IIETA International Information and Engineering Technology Association
Advancing the World of Information and Engineering

TECNICA ITALIANA-Italian Journal of Engineering Science
Vol. 61+1, No. 2, December, 2018, pp. 151-161
Journal homepage: http://iieta.org/Journals/TI-IJES

Stability analysis of ferrothermohaline convection in a Darcy porous medium with Soret and MFD viscosity effects

R. Sekar*, D. Murugan

Department of Mathematics, Pondicherry Engineering College, Pillaichavady, Puducherry 605 014, India

Corresponding Author Email: rsekar@pec.edu

https://doi.org/10.18280/ti-ijes.620214

Received: 2 September 2018
Accepted: 13 October 2018

Keywords:
ferrofluid, Darcy model, Soret effect, MFD viscosity, thermohaline convection, critical magnetic thermal Rayleigh number

ABSTRACT

The present work deals with the theoretical investigation of the effect of Magnetic Field Dependent (MFD) viscosity of Soret-driven thermohaline convection of ferrofluid layer heated from below and salted from above saturating densely packed porous medium subject to a transverse uniform magnetic field. For a fluid layer contained between two free boundaries, an exact solution is obtained using a linear stability analysis. The normal mode method and Darcy model are used in the system. It has been observed that stationary mode is favorable for Darcy model. The critical magnetic thermal Rayleigh number N_C for the onset of instability is also determined for sufficient large values of non-buoyancy magnetization parameter M_1 and results are analyzed graphically. The MFD viscosity parameter is analyzed. The stabilizing behavior for various values of permeability of the porous medium, buoyancy magnetization parameter, ratio of mass transport to heat transport, thermal Rayleigh number and salinity Rayleigh number is studied. The salinity Rayleigh number has a destabilizing behavior for different values of non-buoyancy magnetization parameter, Soret parameter and MFD viscosity. It is also found that the MFD viscosity delays the onset of convection and MFD viscosity effect dominated the convective system.

1. INTRODUCTION

The magnetization of ferromagnetic fluids depends on the magnetic field, the temperature and the density of the fluid. Any variation of these quantities can induce a change in body force distribution in the fluid. These leads to convection in ferrofluids in the presence of magnetic field gradient, known as ferroconvection, which is similar to buoyancy driven convection. Ferroconvection studies were initiated by Cowley and Rosensweig [1]. When a salty fluid gets heated up two types of inter-diffusive phenomena occur. They are known as (a) the Soret effect, (b) Dufour effect. Thermo diffusion, also called the Soret effect, is characterized by the Soret coefficient. Thermo diffusion in a ferrofluid in the presence of a magnetic field was investigated by Voelker and Odenbach [2]. The theory of convective instability of ferrofluid began with Finlayson [3] and extensively continued by Stiles and Kagan [4] and Siddheshwar [5]. Nanjundappa and Shivakumara [6] have investigated a variety of velocity and temperature boundary conditions on the onset of ferroconvection convective instability in an initially quiescent ferrofluid layer in the presence of a uniform magnetic field.

Ferroconvective instability in an anisotropic densely packed magnetic particle, treated as porous medium was studied by Vaidyanathan et al. [7]. Hurle and Jakeman [8] analyzed the Soret driven thermosolutal convection. Shevtsova et al. [9] carried out a study on the onset of convection in Soret driven instability. Soret driven ferrothermohaline convection in the presence and absence of a porous medium was investigated by Vaidyanathan et al. [10-11]. A linear stability analysis on the onset of Soret driven motion in nanoparticles suspension was made by Kim [12]. Hemalatha et al. [13-14] discussed the effect of rotation on Soret driven ferro thermohaline convection of dust particles with and without porous medium. The convection in ferromagnetic fluids is gaining much importance due to astounding physical properities. One such property is viscosity of the ferromagnetic fluid. Fluids with ferromagnetic properties may be formed by colloidal suspension of solid magnetic particles such as magnetite in a parent liquid. Viscosity of the fluid in a magnetic field is predicted by dimensional analysis to be a function of the ratio of hydrodynamic stress to magnetic stress in Rosenswig et al. [15]. The effect of uniform distribution of heat source on the onset of stationary ferroconvection was investigated by Rudraiah et al. [16]. The onset of convection in a sparsely packed porous layer with throughflow was studied by Shivakumara et al. [17]. The onset of surface tension driven convection in a superposed layers of fluid and saturated porous medium was studied by Shivakumara et al. [18]. The boundary and thermal non-equilibrium effects on the onset of Darcy-Brinkman convection in a porous layer was investigated by Shivakumara et al. [19].

The effect of rotation on double diffusive convection in a magnetized ferrofluid with internal angular momentum was studied by Sunil et al. [20]. The effect of a homogeneous magnetic field on the viscosity of a fluid with solid particles possessing intrinsic magnetic moments was investigated by Shliomis [21]. Kaloni and Lou [22] have investigated theoretically the convective instability problem in a thin horizontal layer of magnetic fluid heated from below under alternating magnetic field by considering the quasi stationary model with internal rotation and vortex viscosity. Thermal convection in a ferromagnetic fluid in the presence of a magnetic field dependent viscosity was investigated by Paras

Ram et al. [23-24]. Paras Ram et al. [25-26] discussed the ferrofluid flow with magnetic field dependent viscosity due to rotating disk with and without porous medium. The effect of a magnetic field dependent viscosity on thermosolutal convection ferromagnetic fluid in the presence and absence of porous medium was studied by Sunil et al. [27-28]. It is interesting to study the nature of variable viscosity on fluids. Viscosity may depend on temperature and the magnetic field also. The effect of temperature-dependent viscosity on the threshold of ferroconvective instability in a porous medium using the Brinkman model and Galerkin technique was investigated by Ramanathan and Muchikel [29].

The effect of magnetic field dependent (MFD) viscosity on thermal convection in a ferromagnetic fluid with and without porous medium was investigated by Sunil et al. [30-31]. The effect of magnetic field dependent (MFD) viscosity on ferroconvection in a rotating with and without porous medium was studied by Vaidyanathan et al. [32-33]. The effect of a magnetic field dependent viscosity on the onset of convection in a ferromagnetic fluid layer heated from below and cooled from above in the presence of a vertical magnetic field with constant heat flux has been investigated by Nanjundappa et al. [34]. The effect of a magnetic field dependent (MFD) viscosity on ferroconvection in an anisotropic porous medium analyzed by Ramanathan and Suresh [35]. The comparison of theoretical and computational ferroconvection induced by magnetic field dependet viscosity in an anisotropic porous medium has been analysed by Suresh et al. [36].Malashetty, et al. [37-38] carried out an analytical study of linear and non-linear double diffusive convection with Soret effect in couple-stress liquids with and without Dufour effect.

A nonlinear stability analysis for thermoconvective and duble-diffusive magnetized ferrofluid with MFD viscosity has been investigated by Sunil et al. [39-40]. The effect of a magnetic field dependent viscosity on ferroconvection in an anisotropic porous medium in the presence of a horizontal thermal gradient was studied by Hemalatha and Sivapraba [41]. Sunil et al. [42] have studied theoretically, the effect of magnetic field dependent viscosity on the thermal convection in a ferromagnetic fluid layer with or without dust particles. The effect of a magnetic field dependent viscosity on a Soret driven ferrothermohaline convection in a rotating porous medium was studied by Hemalatha [43]. Sekar and Raju [44-45] have studied the effect of magnetic field dependent viscosity on Soret-driven thermoconvective instability of ferromagnetic fluid in the presence of rotating and saturating an anisotropic porous medium of sparse particle suspension. More recently, the presence and absence of coriolis force on Soret driven ferrothermohaline convection staturating a densely packed anisotropic porous medium was studied by Sekar et al. [46-47].

In this paper, the effect of magnetic field dependent viscosity on Soret-driven ferrothermohaline convection in the presence of densely packed porous medium is studied. The overriding Soret effect on the salinity equation is also studied. Linear stability analysis is used. The conditions for the onset of stationary and oscillatory instabilities are obtained.

2. MATHEMATICAL FORMULATION

An infinitely spread layer of Boussinesq ferromagnetic fluid of thickness ‘d’ in the presence of densely packed porous medium heated from below and salted from above is considered. The temperature and salinity at the bottom and top surfaces z = ±d/2 are T0 ± ΔT/2 and S0 ± ΔS/2, respectively. Both the boundaries are taken to be free and perfect conductors of heat and solute. The Soret effect on the temperature gradient is considered.

The fluid is assumed to be incompressible fluid having a variable viscosity, given by $\eta = \eta_1(1+\delta.B)$ where η_1 is taken as viscosity of the fluid when the applied magnetic field is absent and $B=(B_1,B_2,B_3)$ is the magnetic induction. The variation in the coefficient of the magnetic field dependent viscosity δ has been taken to be isotropic, that is, $\delta=\delta_1=\delta_2=\delta_3$ Hence the component wise n can be written as $\eta_x = \eta_1(1+\delta.B_1)$,$\eta_y = \eta_1(1+\delta.B_2)$ and$\eta_z = \eta_1(1+\delta.B_3)$.

The mathematical equations governing the above investigation are as follows:

The continuity equation for an incompressible fluid is

$$\nabla.\,\mathbf{q}=0 \tag{1}$$

The momentum equation is

$$\rho_o \frac{D\mathbf{q}}{Dt} = -\nabla p + \rho\mathbf{g} + \nabla.(\mathbf{HB}) - \frac{\eta}{k}\mathbf{q} \tag{2}$$

The temperature equation for an incompressible ferrofluid is

$$\left[\rho_o C_{V,H} - \mu_o \mathbf{H}.\left(\frac{\partial \mathbf{M}}{\partial T}\right)_{V,H}\right]\frac{dT}{dt} + \mu_o T\left(\frac{\partial \mathbf{M}}{\partial T}\right)_{V,H}.\frac{d\mathbf{H}}{dt} = K_1\nabla^2 T + \phi \tag{3}$$

The mass flux equation is

$$\frac{DS}{Dt} = K_s\nabla^2 S + S_T\nabla^2 T \tag{4}$$

where ρ_o, $\mathbf{q}=(u,v,w)$, $g=(0,0,-g)$, t, p, μ_0, η, k $\mathbf{H}$,$\mathbf{B}$, $C_{V,H}$, T, $\mathbf{M}$, K_1, S, K_S, S_T and ϕ are the density, filter velocity, acceleration due to gravity, time, pressure, magnetic permeability, viscosity (variable), permeability of the porous medium, magnetic field, magnetic induction, heat capacity at constant volume and magnetic field, temperature, magnetization, thermal conductivity, salinity, mass diffusivity, Soret coefficient and viscous dissipation factor containing second-order terms in velocity, respectively.

Maxwell’s equations are

$$\nabla.\mathbf{B}=0, \nabla\times\mathbf{H}=0 \tag{5}$$

Further B, M and H are related by

$$\mathbf{B}=\mu_0(\mathbf{M}+\mathbf{H}) \tag{6}$$

Using Maxwell’s equation for non-conducting fluids one can assume that the magnetization is aligned with the magnetic field and depends on the magnitude of the magnetic field, temperature and salinity, so that

$$\mathbf{M} = \frac{\mathbf{H}}{H} M(H,T,S) \tag{7}$$

The magnetic equation of state is linearised about the magnetic field H_0, the average temperature T_0 and the average salinity S_0 to become

$$M = M_0 + \chi(H - H_0) - K(T - T_0) + K_2(S - S_0) \tag{8}$$

where $x = (\partial M/\partial H)_{H_0,T_0}$ is the susceptibility, $k = (\partial M/\partial T)_{H_0,T_0}$ is the pyromagnetic coefficient and $k_2 = (\partial M/\partial S)_{H_0,T_0}$ is the salinity magnetic coefficient.

The density equation of state for a Boussinesq two-component fluid is

$$\rho = \rho_0 \left[1 - \alpha_t (T - T_0) + \alpha_s (S - S_0)\right] \tag{9}$$

where α is the thermal expansion coefficient and α_s is the solute analog of α_t.

The basic state is assumed to be the quiescent state and taking the components of the magnetization and magnetic field in the basic state to be $[0,0,M_0(z)]$ and $[0,0,H_0(z)]$ the basic state quantities obtained are:

$$\begin{aligned}
&\mathbf{q} = \mathbf{q}_b = 0, \quad \mathrm{p} = \mathrm{p}_b(\mathrm{z}), \quad \frac{\partial T}{\partial z} = -\beta_t \Rightarrow T_b = T_0 - \beta_t z, \\
&\frac{\partial S}{\partial z} = \beta_S \Rightarrow S_b = S_0 + \beta_s z, \\
&\mathbf{H}_b(Z) = \left[H_0 + \frac{\mathrm{K}(T_b - T_0)}{1+\chi} - \frac{K_2(S_b - S_0)}{1+\chi} \right] \hat{\mathbf{k}}, \\
&\mathbf{M}_b(Z) = \left[M_0 - \frac{\mathrm{K}(T_b - T_0)}{1+\chi} + \frac{K_2(S_b - S_0)}{1+\chi} \right] \hat{\mathbf{k}}.
\end{aligned} \tag{10}$$

where β_t and β_s are non-negative constants and $\hat{k} = (0,0,1)$ is the unit vector along vertical direction.

3. LINEAR STABILITY ANALYSIS

The basic state is disturbed by a small thermal perturbation. Consider a perturbed state such that $\mathbf{q} = \mathbf{q}'$, $\mathrm{p} = \mathrm{p}_b(\mathrm{z}) + \mathrm{p}'$, $\eta = \eta_b(\mathrm{z}) + \eta'$, $T = T_b(\mathrm{z}) + T'$, $\mathbf{H} = \mathbf{H}_b(\mathrm{z}) + \mathbf{H}'$, $\mathbf{M} = \mathbf{M}_b(\mathrm{z}) + \mathbf{M}'$. *where* $\mathbf{q}', \mathrm{p}', \eta', T', \mathbf{H}'$, and $\mathbf{M}'$ are perturbed variables and are assumed to be small. The perturbed state temperature and solute are $\mathrm{T} = T_0 - \beta_t z + T'$ and $\mathrm{S} = S_0 - \beta_s z + S'$ Let the components of the perturbed magnetization and the magnetic field be $(M_1, M_2, M_0(z), M_3)$ and $(H_1, H_2, H_0(z), H_3)$ respectively.

$$H_i' + M_i' = \left(1 + \frac{M_0}{H_0}\right) H_i' \qquad (i = 1,2) \tag{11}$$

$$H_3' + M_3' = (1+\chi) H_3' - KT' + K_2 S' + S_T K T' \tag{12}$$

Let (B_1, B_2, B_3) denote the components of B. Using Eq. (6), one gets the result $B_1 = \mu_0(M_i' + H_i')$ and Eqs. (11) and (12) become

$$B_i = \mu_0 \left(1 + \frac{M_0}{H_0}\right) H_i' \qquad (i = 1,2) \tag{13}$$

$$B_3 = \mu_0 \left[(1+\chi) H_3' - KT' + K_2 S' + S_T K T' + M_0 + H_0\right] \tag{14}$$

When Eq. (5) is used in Eq. (1) and resulting equation is linearized with Bi (i=1, 2, 3) given by Eqs. (13) and (14), we obtain the following components

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu_0 (M_0 + H_0) \frac{\partial H_1'}{\partial z} - \frac{\eta_1}{k} u \tag{15}$$

$$\rho_0 \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu_0 (M_0 + H_0) \frac{\partial H_2'}{\partial z} - \frac{\eta_1}{k} v \tag{16}$$

$$\begin{aligned}
\rho_0 \frac{\partial w}{\partial t} &= -\frac{\partial p}{\partial z} + \mu_0 (M_0 + H_0) \frac{\partial H_3'}{\partial z} - \mu_0 H_3' K \beta_t + \mu_0 H_3' K_2 \beta_s \\
&+ \frac{\mu_0 K^2 \beta_t T'}{1+\chi} (1 - S_T) - \frac{\mu_0 K K_2 \beta_s T'}{1+\chi} (1 - S_T) - \frac{\mu_0 K K_2 \beta_t S'}{1+\chi} \\
&+ \frac{\mu_0 K_2^2 \beta_s S'}{1+\chi} + \rho_0 g \alpha_t T' - \rho_0 g \alpha_s S' - \frac{\eta_1}{k} w - \frac{\eta_1}{k} \delta \mu_0 (M_0 + H_0) w
\end{aligned} \tag{17}$$

Differentiating Eqs. (15)-(17) with respect to x, y and z respectively and adding, the following equation is obtained upon using Eq. (1)

$$\begin{aligned}
\nabla^2 p &= \mu_0 (M_0 + H_0) \frac{\partial}{\partial z} \left(\nabla . \mathbf{H}'\right) + \mu_0 K_2 \beta_s \frac{\partial H_3'}{\partial z} \\
&- \mu_0 K \beta_t \frac{\partial H_3'}{\partial z} + \frac{\mu_0 K^2 \beta_t}{1+\chi} (1 - S_T) \frac{\partial T'}{\partial z} + \frac{\mu_0 K_2^2 \beta_s}{1+\chi} \frac{\partial S'}{\partial z} \\
&- \frac{\mu_0 K K_2 \beta_s}{1+\chi} (1 - S_T) \frac{\partial T'}{\partial z} - \frac{\mu_0 K K_2 \beta_t}{1+\chi} \frac{\partial S'}{\partial z} + \rho_0 g \alpha_t \frac{\partial T'}{\partial z} \\
&- \rho_0 g \alpha_s \frac{\partial S'}{\partial z} - \frac{\eta_1}{k} \delta \mu_0 (M_0 + H_0) \frac{\partial w}{\partial z}
\end{aligned} \tag{18}$$

where H' has the components (H_1', H_2', H_3')

From Eq. (5), $H' = \nabla \emptyset$ where $\emptyset$ is a scalar potential. Elimination of p from Eq. (15) - (17) and using Eq. (18) gives,

$$\begin{aligned}
\rho_0 \frac{\partial}{\partial t} \left(\nabla^2 w\right) &= \mu_0 K_2 \beta_s \frac{\partial}{\partial z} \left(\nabla_1^2 \phi\right) - \mu_0 K \beta_t \frac{\partial}{\partial z} \left(\nabla_1^2 \phi\right) \\
&+ \frac{\mu_0 K^2 \beta_t}{1+\chi} (1 - S_T) \nabla_1^2 T' - \frac{\mu_0 K K_2 \beta_s}{1+\chi} (1 - S_T) \nabla_1^2 T' \\
&+ \frac{\mu_0 K_2^2 \beta_s}{1+\chi} \nabla_1^2 S' - \frac{\mu_0 K K_2 \beta_t}{1+\chi} \nabla_1^2 S' + \rho_0 g \alpha_t \nabla_1^2 T' \\
&- \rho_0 g \alpha_s \nabla_1^2 S' - \frac{\eta_1}{k} \nabla^2 w - \frac{\eta_1}{k} \delta \mu_0 (M_0 + H_0) \nabla_1^2 w
\end{aligned} \tag{19}$$

where $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$

4. NORMAL MODE ANALYSIS

We now proceed to a normal mode analysis of the above stability problem. Let us take.

$$f(x, y, z, t) = f(z, t) e^{i(k_x x + k_y y)},$$
$$\phi = \phi(z, t) e^{i(k_x x + k_y y)}, \; w = w(z, t) e^{i(k_x x + k_y y)},$$
$$T' = \theta(z, t) e^{i(k_x x + k_y y)}, \; S' = S(z, t) e^{i(k_x x + k_y y)} \tag{20}$$

with the wave number

$$k_0^2 = k_x^2 + k_y^2$$

Using Eq. (20) in Eq. (19), one gets

$$\rho_0 \frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w = \frac{\mu_0 K_2 \beta_s}{1+\chi}\left[(1+\chi)\frac{\partial \phi}{\partial z} + K_2 S\right] k_0^2$$
$$+ \frac{\mu_0 K \beta_t}{1+\chi}\left[(1+\chi)\frac{\partial \phi}{\partial z} - K\theta(1-S_T)\right] k_0^2 + \rho_0 g \alpha_s k_0^2 S$$
$$- \rho_0 g \alpha_t k_0^2 \theta - \frac{\mu_0 K K_2}{1+\chi}\left[\beta_s (1-S_T)\theta - \beta_t S\right] k_0^2$$
$$- \frac{\eta_1}{k}\left(\frac{\partial^2}{\partial z^2} - k_0^2\right) w + \frac{\eta_1}{k} k_0^2 \delta \mu_0 (M_0 + H_0) w \tag{21}$$

The temperature equation is

$$\rho_0 C_{V,H} \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) = K_1 \left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\theta$$
$$+ \left[\rho_0 C_{V,H} \beta_t - \frac{\mu_0 K^2 T_0^2 \beta_t}{1+\chi} + \frac{\mu_0 K K_2 T_0 \beta_s}{1+\chi}\right] w \tag{22}$$

where $\rho_0 C = \rho_0 C_{V,H} + \mu_0 K H_0$

The salinity equation is

$$\frac{\partial S}{\partial t} + \beta_s w = K_s \left(\frac{\partial^2}{\partial z^2} - k_0^2\right) S + S_T \left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\theta \tag{23}$$

The magnetic potential equation is

$$(1+\chi)\frac{\partial^2 \phi}{\partial z^2} - \left(1 + \frac{M_0}{H_0}\right) k_0^2 \phi - K\frac{\partial \theta}{\partial z} + K_2 \frac{\partial S}{\partial z} + S_T K \frac{\partial \theta}{\partial z} = 0 \tag{24}$$

The non-dimensional numbers can be written using

$$t^* = \frac{\nu t}{d^2}, \; w^* = \frac{wd}{\nu}, \; T^* = \left(\frac{K_1 a R^{1/2}}{\rho_0 C_{V,H} \beta_t \nu d}\right)\theta, k^* = \frac{k}{d^2},$$
$$\phi^* = \left(\frac{(1+\chi) K_1 a R^{1/2}}{K \rho_0 C_{V,H} \beta_t \nu d^2}\right)\phi, z^* = \frac{z}{d}, D = \frac{\partial}{\partial z^*}, a = k_0 d,$$
$$S^* = \left(\frac{K_s a R_s^{1/2}}{\rho_0 C_{V,H} \beta_s \nu d}\right) S, \; \nu = \frac{\eta}{\rho_0}, \delta^* = \mu_0 \delta H_0 (1+\chi)$$

Then Eqs. (21) – (24) become

$$\left.\begin{aligned} &\left(\frac{\partial}{\partial t^*} + \frac{1}{k^*}\right)(D^2 - a^2) w^* = a R^{1/2} M_1 M_5 D\phi^* \\ &+ a R^{1/2}\left[M_1 D\phi^* - (1 + M_1(1-S_T)) T^*\right] + \frac{a^2}{k^*} M_3 \delta^* w^* \\ &- a R^{1/2} M_1 M_5 (1-S_T) T^* + a R_S^{1/2}\left[1 + M_4 + \frac{M_4}{M_5}\right] S^*, \end{aligned}\right\} \tag{25}$$

$$P_r \left[\frac{\partial T^*}{\partial t^*} - M_2 \frac{\partial}{\partial t^*}(D\phi^*)\right] = (D^2 - a^2) T^*$$
$$+ a R^{1/2} (1 - M_2 - M_2 M_5) w^*, \tag{26}$$

$$P_r \frac{\partial S^*}{\partial t^*} = \tau (D^2 - a^2) S^* - a R_S^{1/2} M_6 w^*$$
$$+ S_T \left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2} (D^2 - a^2) T^*, \tag{27}$$

$$D^2 \phi^* - M_3 a^2 \phi^* - (1-S_T) D T^* + \frac{M_5}{M_6}\left(\frac{R}{R_S}\right)^{1/2} D S^* = 0, \tag{28}$$

where the non-dimensional parameters used are

$$M_1 = \frac{\mu_0 K^2 \beta_t}{(1+\chi)\rho_0 g \alpha_t}, M_2 = \frac{\mu_0 K^2 T_0}{(1+\chi)\rho_0 C_{v,H}}, M_5 = \frac{K_2 \beta_s}{K \beta_t},$$
$$M_3 = \frac{1 + M_0 / H_0}{(1+\chi)}, M_4 = \frac{\mu_0 K^2 \beta_s}{(1+\chi)\rho_0 g \alpha_s}, \; M_6 = \frac{K_S}{K_1},$$
$$P_r = \frac{\eta C_{v,H}}{K_1}, \tau = \rho_0 C_{v,H}\left(\frac{K_S}{K_1}\right), R_S = \frac{\rho_0 C_{v,H} \beta_s \alpha_s g d^4}{\nu K_S},$$
$$R = \frac{\rho_0 C_{v,H} \beta_t \alpha_t g d^4}{\nu K_1}, \tag{29}$$

where R_S is the salinity Rayleigh number, R is the thermal Rayleigh number, P_r is the Prandtl number and other parameters describe non-dimensional parameters.

5. ANALYSIS OF SOLUTION AT FREE BOUNDARIES

The boundary conditions on velocity, temperature and salinity are

$$w^* = D^2 w^* = T^* = D\phi^* = S^* = 0 \quad at \quad z^* = \pm 1/2. \tag{30}$$

The exact solutions satisfying above equation (30) are

$$w^* = Ae^{\sigma t^*}\cos\pi z^*, T^* = Be^{\sigma t^*}\cos\pi z^*,$$
$$S^* = Ce^{\sigma t^*}\cos\pi z^*, D\phi^* = Ee^{\sigma t^*}\cos\pi z^*,$$
$$\phi^* = \frac{E}{\pi}e^{\sigma t^*}\sin\pi z^* \tag{31}$$

where A, B, C and E are constants. These functions substituted in the set of Eqs. (25)-(28) give the following four linear homogeneous algebraic equations in the constant A, B, C and E:

$$\left.\begin{aligned}&\left[\left(\sigma+\frac{1}{k}\right)(\pi^2+a^2)+\frac{1}{k}a^2M_3\delta\right]A\\ &-aR^{1/2}\left[1+M_1(1-S_T)+M_1M_5(1-S_T)\right]B\\ &+aR_S^{1/2}(1+M_4+M_4M_5^{-1})C+aR^{1/2}\ M_1(1+M_5)E=0,\end{aligned}\right\} \tag{32}$$

$$aR^{1/2}(1-M_2-M_2M_5)A-(\pi^2+a^2+P_r\sigma)B+P_r\sigma M_2E=0, \tag{33}$$

$$aR_S^{1/2}M_6A+S_T\left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2}(\pi^2+a^2)B + \left[\tau(\pi^2+a^2)+\sigma P_r\right]C=0, \tag{34}$$

$$-R_S^{1/2}\pi^2(1-S_T)B+R^{1/2}\pi^2M_5M_6^{-1}C+R_S^{1/2}(\pi^2+a^2M_3)E=0, \tag{35}$$

For the existence of non-trivial Eigen functions, the determinant of the co-efficients of A, B, C and E in Eqs. (32) – (35) must vanish. Following the techniques and analysis of Vaidyanathan et al. [32], Eqs. (32) – (35) lead to

$$U\sigma^3+V\sigma^2+W\sigma+X=0 \tag{36}$$

$$U=\left(\pi^2+a^2\right)\left[\left(1-S_T\right)\pi^2M_2-\left(\pi^2+a^2M_3\right)\right]P_r^2$$

$$\begin{aligned}V=&\frac{1}{k}\left(\pi^2+a^2+a^2M_3\delta\right)\left[\left(1-S_T\right)\pi^2M_3-\left(\pi^2+a^2M_3\right)\right]P_r^{\ 2}\\ &+\left(\pi^2+a^2\right)^2\left[\pi^2M_2\left\{S_T\left(\frac{M_5}{M_6}\right)^2+\tau\left(1-S_T\right)\right\}-\left(\pi^2+a^2M_3\right)\left(1+\tau\right)\right]P_r\end{aligned}$$

$$\begin{aligned}W=&\frac{1}{k}\left(\pi^2+a^2+a^2M_3\delta\right)\left(\pi^2+a^2\right)\\ &\left[\pi^2M_2\left\{S_T\left(\frac{M_5}{M_6}\right)^2+\tau\left(1-S_T\right)\right\}-\left(\pi^2+a^2M_3\right)\left(1+\tau\right)\right]P_r\\ &+a^2RP_r\left[1+M_1\left(1+M_5\right)\left(1-S_T\right)\right]\\ &\left[\left(1-M_2-M_2M_5\right)\left(\pi^2+a^2M_3\right)+\pi^2M_2M_5\right]\\ &+a^2R_sP_r\left(1+M_4+M_4M_5^{-1}\right)M_6\\ &\left[\left(\pi^2+a^2M_3\right)-\pi^2M_2\left(1-S_T\right)\right]\\ &+a^2RP_r\pi^2M_1\left(1+M_5\right)\left[\left(1-M_2-M_2M_5\right)\left(1-S_T\right)+M_5\right]\\ &-\tau\left(\pi^2+a^2\right)^3\left(\pi^2+a^2M_3\right)\end{aligned}$$

$$\begin{aligned}X=&-\frac{1}{k}\tau\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)^2\left(\pi^2+a^2+a^2M_3\delta\right)\\ &+a^2R\tau\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\\ &\left(1-M_2-M_2M_5\right)\left[1+\left(1-S_T\right)M_1\left(1+M_5\right)\right]\\ &+a^2R\left(\pi^2+a^2\right)M_1\left(1+M_5\right)\pi^2\\ &\left[\left(1-M_2-M_2M_5\right)\left\{S_T\left(\frac{M_5}{M_6}\right)^2+\tau\left(1-S_T\right)\right\}+M_5\right]\\ &+a^2R_s\left(\pi^2+a^2M_3\right)\left(\pi^2+a^2\right)\left(1+M_4+M_4M_5^{-1}\right)\\ &\left[\left(1-M_2-M_2M_5\right)S_T\left(\frac{M_5}{M_6}\right)+M_6\right]\end{aligned}$$

6. STATIONARY CONVECTION

For obtaining stationary instability, the time-independent term X = 0. Eq. (36) helps one to obtain Eigen value R_{SC} for which a solution exists;

$$R_{SC}=\frac{N_r}{D_r},$$

where

$$N_r=\left(\pi^2+a^2\right)\left(\pi^2+a^2+a^2M_3\delta\right)-a^2kR_s\tau^{-1}$$
$$\left(1+M_4+M_4M_5^{-1}\right)\left[\left(1-M_2-M_2M_5\right)S_T\left(\frac{M_5}{M_6}\right)+M_6\right]$$

And

$$\begin{aligned}D_r=&a^2k\left(1-M_2-M_2M_5\right)\left[1+\left(1-S_T\right)M_1\left(1+M_5\right)\right]\\ &-\pi^2\left[\frac{a^2kM_1\left(1+M_5\right)}{\pi^2+a^2M_3}\right]\\ &\left[\left(1-M_2-M_2M_5\right)\left\{S_T\left(\frac{M_5}{M_6}\right)^2\tau^{-1}+\left(1-S_T\right)\right\}+M_5\tau^{-1}\right]\end{aligned}$$

For M_1 very large, one gets the results for the magnetic mechanism, and the critical thermo magnetic Rayleigh number for stationary mode is obtained using

$$N_{SC}=R_{SC}M_1=\frac{N_r}{D_r},$$

where

$$N_r=\left(\pi^2+a^2\right)\left(\pi^2+a^2+a^2M_3\delta\right)-a^2kR_s\tau^{-1}$$
$$\left(1+M_4+M_4M_5^{-1}\right)\left[\left(1-M_2-M_2M_5\right)S_T\left(\frac{M_5}{M_6}\right)+M_6\right]$$

And

$$D_r = a^2k\left[(1-M_2-M_2M_5)(1-S_T)(1+M_5)\right] - \pi^2\left[\frac{a^2k(1+M_5)}{\pi^2+a^2M_3}\right]$$
$$\left[(1-M_2-M_2M_5)\left\{S_T\left(\frac{M_5}{M_6}\right)^2\tau^{-1}+(1-S_T)\right\}+M_5\tau^{-1}\right]$$

7. OSCILLATORY CONVECTION

The conditions for the onset of oscillatory stabilities are obtained as follows: Taking $\sigma = i\sigma$ and $\sigma^2 > 0$, in Eq. (36) and following the analysis and techniques of Vaidyanathan et al. [32], the critical Rayleigh number for oscillatory mode has been calculated using

$$R_{OC} = \frac{U_1U_3 + (U_1U_2 + V_1V_3)\sigma^2 + V_1V_2\sigma^4}{U_1^2 + V_1^2\sigma^2}$$

where

$$U_1 = a^2(\pi^2+a^2)\tau(1-M_2-M_2M_5)(\pi^2+a^2M_3)\{1+(1-S_T)M_1(1+M_5)\}$$
$$+a^2(\pi^2+a^2)M_1(1+M_5)\pi^2\left\{(1-M_2-M_2M_5)\left[S_T\left(\frac{M_5}{M_6}\right)^2+\tau(1-S_T)\right]+M_5\right\}$$

$$U_2 = \frac{1}{k}(\pi^2+a^2+a^2M_3\delta)\left[(\pi^2+a^2M_3)-\pi^2M_2(1-S_T)\right]P_r^2$$
$$+(\pi^2+a^2)^2\left[\begin{array}{c}\pi^2M_2\left\{S_T\left(\frac{M_5}{M_6}\right)^2+\tau(1-S_T)\right\}\\ -(\pi^2+a^2M_3)(1+\tau)\end{array}\right]P_r$$

$$U_3 = (\pi^2+a^2M_3)(\pi^2+a^2)\left[\begin{array}{c}a^2R_s\left(1+M_4+\frac{M_4}{M_5}\right)\\ \left\{(1-M_2-M_2M_5)S_T\left(\frac{M_5}{M_6}\right)+M_6\right\}\\ -\frac{1}{k}\tau(\pi^2+a^2+a^2M_3\delta)(\pi^2+a^2)\end{array}\right]$$

$$V_1 = a^2P_r\left[1+M_1(1+M_5)(1-S_T)\right]\left[(1-M_2-M_2M_5)(\pi^2+a^2M_3)+\pi^2M_2M_5\right]$$
$$+a^2\pi^2P_rM_1(1+M_5)\left[(1-M_2-M_2M_5)(1-S_T)+M_5\right]$$

$$V_2 = (\pi^2+a^2)\left[\pi^2M_2(1-S_T)-(\pi^2+a^2M_3)\right]P_r^2$$

$$V_3 = \frac{1}{k}(\pi^2+a^2)(\pi^2+a^2+a^2M_3\delta)\left[\pi^2M_2\left\{S_T\left(\frac{M_5}{M_6}\right)^2+\tau(1-S_T)\right\}-(\pi^2+a^2M_3)(1+\tau)\right]P_r$$
$$+a^2R_sM_6P_r\left(1+M_4+\frac{M_4}{M_5}\right)\left[(\pi^2+a^2M_3)-\pi^2M_2(1-S_T)\right]$$
$$-\tau(\pi^2+a^2)^3(\pi^2+a^2M_3)$$

$$\sigma^2 = \frac{U_3V_1-U_1V_3}{U_1V_2-U_2V_1}$$

R_{OC} and R_{SC} are critical Rayleigh numbers for oscillatory and stationary convection system. If $R_{OC}>R_{SC}$, the system stabilizes through stationary mode. If $R_{OC} < R_{SC}$, the system stabilizes through oscillatory mode.

8. METHODS OF SOLUTIONS

The Soret-driven thermoconvective instability of ferromagnetic fluid layer heated from below and salted from above densely packed porous medium with MFD viscosity is considered using Darcy model. Perturbation method is applied and Normal mode analysis is adopted. In the perturbation method, due to the application of magnetic field, the system is perturbed from the basic state (quiescent state). Accordingly the governing and other equations are modified (as seen from Eqs. 11 to 17). Linear stability analysis is considered. Then Normal mode analysis is taken (as seen from Eq. 20), Non-dimensional analysis is carried out and the exact solutions satisfying the appropriate boundary conditions are taken yielding to algebraic equations. For getting non-trivial solution for the system of linear homogeneous equations, the coefficients of the dynamic variables are equated to zero and on simplification, the expression for $R_{SC}$is obtained. Varying the values of the parameters in the allowable range and getting the corresponding R_{SC}values, the stability pattern is discussed.

9. RESULTS AND DISCUSSIONS

Before discussing the significant results of the convective system, we turn our attention to the possible range of values of various parameters arising in the study. The Soret parameter S_T is assumed to take values from -0.002 to 0.002, the salinity Rayleigh number R_S is varied from -500 to 500 and the magnetization parameter M_3 is allowed to take values from 5 to 25. The Prandtl number Pr is assumed to be 0.01. The ratio of mass transport to heat transport τ is assumed to be 0.03, 0.05, 0.07, 0.09 and 0.11. The coefficient of MFD viscosity δ is assumed from 0.01 to 0.09. The magnetization parameter M_1 is assumed to be 1000. For these fluids, M_2 will have a negligible value and hence is taken as zero and permeability of porous medium k is assumed to take the values from 0.001, 0.003, 0.005, 0.007 and 0.009.

Figures 1, 2 and 3 represent the variation of R_{SC} versus R_S for different values of M_1 , S_T and δ When the salinity Rayleigh number $R_S$increases from -500 to 500, the critical magnetic Rayleigh number R_{SC} decreases. Therefore the system gets a destabilizing behaviour. It is observed from figure 1 that the magnetization parameter M_3 is found to destabilize the system. Also, the stabilizing trend of Soret parameter S_T and MFD viscosity δ are also seen from figures 2 and 3.

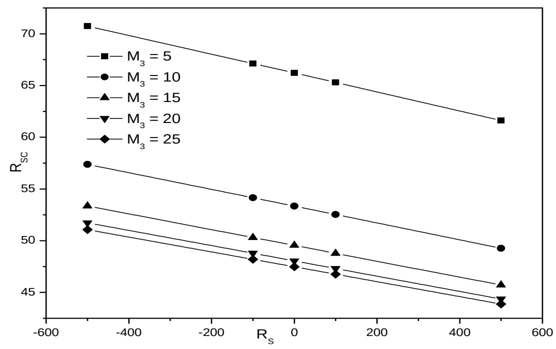

Figure 1. Variation of R_{SC}versus R_S for various M_3, $\tau = 0.003, \mathrm{k} = 0.001, \delta = 0.01$ and $S_T = -0.002$

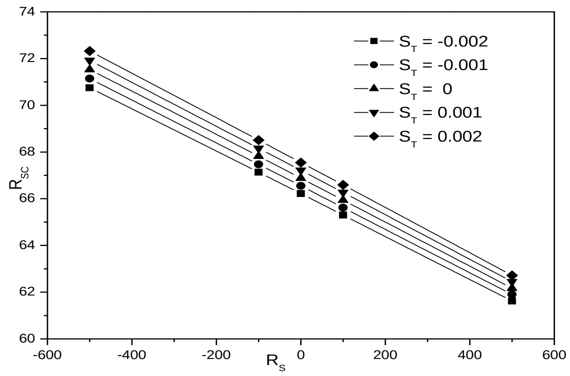

Figure 2. Variation of R_{SC}versus R_S for various S_T, $\tau = 0.003, \mathrm{k} = 0.001, \delta = 0.01$ and $M_3 = 5$

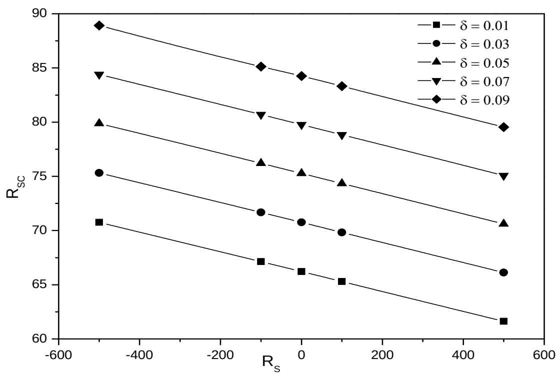

Figure 3. Variation of R_{SC}versus R_S for various δ, $\tau = 0.003, \mathrm{k} = 0.001, \delta = 0.01$ and $M_3 = 5$

Figures 4 and 5 indicate the variation of the critical magnetic Rayleigh number R_{SC} with respect to the Soret parameter S_T for various R_S *and* δ . It is found that the increase in Soret effect stabilizes the system, thereby delaying the onset of convection. Both figures exhibit a stabilizing trend. This is due to the fact that the modulation of the salinity gradient by temperature gradient promotes stabilization. Positive values of S_T stabilize the system which is more pronounced. The destabilizing trend of R_S is seen from Fig. 4 and stabilizing behavior of δ is seen from Fig.5, as would mean adding salt from the top.

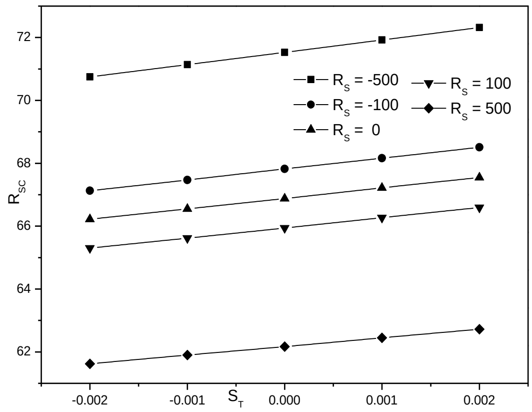

Figure 4. Variation of R_{SC}versus S_T for various R_S, $\tau = 0.003, \mathrm{k} = 0.001, \delta = 0.01$ and $M_3 = 5$

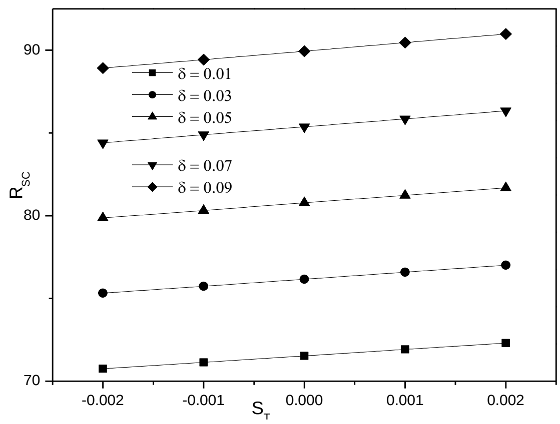

Figure 5. Variation of R_{SC}versus S_T for various δ, $\tau = 0.003, \mathrm{k} = 0.001, R_S = -500$ and $M_3 = 5$

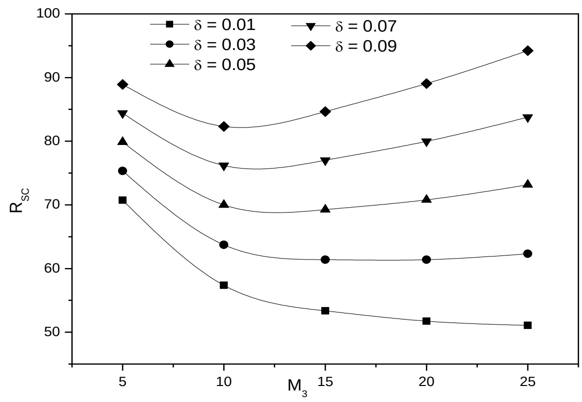

Figure 6. Variation of R_{SC}versus M_3 for various δ, $\tau = 0.003, \mathrm{k} = 0.001, R_S = -500$ and $S_T = -0.002$

Figure 6 gives the variation of the critical Rayleigh number R_{SC}versus the non-buoyancy magnetization parameter M_3 for different MFD viscosity parameter δ It is seen from the figure that as the value of M_3 increases from 5 to 25, the value of R_{SC}decreases for small value $\delta = 0.01$, thus the convective system has a destabilizing effect for $\delta = 0.01$ whereas for higher values of δ (0.05, 0.07 and 0.09), R_{SC} gets increasing values. In this situation, the system has a stabilizing behavior which is increasing slowly.

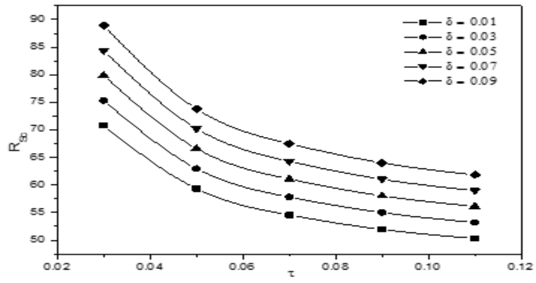

Figure 7. Variation of R_{SC}versus τ for various δ, $R_S = -500, \mathrm{k} = 0.001,\ S_T = -0.002$ and $M_3 = 5$

Figure 7 shows the variation of critical magnetic Rayleigh number R_{SC}versus the mass transport to heat transport τfor different δ . It is seen from this figure that the system destabilizes as the mass transport to heat transport τ increases. This is shown by a fall in R_{SC}values. It is observed from the figure that the magnetic field dependent viscosity δ is found to stabilize the system.

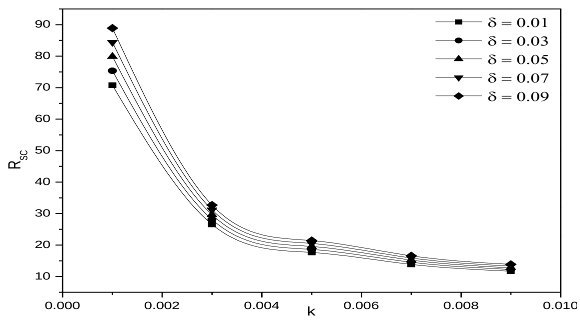

Figure 8. Variation of R_{SC}versus k for various δ, $R_S = -500, \tau = 0.003,\ S_T = -0.002$ and $M_3 = 5$

Figure 8 represents the variation of critical magnetic Rayleigh number R_{SC} versus permeability of the porous medium k for different δ It is clear that the system destabilizes as the permeability of the porous medium k increases. This is indicated by a decrease in R_{SC}values. The reason is that as the pore size increases, it becomes easier for the flow to destabilize the system. It is observed from the figure that the magnetic field dependent viscosity δ is found to stabilize the system.

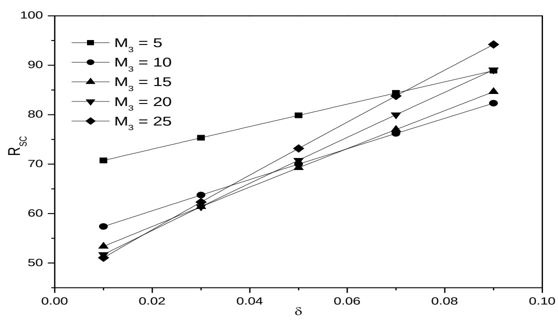

Figure 9. Variation of R_{SC}versus δ for various M_3, $R_S = -500, \tau = 0.003,\ S_T = -0.002$ and $k = 0.001$

From Fig. 9, illustrates that as M_3 increases, the values of R_{SC} decreases for small values of δwhereas for higher values of δ, R_{SC} decreases for lowest values of M_3, and then increases for higher values of M_3. The same trend is seen from figure 6. The destabilizing trend of R_S, k and τ is also seen from Figs. 11, 12 and 13. But stabilizing behavior of $S_T$is seen from figure 10.

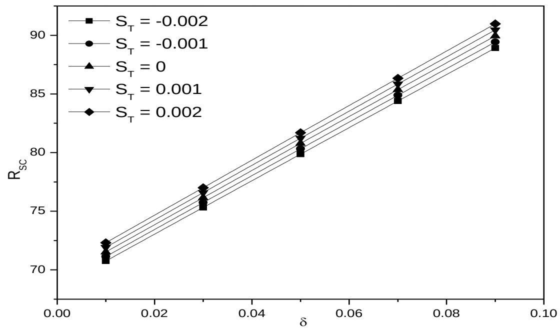

Figure 10. Variation of R_{SC}versus δ for various S_T, $R_S = -500, \tau = 0.003,\ M_3 = 5$ and $k = 0.001$

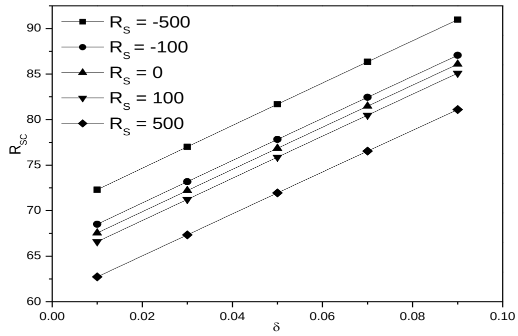

Figure 11. Variation of R_{SC}versus δ for various R_S, $S_T = -0.002, \tau = 0.003,\ M_3 = 5$ and $k = 0.001$

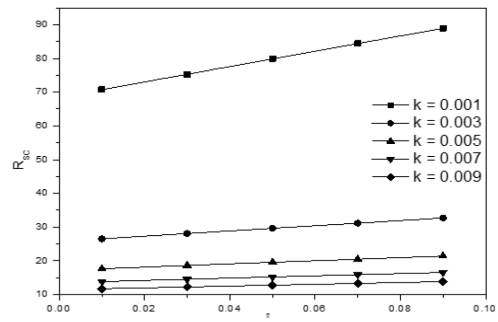

Figure 12. Variation of R_{SC}versus δ for various k, $R_S = -500\ , S_T = -0.002, \tau = 0.003,$ and $M_3 = 5$

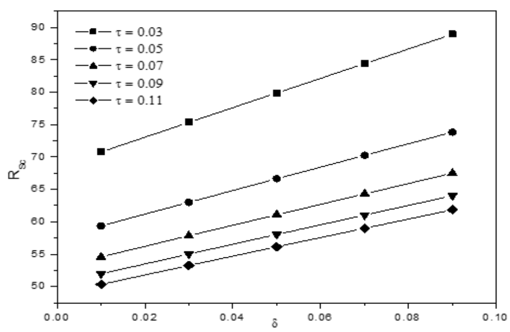

Figure 13. Variation of R_{SC}versus δ for various τ, $R_S = -500\ , S_T = -0.002,\ M_3 = 5$ and $k = 0.001$

Figures 9-13 investigate the variation of R_{SC} versus δfor different values of M_3, S_T,k and τ. From Figs. 9 - 13, one can find that as the coefficient of MFD viscosity is increased from 0.01 to 0.09, the critical magnetic Rayleigh number increases. This means that the system is stabilized through viscosity variation with respect to magnetic field. This leads to the conclusion that the MFD viscosity delays the onset of convection for ferrofluid in a densely distributed porous medium.

10. CONCLUSIONS

The linear stability of thermohaline convection in a ferrofluid layer heated from below and salted from above saturating a porous medium subjected to a transverse uniform magnetic field has been considered with effect of Soret and magnetic field dependent (MFD) viscosity using Darcy model. In this investigation, the effect of various parameters like permeability of the porous medium, Soret parameter, MFD viscosity parameter, non – buoyancy magnetization, buoyancy magnetization, Prandtl number, ratio of mass transport to heat transport, Rayleigh number and salinity Rayleigh number on the onset of convection have been calculated. The thermal critical magnetic Rayleigh numbers for the onset of instability are also determined for both stationary and oscillatory modes, for sufficient large values of buoyancy magnetization parameter M_1 and results are depicted graphically. Furthermore, the principle of exchange of stability is applied to find out the mode of attaining instability.

In conclusion, we see that convection can encourage in a ferromagnetic fluid by means of spatial variation in magnetization, which is induced when the magnetization of the ferrofluid depends on temperature and salinity. For the stationary convection, MFD viscosity has always a stabilizing effect, whereas the permeability of the porous medium has always a destabilizing effect on the onset of convection. In the absence of MFD viscosity ($\delta = 0$) magnetization has always a destabilizing effect. In the presence of MFD viscosity, nothing specific can be said, since there is competition between the destabilizing role of the magnetization M_3 and the stabilizing role of the MFD viscosity δ

The coefficient of MFD viscosity δ effect has a destabilizing behavior for various values of R_S, τM_3 and k which are observed in Figures 1 - 3, 6 - 13. But, the convective system has a stabilizing effect which is analyzed in Figures 4 and 5 for Soret parameterS_T. It is evident from Figure 9 that lower values of $R_{SC}$are needed for the onset of convection with increase in M_3 for smaller values of δ whereas higher values of R_{SC} are needed for the onset of convection with increase in M_3 for smaller values of δ justifying the competition between the destabilizing effect of the magnetization M_3 and the stabilizing effect of the MFD viscosityδ Thus magnetization destabilizes the system and coefficient of MFD viscosity stabilizes the system for both modes. This leads to the conclusion that the MFD viscosity delays the onset of convection for ferrofluid saturating a densely distributed porous medium.

ACKNOWLEDGMENT

The author D. Murugan is thankful to the DC member Dr. R. Vasanthakumari, Director, Kanchi Mamunivar for Post Graduate Studies, Puducherry, India, for her constant encouragement.

REFERENCES

[1] Cowley MD, Rosensweig RE. (1967). The interfacial stability of ferromagnetic fluid. Journal of Fluid Mechanics 30: 671-688.

[2] Voelker T, Odenbach S. (2005). Thermo diffusion in ferrofluids in the presence of a magnetic field. Physics Fluids 17: 037104.

[3] Finlayson BA. (1970). Convective instability of ferromagnetic fluids. Journal of Fluid Mechanics 40: 753-767.

[4] Stiles PJ, Kagan MJ. (1990). Thermoconvective instability of a ferrofluid in a strong magnetic field. Journal Colloid and Interface Science 134: 435-449.

[5] Siddheshwar PG. (1995). Convective instability of ferromagnetic fluids bounded by fluid-permeable, magnetic boundaries. Journal of Magnetism and Magnetic Materials 149: 148–150.

[6] Nanjundappa CE, Shivakumara IS. (2008). Effect of velocity and temperature boundary conditions on convective instability in a ferrofluid layer. ASME Journal of Heat Transfer 130: 1045021-1045025.

[7] Vaidyanathan G, Sekar R, Ramanathan A. (2002). Ferroconvection in an anisotropic densely packed porous medium. Indian Journal of Chemical Technology 9: 446-449.

[8] Hurle DTJ, Jakeman EJ. (1971). Soret driven thermo solutal convection. Journal of Fluid Mechanics 47: 667-687. https://doi.org/10.1017/S0022112071001319

[9] Shevtsova MD, Melnikov E, Clegros J. (2006). Onset of convection in Soret driven instability. Physical Review E 73: 047302. https://doi.org/10.1103/physRevE.73.047302

[10] Sekar R, Vaidyanathan G, Hemalatha R, Senthilnathan S. (2006). Effect of sparse distribution pores in Soret-driven ferrothermohaline convection. Journal of Magnetism and Magnetic Materials 302: 20–28.

[11] Vaidyanathan G, Sekar R, Hemalatha R, Vasanthakumari R, Senthilnathan S. (2005). Soret-driven ferro thermohalineconvection. Journal of Magnetism and Magnetic Materials 288: 460–469.

[12] Kim MC. (2011). Linear stability analysis on the onset of Soret driven motion in nanoparticles suspension. European Physical Journal 34: 27.

[13] Hemalatha R, Sekar R, Vaidyanathan G. (2012). Effect of dust particles on a Soret-driven ferro thermohaline convection in a rotating medium. International Journal of Applied Mechanics and Engineering 17(2): 367-381.

[14] Hemalatha R, Sekar R, Vaidyanathan G. (2011). Effect of rotaion on a Soret-driven thermohaline convection in a dusty ferrofluids saturating a porous medium. International Journal ofApplied Mechanics and Engineering 16(4): 1021-1036.

[15] Rosensweig RE, Kaiser R, Miskolczy G. (1969). Viscosity of magnetic fluid in a magnetic field. Journal Colloid and Interface Science 29(4): 680-686.

[16] Rudraiah N, Sekhar GN. (1991). Convection in magnetic fluids with internal heat generation. ASME Journal of Heat Transfer 113: 122-127. https://doi.org/10.1115/1.2910514

[17] Shivakumara IS, Nanjundappa CE. (2001). Onset of convection in a sparsely packed porous layer with throughflow. Archivesof Mechanics 53: 219–241.

[18] Shivakumara IS, Suma SP, Chavaraddi KB. (2006). Onset of surface tension driven convection in a superposed layers of fluid and saturated porous medium. Archivesof Mechanics 58: 71–92.
[19] Shivakumara IS, Mamatha AL, Ravisha M. (2010). Boundary and thermal non-equilibrium effects on the onset of Darcy-Brinkman convection in a porous layer. Journal of Engineering Mathematics 67: 317-328. https://doi.org/ 10.1007/s10665-010-9362-3
[20] Sunil P, Chand, Mahajan A, Sharma P. (2011). Effect of rotation on double diffusive convection in a magnetized ferrofluid with internal angular momentum. Journal of Applied Fluid Mechanics 4(4): 43-52.
[21] Shliomis MI. (1972). Effect of viscosity of magnetic suspensions. Soviet physics FETP 34: 1291-1294.
[22] Kaloni PN, Lou JX. (2004). Convective instability of magnetic fluids. Physical Review E 70: 026313- 026324.
[23] Ram P, Bhandari A, Sharma K. (2010). Effect of magnetic field dependent viscosity on revolving ferrofluid. Journal of Magnetism and Magnetic Materials 322(21): 3476–3480. https://doi.org/10.1016/j.jmmm.2010.06.048
[24] Ram P, Sharma K. (2011). Revolving ferrofluid flow under the influence of magnetic field dependent viscosity and porosity with rotating disk. Journal of Electromagnetic Analysis andApplications 3(9): 378-386. https://doi.org/10.4236/jemaa.2011.39060
[25] Ram P, Kumar V. (2012). Ferrofluid flow with magnetic field dependent viscosity due to rotating disk in porous medium. International Journal of Applied Mechanics 4(4): 12500411-125004118. https://doi.org/10.1142/S175882511250041X
[26] Ram P, Sharma K. (2014). Effect of rotation and MFD viscosity on ferrofluid flow with rotating disk. Indian Journal of Pure and Applied Physics 52: 87-92.
[27] Sunil, Sharma RC. (2005). Effect of magnetic field dependent viscosity on thermosolutal convection in ferromagnetic fluid saturating a porous medium. Transport in Porous Media 60(3): 251-274. https://doi.org/ 10.1007/s11242-004-5739-y
[28] Sunil A, Sharma RC. (2005). Effect of magnetic field dependent viscosity on thermosolutal convection in ferromagnetic fluid. Journal of Applied Mathematics and Computing 163(3): 1197-1214. https://doi.org/10.1016/j.amc.2004.06.006
[29] Ramanathan A, Muchikel N. (2006). Effect of Temperature-Dependent viscosity on Ferroconvection in a porous medium. International Journal ofApplied Mechanics and Engineering 11(1): 93-104.
[30] Sunil A, Sharma, Sharma D, Kumar P. (2008). Effect of magnetic field dependent viscosity on thermal convection in a ferromagnetic fluid. Chemical Engineering and Communication 195: 571-583. https://doi.org/10.1080/00986440701707719
[31] Sunil P, Bharti K, Sharma D, Sharma RC. (2004). The Effect of magnetic field dependent viscosity on thermal convection in a ferromagnetic fluid in a porous medium. Verlag der Zeitschrift fur Naturforschung 195: 397-406.
[32] Vaidynathan G, Sekar R, Vasanthakumari R, Ramanathan A. (2002). The effect of magnetic field dependent viscosity on ferroconvection in a rotating sparsely distributed porous medium. Journal of Magnetism and Magnetic Materials 250: 65–76. https://doi.org/ 10.1016/S0304-8853(02)00355-4
[33] Vaidynathan G, Sekar R, Ramanathan A. (2002). Effect of magnetic field dependent viscosity on ferroconvection in rotating medium. Indian Journal of Pure and Applied Physics 40: 159-165.
[34] Nanjundappa CE, Shivakumara IS, Srikumar K. (2009). Effect of MFD viscosity on the onset of ferromagnetic fluids layer heated from below and cooled from above with constant heat flux. Measurement Science Review 9(3): 75-80. https://doi.org/10.2478/v10048-009-0010-x
[35] Ramanathan A, Suresh G. (2004). Effect of magnetic field dependent viscosity and anisotropic porous medium on ferroconvection. International Journal of Engineering Science 42: 2411-425. https://doi.org/10.1016/S0020-7225(02)00273-2
[36] Suresh G, Vasanthakumari R. (2009). Comparison of theoretical and computational ferroconvection induced by magnetic field dependet viscosity in an anisotropic porous medium. International Journal of Recent Trends in Engineering 1(1): 41-45.
[37] Malashetty MS, Gaikwad SN, Swamy M. (2006). An analytical study of linear and non-linear double diffusive convection with Soret effect in couple-stress liquids. International Journal of Thermal Science 45(9): 897. https://doi.org/10.1016/j.ijthermalsci.2005.12.005
[38] Gaikwad SN, Malashetty MS, Ramaprasad K. (2007). An analytical study of linear and non-linear double diffusiveconvection with Soret and Dufour effects in couple-stress liquids. International Journal of Nonlinear Mechanics 42(7): 903. https://doi.org/10.1016/j.ijnonlinmec.2007.03.009
[39] Sunil P, Sharma, Mahajan A. (2008). A nonlinear stability analysis for thermoconvective magnetized ferrofluid with magnetic field dependent viscosity. International Communications in Heat and Mass Transfer 35: 1281-1287. https://doi.org/10.1016/j.icheatmasstransfer.2008.08.004
[40] Sunil, Mahajan A. (2009). A nonlinear stability analysis of a double diffusive magnetized ferrofluid with magnetic field dependent viscosity. Journal of Magnetism and Magnetic Materials 321: 2810–2820. https://doi.org/10.1016/j.jmmm.2009.04.016
[41] Hemalatha R, Sivapraba N. (2012). Effect of magnetic field dependent viscosity on ferroconvection in a sparsely distributed anisotropic porous medium in the presence of horizontal thermal gradient. Indian Journal of Pure and Applied Physics 50: 907-914.
[42] Sunil A, Sharma R, Shandil G. (2008). Effect of magnetic field dependent viscosity on ferroconvection in the presence of dust particles. Journal of Applied Mathematics and Computing 27: 7-22. https://doi.org/10.1007/S12190-008-0055-2
[43] Hemalatha R (2014). Study of magnetic field dependent viscosity on a Soret driven ferrothermohaline convection in a rotating porous medium. International Journal ofApplied Mechanics and Engineering 19(1): 61-77. https://doi.org/10.2478/ijame-2014-0006
[44] Sekar R, Raju K. (2013). Effect of magnetic field dependent viscosity on Soret-driven thermoconvective instability of ferromagnetic fluid in the presence of rotating anisotropic porous medium of sparse particle suspension. International Journal of Mathematical Sciences 12: 13–31.

[45] Sekar R, Raju K. (2014). Effect of magnetic field dependent viscosity on Soret driven ferrothermohaline convectionsaturating aanisotropic porous medium of sparse particle suspension. World Journal of Engineering 11(3): 213-228. https://doi.org/10.1260/1708-5284.11.3.213

[46] Sekar R, Murugan D, Raju K. (2016). Densely Distribution Pores and Coriolis Force on Thermohaline Convection in a Ferrofluid with Soret and Anisotropy Effects. International Journal ofAdvancedScience and Technology 87: 15-30.

[47] Sekar R, Murugan D, Raju K. (2016). Ferrothermoconvective instability in Soret driven convection saturating a densely packed anisotropic porous medium. International Journal ofApplied Mathematics, Electronics and Computers 4(2): 58-64.