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Hydromagnetic flow of power law fluid in a porous medium with energy dissipation: A numerical approach

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ABSTRACT

A time independent boundary layer flow through a porous medium has been studied with energy dissipation and transverse magnetic field. The constitutive equation of the fluid flow is based on power law fluid flow. Body forces generated in the fluid flow are Lorentz force, Buoyancy and Darcy's force. Surface of the porous medium is characterized by velocity and thermal slip factors. In the mathematical analysis, we have used similarity transformation (for conversion of partial differential equations into ordinary) and MATLAB built in bvp4c solver method. Effects of various flow parameters are represented by the graphs and effort has been made to discuss the results.

1. INTRODUCTION

Flows of non-Newtonian fluids are more familiar and valuable than Newtonian fluid due to their importance in engineering and science. In view of differences in Newtonian and non-Newtonian fluids, various models of non-Newtonian fluids have been proposed. In the present investigation we have taken power law fluid model because of its simplicity in mathematics. Also this model can describe the flow behavior of non-Newtonian fluid over the most important range of shear stress. Beavers and Joseph [1] was first proposed the slip flow condition at the boundary. The flow behaviors of boundary layer flow of non-Newtonian fluids are given in [2-11].

Boundary layer flow problems governed by power-law fluid model have been studied by Acrious [12] and Schowalter [13]. Lee and Ames [14] have obtained the similarity solutions of fluid flow using non-Newtonian fluid model. Anderson et al. [15] and Mahmoud et al. [16] have studied the flow behaviours of hydro-magnetic power-law fluid flow past a continuous moving surface. Cheng [17) has studied the natural convection flow problem using power law fluid model with thermal and mass stratifications. Pal et al. [18] have investigated the influences of thermal diffusion and diffusion-thermo on hydro-magnetic flow of power-law fluid over an inclined plate with variable thermal conductivity. Yazdi et al. [19] have analysed the effects of slip velocity on fluid flow guided by non-Newtonian fluid model over a moving permeable surface with internal heat generation or absorption. Hayat [20] has studied the flow of nano-fluid governed by power-law model over a stretching surface with Joule heating. Analysis of heat transfer on slip flow of a power-law fluid in a Darcy's porous medium with plate suction/injection has been done by Aziz et al. [21] and we have extended the work of [21] to MHD case.

The objective of the present study is to study the hydro-magnetic fluid flow guided by Power law model though a porous medium with energy dissipation. Heat is generated within the through the energy dissipation due to viscosity. The model of the problem is framed using conservation principles of mass, momentum and energy with constitutive equation of Power law fluid model. This problem finds application in polymer processing industries, food industries etc.

2. MATHEMATICAL FORMULATIONS

A two dimensional time independent flow of power law fluid past a semi- infinite porous plate in a porous medium has been investigated with free convection. Let x axis be coincide along the surface and y axis be taken along the direction at right angles to the surface. Physical description of the problem is shown by figure 1. Application of magnetic field of uniform strength B0 (using Gauss's law of magnetism) generates Lorentz force $\sigma B_0^2 u$. Using Boussinesq approximation and boundary layer assumption, the pressure gradient term is replaced by $g\beta(T-T_\infty)+\frac{1}{pk}U_\infty$. The Darcy's term due to the permeability of porous medium is exhibited by $-\frac{1}{pk}u$ Inertia and viscous forces of the governing fluid motion are given by $u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}$ and $\frac{1}{\rho}\frac{\partial}{\partial y}\tau_{xy}$ respectively in (2.2). Again, in the energy equation (2.3), convection and conduction are characterized by $u\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}$ and $\frac{k_T}{\rho C_p}\frac{\partial^2 T}{\partial y^2}$ respectively. Also, Mechanism of heat generation due to energy dissipation by viscosity is given by $\frac{k}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^{n+1}$ in (2.3).

Using the boundary layer approximations and Boussinesq approximation, the continuity, momentum and energy equations are written in usual notation as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\tau_{xy}-\frac{\sigma B_0^2 u}{\rho}+g\beta(T-T_\infty)-\frac{1}{pk}(u-U_\infty) \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_T}{\rho C_p}\frac{\partial^2 T}{\partial y^2}+\frac{k}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^{n+1} \quad (3)$$

The boundary conditions of the problem are:

$$y=0: u=L_1\frac{\partial u}{\partial y}, v=v_w, T=T_W+D_1\frac{\partial T}{\partial y}$$
$$\&y\to\infty: u\to u_w, T\to T_W \quad (4)$$

where, $L_1=L(Re_x)^{\frac{1}{2}}$ and $D_1=D(Re_x)^{\frac{1}{2}}$

The shear stress component τxy in equation (2. 2) for the power-law model is defined as(see (29))

$$\tau_{xy}=\frac{k_i}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) \quad (5)$$

where, n < , =, > 1 correspond to shear thinning, Newtonian and shear thickening cases respectively.

Using the expression of shear stress components (2.5) in (2.2), we get

$$u\frac{\partial_u}{\partial_x}+v\frac{\partial u}{\partial_y}=\frac{k_i}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n-\frac{\sigma B_0^2 u}{\rho}+g\beta(T-T_\infty)-\frac{1}{pk}(u-U_\infty) \quad (6)$$

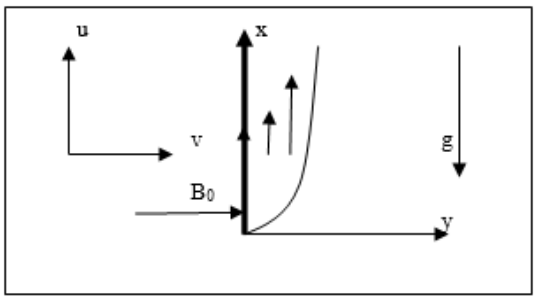

Figure 1. Physical description of the problem

3. METHOD OF SOLUTION

The stream function ψ is introduced in (2.6) and (2.3) and its relation with the velocity components are given as $u=\frac{\partial\varphi}{\partial_y}, v=\frac{\partial\varphi}{\partial_x}$

Introducing the following similarity variables into the equations (2.6) and (2.3),

$$\varphi=Lu-U_\infty\left(\frac{x}{LRe}\right)^{\frac{1}{n+1}}f(n), n=\left(\frac{LRe}{x}\right)^{\frac{1}{n+1}}\frac{Y}{L}, u=u-U_\infty f'(n), \theta=\frac{T-T_\infty}{T_\infty-T_W} \quad (7)$$

we get the following set of differential equations:

$$-\frac{1}{n+1}ff''=n(f'')^{n-1}f'''-k_p(f'-1)-M(f'-1)+Gr\theta \quad (8)$$

$$\theta''=-\frac{Pr}{n+1}f\theta'-PrEc(f'')^{n+1} \quad (9)$$

where $Pr=\frac{k_T x\left(\frac{ReL}{x}\right)^{\frac{1}{n+1}}}{\rho C_p}$, $Ec=\frac{KU_\infty^n Re}{\rho C_p L^n(T_W-T_\infty)}$, $M=\frac{\sigma B_0^2 x}{\rho U_\infty}$, $k=\frac{kx\left(\frac{ReL}{x}\right)^{\frac{n-1}{n+1}}U_\infty{}^{n-2}}{\rho K_p L^{n-1}}$.

And $Ec=\frac{g\beta(T_W-T_\infty)x}{U_\infty{}^2}$

The corresponding boundary conditions are

$$f=S, f'=lf'', \theta=1+d\theta': n=0\ \&$$
$$f'\to 1, \theta\to 0: n\to\infty \quad (10)$$

where, $S=\frac{(n+1)v_W}{U_W}(Re_x)^{\frac{1}{2}}$, $l=\frac{L\rho U_\infty}{k}$& $d=\frac{D\rho U_\infty}{k}$

The non-linear coupled ordinary differential equations (3.2) & (3.3) together with boundary conditions (3.4) are solved using MATLAB built in bvp4c solver method.

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4. RESULTS AND DISCUSSIONS

Influences of magnetic parameter, energy dissipation factor (Eckert number) and power law index on governing power law fluid motion through porous medium with energy dissipation have been studied numerically with different values of flow parameters. In our discussions, we have kept some parameters are fixed as Kp=0.3,Gr=5, l=0.1, d=0.1.

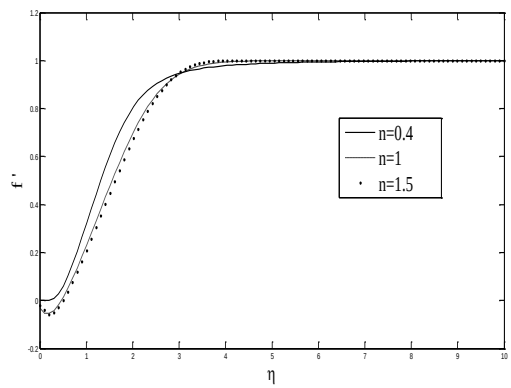

Figure 2. Velocity against displacement variable kd=0.3; G=5; Pr=7; M=2; S=0.3, l=0.1, d=0.1

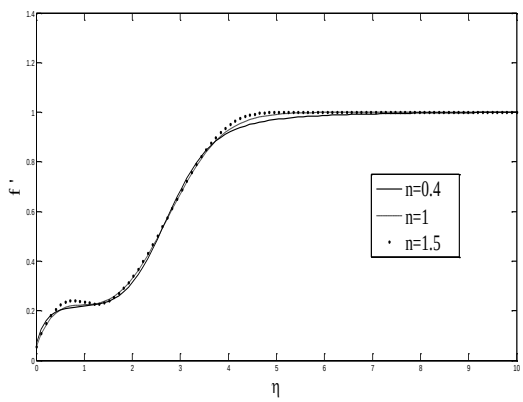

Figure 3. Velocity against displacement variable kd=0.3; G=5; Pr=7; M=2; S=-0.3, l=0.1, d=0.1

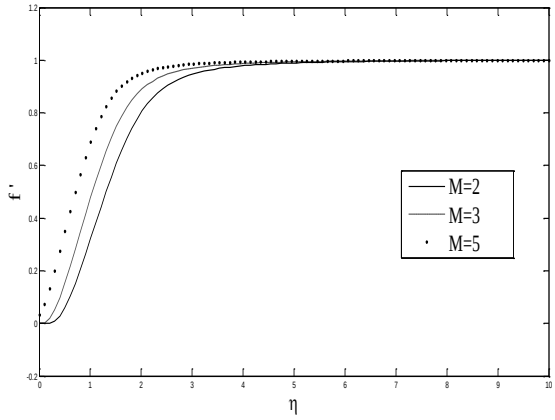

Figure 4. Velocity against displacement variable kd=0.3; G=5; Pr=7; n=0.4; S=0.3, l=0.1, d=0.1

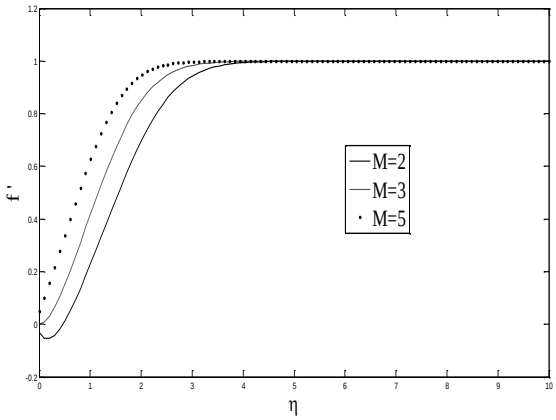

Figure 5. Velocity against displacement variable kd=0.3; G=5; Pr=7; n=1; S=0.3, l=0.1, d=0.1

Figures 2 to 5 depict the pattern of velocity profile against the displacement variable. The figures reveal the fact that boundary layer is seen in the region $0 \leq n \leq 5$. It is seen from figures 2 and 3 that effect of power law index is prominent during heat source parameter (figure 2) than heat sink parameter (figure 3). Growth in power law index subdues the speed of fluid flow during the generation of heat (case of heat source).

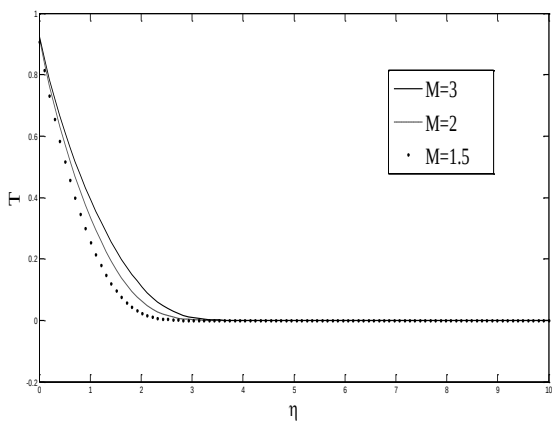

Figure 6. Temperature against displacement variable kd=0.3; G=5; Pr=7; n=0.4; S=0.3, l=0.1, d=0.1

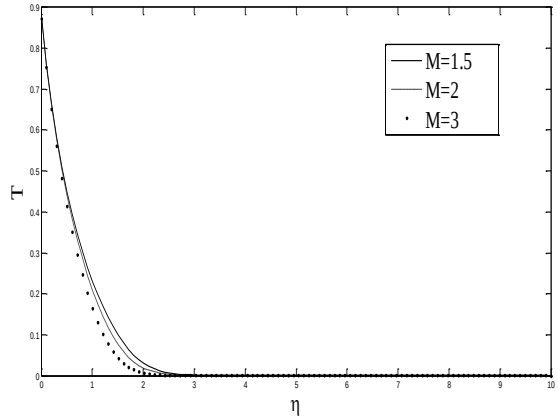

Figure 7. Temperature against displacement variable kd=0.3; G=5; Pr=7; n=1.5; S=0.3, l=0.1, d=0.1

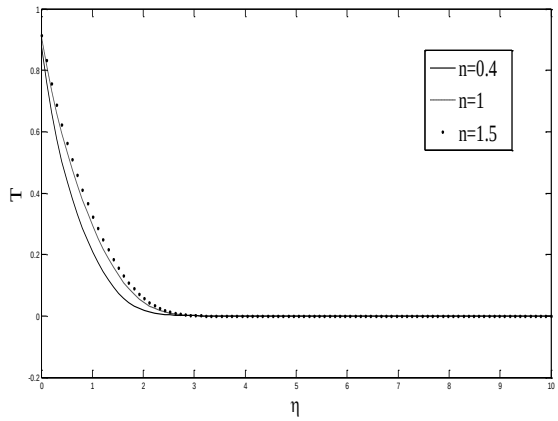

Figure 8. Temperature against displacement variable kd=0.3; G=5; Pr=7; M=2; S=0.3, l=0.1, d=0.1

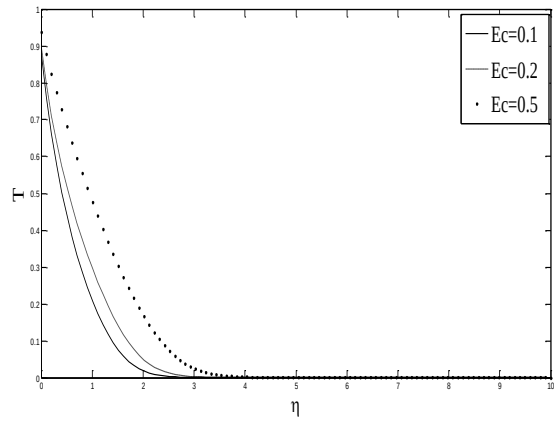

Figure 9. Temperature against displacement variable kd=0.3; G=5; Pr=7; M=2; n=0.4, S=0.3, l=0.1, d=0.1

Figure 4 and 5 represents the velocity profile of shear thinning and the shear thickening cases under the influence of magnetic parameter. It is notified that the growth in magnetic parameter enhances the velocity of fluid motion.

Figure 6 to 9 show pattern of temperature field of power law fluid motion against the displacement variable. The figures state that there is a continuous reduction in temperature across the boundary layer until it reaches the thermal stability stage. It is also noticed that an increment in magnetic parameter decreases the temperature in both these cases (figure 6 & 7). It

may be concluded from figure 8 that increment of temperature in shear thickening case is superior to shear thinning case. Physically it may be interpreted that during shear thickening cases, fluid flow is retarded and mechanical energy reduces and consequently thermal energy increases. Influence of Eckert number on temperature field is shown by figure 9 and it is revealed that enhancement of Eckert number increases the temperature of fluid.

5. CONCLUSIONS

Some of the points are concluded from the above study and are given as follows:

(1) Fluid flow is accelerated during the enhancement of magnetic parameter.

(2) Temperature is enhanced during shear thinning cases for increasing values of magnetic parameter.

(3) Eckert number helps to increase the temperature of fluid flow.

(4) There is an increment in temperature in shear thickening cases than shear thinning and Newtonian cases.

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NOMENCLATURE

u	velocity component along x direction, $m.s^{-1}$
v	Velocity component along y direction, $m.s^{-1}$
T	Temperature of fluid, K
U_∞	Free stream velocity, $m.s^{-1}$
T_∞	Free stream temperature, K
B_0	Strength of magnetic field, $kg.s^{-2}.A^{-1}$
C_p	Specific heat at constant pressure, $m^2s^{-1}K^{-1}$
k	Permeability of porous medium, m^2
K_T	Thermal conductivity, $W.m^{-1}K^{-1}$
L_1	Velocity slip parameter, m^{-1}
D_1	Thermal slip parameter, m^{-1}
T_w	temperature of fluid at the surface, K
v_w	section ($v_w < 0$) or injection ($v_w > 0$) velocity at the surface, ms^{-1}
k_i	consistency of power law fluid, m^2s^{-1}
n	power-law index
Pr	Prandtl number
Ec	Eckert number
M	magnetic parameter
Kp	dimensionless permeability parameter
Gr	Grashoff number
L	a constant having dimension of velocity slip
D	a constant having dimension of thermal slip
Re_x	Local Reynolds number
S	dimensionless suction/injection velocity
l	dimensionless velocity slip parameter
d	dimensionless thermal slip parameter
g	gravitational acceleration, $m.s^{-2}$
k	thermal conductivity, $W.m^{-1}. K^{-1}$
Nu	local Nusselt number along the heat source

Greek symbols

ρ	density of the fluid, $kg.m^{-3}$
β	co-efficient of volume expansion, $mol^{-1}m^3$
σ	Electrical conductivity, $m^{-3}kg^{-1}s^3A^2$
τ_{xy}	shear stress components
ψ	stream function