from Figure 9 that lower values of Rs are needed for the onset of convection with increase in M3 for smaller values of δ whereas higher values of Rs are needed for the onset of convection with increase in M3 for smaller values of δ justifying the competition between the destabilizing effect of the magnetization M3 and the stabilizing effect of the MFD viscosity δ. Thus magnetization destabilizes the system and coefficient of MFD viscosity stabilizes the system for both modes. This leads to the conclusion that the MFD viscosity delays the onset of convection for ferrofluid saturating a densely distributed porous medium.

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