









unknown integration constants. The algebraic eigenvalue – eigenvector problem resulting from the enforcement of boundary conditions yielded the matrix equation – Equation (37). Further use of the boundary conditions simplified the algebraic eigenvalue – eigenvector problem as Equation (43).

The characteristic buckling equation obtained from the requirement of non – trivial solution gave the determinantal equation – Equation (44). Expansion and simplification of the determinantal equation yielded the characteristic buckling equation as Equation (47). The characteristic buckling equation is found to be a transcendental equation in terms of  $\alpha_1$  and  $\alpha_2$  which are each functions of the buckling load  $N_0$  as given in Equations (22) and (23). Solution of the buckling equation yielded the buckling loads for various values of the plate aspect ratio  $a/b$  as presented in Table 1. The critical buckling stresses as were further calculated for various plate aspect ratios for Kirchhoff plate with simply supported edges ( $x = 0$  and  $x = a$ ) and clamped edges ( $y = 0$  and  $y = b$ ) and for plate material properties given by  $E = 58.8 \times 10^9 \text{ N/m}^2$ ,  $h = 0.02b$  and  $\mu = 0.30$ , and presented in Table 2.

## 7. CONCLUSIONS

The following conclusions can be made from the study:

- (i) The single Fourier sine transform method is an ideal mathematical/analytical tool for solving the boundary value problem of elastic buckling of Kirchhoff plates with two opposite simply supported edges ( $x = 0, x = a$ ) and two opposite clamped edges ( $y = 0, y = b$ ).
- (ii) The single Fourier sine transformation applied in the  $x$ -coordinate direction simplified the problem since the Dirichlet boundary conditions are satisfied in that direction, thus simplifying the Fourier sine transformation of partial derivatives with respect to  $x$ .
- (iii) The single finite Fourier sine transformation simplifies the boundary value problem of elastic buckling to an algebraic eigenvalue – eigenvector problem, which is easier to solve.
- (iv) The characteristic buckling equations obtained by enforcement of boundary conditions are the same as the characteristic buckling equations obtained by other scholars who applied Navier’s double trigonometric series methods, and total potential energy minimization methods.
- (v) The critical buckling loads obtained were exact solutions and were the same as critical buckling loads obtained by Timoshenko and other researchers who used total potential energy minimization methods.

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## NOMENCLATURE

$N_{xx}$	uniform compressive force in the $x$ – direction
$N_{yy}$	uniform compressive force in the $y$ – direction
$N_{xy}$	twist
$w(x, y)$	deflection
$x, y$	in-plane Cartesian coordinates
$D$	flexural rigidity of plate
$E$	Young’s modulus of elasticity of the plate material
$\mu$	Poisson’s ratio of plate material
$h$	plate thickness
$N_0$	uniform compressive force in the $x$ direction
$M_{xx}$	bending moment
$W(k,y)$	finite Fourier sine transform of $w(x,y)$
$k$	finite Fourier sine transform variable
$m$	integer
$\beta_m$	parameter defined to relate with $m$ and $a$
ODE	ordinary differential equation
$C$	clamped edge
$S$	simply supported edge
CCSS	plate with two opposite edges clamped and two opposite edges simply supported

## Subscripts

$cr$  critical

## Mathematical symbols

$\int$	integration sign or integral
$\frac{\partial}{\partial x}$	partial derivative with respect to $x$
$\frac{\partial^2}{\partial x \partial y}$	mixed partial derivative