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Free convective flow of two immiscible memory fluids in an inclined channel with energy dissipation

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ABSTRACT

Magnetohydrodynamic (MHD) convection flow problem of two immiscible fluids through an inclined channel in presence of a magnetic field and energy dissipation has been investigated. Both fluids are assumed to be Non-Newtonian. The channel walls are maintained at different temperatures. The resulting coupled and non-linear equations of momentum and energy are solved analytically using the regular perturbation method. Results are computed using MATLAB computer programming. The influence of various parameters on shearing stresses and rate of heat transfers (Nusselt numbers) are discussed with the numerical values.

1. INTRODUCTION

Memory fluid flow through porous medium in presence of transverse magnetic field has various applications in the field of engineering like chemical, agriculture and petroleum technology etc. Walter has demonstrated the constitutive model of memory fluid with short relaxation memories. Various polymers used in chemical industries resemble the constitutive model given by Walters. Due to its extensive applications in engineering and technology, many researchers [1-4] have studied fluid flow problems using Walters liquid model for short relaxation memories.

Time independent hydromagnetic flow of conducting fluid through a channel under transverse magnetic field has been studied by Hartmann [5]. Seigal [6] have carried out the problem of fully developed hydromagnetic laminar flow of a conducting liquid between two parallel plates. Heat transfer phenomena in MHD flow between vertical parallel plates has been investigated by various researchers Osterle and Young [7], Perlmutter and Seigal [8], Romig [9] and Umavathi [10].

Multi-layered-fluid flow has many applications in the field of plasma physics, aeronautics, geo-physics and petroleum industries etc. Influences of transverse magnetic field on two-phase fluid flows have been investigated by Thome [11], Lohrasbi and Sahai [12], Malashetty and Leela [13]. Laminar flow problems governed by two electrically conducting immiscible fluids with heat generation/absorption through infinitely long porous and nonporous channels have been studied by Chamkha [14]. Fluid flow through an open inclined channel has been investigated by Prakash [15]. Verma and Vyas [16], Malashetty and Umavathi [17], Malashetty et al. [18] and Wang and Robillard [19] have analyzed the flow problem of viscous fluid through an inclined channel. Chamkha et al. [20] have discussed the problem of double-diffusive natural convection in inclined finned triangular porous enclosures for thermal and concentration boundary conditions in presence of heat generation/absorption.

Zaidi and Ahmad [21] have investigated the flow problem of two immiscible fluids in an inclined channel with heat generation/absorption. We have extended the work of [21] using non-Newtonian fluid model because of the vast applications of non-Newtonian fluid model in engineering science and technology.

2. MATHEMATICAL FORMULATIONS

A steady, hydro-magnetic, laminar and fully developed flow of two immiscible non-Newtonian fluids through an inclined channel of infinite length has been considered with external heat agent. The walls are maintained at different temperatures T_1 and T_2 making an angle α (small) with the vertical. A uniform magnetic field of strength B_0 is applied along the transverse direction to the fluid flow. The regions $0 \le y \le h1\ (Region - I)$ and $-h1 \le y \le 0\ (Region - II)$ are occupied by the viscous incompressible and electrically conducting Non-Newtonian fluids. The geometry of the problem is given by figure 1.

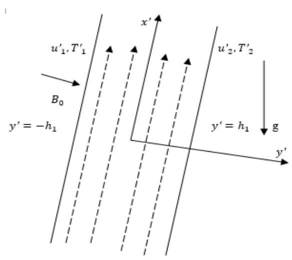

Figure 1. Geometry of the problem

The velocities and temperatures of two fluids along the direction of motion are given by u'_1, u'_2, T'_1, T'_2 respectively,

g acceleration due to gravity, α be the angle of inclination of the surface along the vertical, B_0 strength of uniform magnetic field ($\nabla.\vec{B} = 0 \Rightarrow \vec{B} = B_0$, $\vec{B}$ magnetic field vector), σ_i electrical conductivities, ρ_i densities, β_i co-efficient of volume expansions, k_i thermal conducitivites, k_{0i} visco-elasticities, c_{p_i} specific heat capacities, Q_i external heat agents of two fluids (i=1, 2).

The fluid flow is guided by following assumptions.

- All the fluidic properties are constants except the variation of density in the buoyancy term (Buossinesq Approximation).
- Application of transverse magnetic field generates a Lorentz force and is given by $-\sigma_i\ \beta_0^2\ u_i^{'}$
- Mechanical energy is dissipated into thermal energy by energy dissipation and is given by $\mu_i\left(\frac{du_i^{'}}{dy_i^{'}}\right)^2 - k_{0_i}\nu_0\frac{du_i^{'}}{dy_i^{'}}\frac{d^2u_i^{'}}{dy_i^{'2}}$
- Conservation of Mass principle: $\frac{\partial u_i^{'}}{\partial x^{'}} = 0 \Rightarrow u_i^{'} = u_i^{'}(y)$

The governing equations of motions using conservation principle of momentum and energy are given as follows:

$$\rho\nu_0\ \frac{du_i^{'}}{dy^{'}} = -\sigma_i\ \beta_0^2\ u_i^{'} + \rho_i\ g\beta_i\left(T_i^{'} - T_s\right)Cos\ \alpha + \mu\frac{d^2u_i^{'}}{dy^{'2}} - k_0\nu_0\frac{d^3u_i^{'}}{dy^{'3}} \quad (2.1)$$

$$\rho_i c_{p_i}\left[\gamma_0\frac{dT_i^{'}}{dy_i^{'}}\right] = k_i\frac{d^2T_i^{'}}{dy_i^{'}} + \mu_i\left(\frac{du_i^{'}}{dy_i^{'}}\right)^2 - k_{0_i}\nu_0\frac{du_i^{'}}{dy_i^{'}}\frac{d^2u_i^{'}}{dy_i^{'2}} \pm Q_i\left(T_i^{'} - T_s\right) \quad (2.2)$$

The boundary and interface conditions of the problem on velocity and temperature are given as follows:

$$u'_1(h_1) = 0, u'_2(-h_1) = 0, u'_1(0) = u'_2(0),\ \mu_1\frac{du'_1}{dy} = \mu_2\frac{du'_2}{dy}\ at\ y' = 0 \quad (2.3)$$

$$T^{'}_1(h_1) = T_1, T^{'}_2(-h_1) = T_2\ , T^{'}_1(0) = T_2(0),\ K_1\frac{dT'_1}{dy} = K_2\frac{dT'_2}{dy}\ at\ y' = 0 \quad (2.4)$$

3. METHOD OF SOLUTION

The following non-dimensional variables are used in (2.1) and (2.2),

$$y_i = \frac{y_i^{'}}{h_i}, u_i = \frac{u_i^{'}}{V_0}, \theta_i = \frac{\left(T_i^{'} - T_s\right)}{\left(T_i - T_s\right)}$$

The dimensionless equations are:

$$e_i u_i^{'''} + \frac{1}{Re_i}\ u_i^{''} + u_i^{'} + M_i u_i = -Gr_i\theta_i Cos\ \alpha \quad (3.1)$$

$$\theta_i^{''} = Re_i Pr_i\theta_i^{'} - Pr_i Ec_i\left(u_i^{'}\right)^2 + e_i Ec_i Pr_i Re_i u_i^{'}u_i^{''} \pm Q_{Hi}\theta_i \quad (3.2)$$

where, the dimensionless parameters appeared in the above equations are given as follows:

$$M_i = \frac{\sigma_i\beta_0^2 h_i}{\rho_i V_0}, Re_i = \frac{V_0 h_i}{\gamma_i}, \gamma_i = \frac{\mu_i}{\rho_i}\ , Q_{H_i} = \frac{Q_i h_i^2}{K_i}$$

$$Gr_i = \frac{g\beta_i h_i(T_i - T_s)}{V_0^2}, \frac{1}{Pr_i} = \frac{K_{0i}}{\rho_i Cp_i\gamma_i}$$

$$e_i = \frac{K_{0i}}{\rho_i h_i^2}, E_{c_i} = \frac{V_0^2}{Cp_i(T_i - T_s)}$$

The dimensionless boundary conditions are

$$u_1(1) = 0, u_2(-1) = 0, u_1(0) = u_2(0), \frac{du_1}{dy} = \left(\frac{1}{mh}\right)\frac{du_2}{dy}\ at\ y = 0 \quad (3.3)$$

$$\theta_1(1) = 1,\ \theta_2(-1) = 0, \theta_1(0) = \theta_2(0), \frac{d\theta_1}{dy} = \left(\frac{1}{kh}\right)\frac{d\theta_2}{dy}\ at\ y = 0 \quad (3.4)$$

The resulting governing equations (3.1) and (3.2) are coupled and non-linear. So, regular perturbation method has been used. To use perturbation scheme, a small perturbation parameter has been selected. In this problem, we have chosen the parameter is as Eckert number, which may be treated as very small for incompressible viscous fluid flows. The perturbation scheme is given as follows

$$u_i = u_{i0} + Ec_i u_{i1} \quad \& \quad \theta_i = \theta_{i0} + Ec_i\theta_{i1} \quad (3.5)$$

Using (3.5) in equations (3.1) and (3.2) and then equating the co-efficient of E_{c_i} and constant terms on both the sides, we get

$$\frac{du_{i1}}{dy_i} = -M_i u_{i1} + Gr_i\theta_{i1}\ Cos\ \alpha + \frac{1}{Re_i}\frac{d^2u_{i1}}{dy_i^2} - e_i\frac{d^3u_{i1}}{dy_i^3} \quad (3.6)$$

$$\frac{du_{i0}}{dy_i} = -M_i u_{i0} + Gr_i\theta_{i0}\ Cos\ \alpha + \frac{1}{Re_i}\frac{d^2u_{i0}}{dy_i^2} - e_i\frac{d^3u_{i0}}{dy_i^3} \quad (3.7)$$

$$\frac{d\theta_{i0}}{dy_i} = \frac{1}{Re_i Pr_i}\frac{d^2\theta_{i0}}{dy_i^2} \pm \frac{Q_H\theta_{i0}}{Re_i Pr_i} \quad (3.8)$$

$$\frac{d\theta_{i1}}{dy_i} = \frac{1}{Re_i Pr_i}\frac{d^2\theta_{i1}}{dy_i^2} + \frac{1}{Re_i}\frac{du_{i0}}{dy_i} - e_i\frac{du_{i0}}{dy_i}\frac{d^2u_{i0}}{dy_i^2} \pm \frac{Q_H\theta_{i1}}{Re_i Pr_i} \quad (3.9)$$

For solving the equations, (3.6) – (3.9), we use the following boundary conditions:

$$u_{1,0}(1) = 0, u_{2,0}(-1) = 0, u_{1,0}(0) = u_{2,0}(0), \frac{du_{1,0}}{dy} = \left(\frac{1}{mh}\right)\frac{du_{2,0}}{dy}\ at\ y = 0$$

$$\theta_{1,0}(1) = 1, \theta_{2,0}(-1) = 0, \theta_{1,0}(0) = \theta_{2,0}(0), \frac{d\theta_{1,0}}{dy} = \left(\frac{1}{kh}\right)\frac{d\theta_{2,0}}{dy}\ at\ y = 0$$

$$u_{1,1}(1) = 0, u_{2,1}(-1) = 0, u_{1,1}(0) = u_{2,1}(0)\frac{du_{1,1}}{dy} = \left(\frac{1}{mh}\right)\frac{du_{2,1}}{dy}\ at\ y = 0$$

$$\theta_{1,1}(1) = 1, \theta_{2,1}(-1) = 0, \theta_{1,1}(0) = \theta_{2,1}(0), \frac{d\theta_{1,1}}{dy} = \left(\frac{1}{kh}\right)\frac{d\theta_{2,1}}{dy}\ at\ y = 0$$

Ordinary differential is solved using the above mentioned boundary conditions and the solutions are given as follows:

$$u_1 = u_{10} + Ec_1 u_{11},\ u_2 = u_{20} + Ec_2 u_{21}, \theta_1 = \theta_{10} + Ec_1\theta_{11}, \theta_2 = \theta_{20} + Ec_2\theta_{21}$$

Where

$$u_{10} = C_5 e^{\alpha_5 y} + C_6 e^{\alpha_6 y} + M_6 e^{\alpha_1 y} + M_7 e^{\alpha_2 y},$$

$$u_{20} = C_8 e^{\alpha_3 y} + C_9 e^{\alpha_9 y} + M_8 e^{\alpha_3 y} + M_9 e^{\alpha_4 y}$$

$$u_{11} = C_{15} e^{\alpha_5 y} + C_{16} e^{\alpha_6 y} + M_{35} e^{\alpha_1 y} + M_{36} e^{\alpha_2 y} + M_{37} e^{2\alpha_5 y} + M_{38} e^{2\alpha_6 y} + M_{39} e^{2\alpha_1 y} + M_{40} e^{2\alpha_2 y} + M_{41} e^{\overline{\alpha_1+\alpha_5} y} + M_{42} e^{\overline{\alpha_2+\alpha_5} y} + M_{43} e^{\overline{\alpha_5+\alpha_6} y} + M_{44} e^{\overline{\alpha_1+\alpha_6} y} + M_{45} e^{\overline{\alpha_2+\alpha_6} y} + M_{46} e^{\overline{\alpha_1+\alpha_2} y}$$

$$u_{21} = C_{17} e^{\alpha_8 y} + C_{18} e^{\alpha_9 y} + M_{47} e^{\alpha_5 y} + M_{48} e^{\alpha_4 y} + M_{49} e^{2\alpha_8 y} + M_{50} e^{2\alpha_9 y} + M_{51} e^{2\alpha_3 y} + M_{52} e^{2\alpha_4 y} + M_{53} e^{\overline{\alpha_8+\alpha_9} y} + M_{54} e^{\overline{\alpha_3+\alpha_8} y} + M_{55} e^{\overline{\alpha_4+\alpha_8} y} + M_{56} e^{\overline{\alpha_3+\alpha_9} y} + M_{57} e^{\overline{\alpha_4+\alpha_9} y} + M_{59} e^{\overline{\alpha_3+\alpha_4} y} \theta_{10} = C_1 e^{\alpha_1 y} + C_2 e^{\alpha_2 y}, \theta_{20} = C_3 e^{\alpha_3 y} + C_4 e^{\alpha_4 y}$$

$$\theta_{11} = C_{13} e^{\alpha_1 y} + C_{14} e^{\alpha_2 y} + M_{25} e^{2\alpha_5 y} + M_{26} e^{2\alpha_6 y} + M_{27} e^{2\alpha_1 y} + M_{28} e^{2\alpha_2 y} + M_{29} e^{\overline{\alpha_1+\alpha_5} y} + M_{30} e^{\overline{\alpha_2+\alpha_5} y} + M_{31} e^{\overline{\alpha_5+\alpha_6} y} + M_{32} e^{\overline{\alpha_1+\alpha_6} y} + M_{33} e^{\overline{\alpha_2+\alpha_6} y} + M_{34} e^{\overline{\alpha_1+\alpha_2} y}$$

$$\theta_{21} = C_{11} e^{\alpha_3 y} + C_{12} e^{\alpha_4 y} + M_{15} e^{2\alpha_8 y} + M_{16} e^{2\alpha_9 y} + M_{17} e^{2\alpha_3 y} + M_{18} e^{2\alpha_4 y} + M_{19} e^{\overline{\alpha_8+\alpha_9} y} + M_{20} e^{\overline{\alpha_3+\alpha_8} y} + M_{21} e^{\overline{\alpha_4+\alpha_8} y} + M_{22} e^{\overline{\alpha_3+\alpha_9} y} + M_{23} e^{\overline{\alpha_4+\alpha_9} y} + M_{24} e^{\overline{\alpha_3+\alpha_4} y}$$

The other constants appeared in the equations or solutions are not presented here for the sake of brevity.

4. RESULTS AND DISCUSSIONS

Free convective flows of two immiscible memory fluids characterized by Walters liquid models have been considered in an inclined channel with energy dissipation.

Shearing stresses (σ_1 & σ_2) and the Nusselt number (Nu_1 & Nu_2) of the two fluids are given by:

$$\sigma_1 = (C_5\alpha_5 + C_6\alpha_6 + M_6\alpha_1 + M_7\alpha_2) + Ec\{C_{15}\alpha_5 + C_{16}\alpha_6 + M_{35}\alpha_1 + M_{36}\alpha_2 + 2M_{37}\alpha_5 + 2M_{38}\alpha_6 + 2M_{39}\alpha_1 + 2M_{40}\alpha_2 + M_{41}(\alpha_1+\alpha_5) + M_{42}(\alpha_2+\alpha_5) + M_{43}(\alpha_5+\alpha_6) + M_{44}(\alpha_1+\alpha_6) + M_{45}(\alpha_2+\alpha_6) + M_{46}(\alpha_1+\alpha_2)\} - e_1[C_5\alpha_5^2 + C_6\alpha_6^2 + M_6\alpha_1^2 + M_7\alpha_2^2 + E\{C_{15}\alpha_5^2 + C_{16}\alpha_6^2 + M_{35}\alpha_1^2 + M_{36}\alpha_2^2 + 4M_{37}\alpha_5^2 + 4M_{38}\alpha_6^2 + 4M_{39}\alpha_1^2 + 4M_{40}\alpha_2^2 + M_{41}(\alpha_1+\alpha_5)^2 + M_{42}(\alpha_2+\alpha_5)^2 + M_{43}(\alpha_5+\alpha_6)^2 + M_{44}(\alpha_1+\alpha_6)^2 + M_{45}(\alpha_2+\alpha_6)^2 + M_{46}(\alpha_1+\alpha_2)^2\}]$$

$$\sigma_2 = (C_8\alpha_3 + C_9\alpha_9 + M_8\alpha_3 + M_9\alpha_4) + Ec\{C_{17}\alpha_8 + C_{18}\alpha_9 + M_{47}\alpha_3 + M_{48}\alpha_4 + 2M_{49}\alpha_8 + 2M_{50}\alpha_9 + 2M_{51}\alpha_3 + 2M_{52}\alpha_4 + M_{53}(\alpha_8+\alpha_9) + M_{54}(\alpha_3+\alpha_8) + M_{55}(\alpha_4+\alpha_8) + M_{56}(\alpha_3+\alpha_9) + M_{57}(\alpha_4+\alpha_9) + M_{58}(\alpha_3+\alpha_4)\} - e_1[C_8\alpha_3^2 + C_6\alpha_6^2 + M_8\alpha_3^2 + M_9\alpha_4^2 + E\{C_{17}\alpha_8^2 + C_{18}\alpha_9^2 + M_{47}\alpha_3^2 + M_{48}\alpha_4^2 + 4M_{49}\alpha_8^2 + 4M_{50}\alpha_9^2 + 4M_{51}\alpha_3^2 + 4M_{52}\alpha_4^2 + M_{53}(\alpha_8+\alpha_9)^2 + M_{54}(\alpha_3+\alpha_8)^2 + M_{55}(\alpha_4+\alpha_8)^2 + M_{56}(\alpha_3+\alpha_9)^2 + M_{57}(\alpha_4+\alpha_9)^2 + M_{58}(\alpha_3+\alpha_4)^2\}]$$

$$Nu_1 = (C_1\alpha_1 + C_2\alpha_2) + Ec\{C_{13}\alpha_1 + C_{14}\alpha_2 + 2M_{25}\alpha_5 + 2M_{26}\alpha_6 + 2M_{27}\alpha_1 + 2M_{28}\alpha_2 + M_{29}(\alpha_5+\alpha_1) + M_{30}(\alpha_5+\alpha_2) + M_{31}(\alpha_5+\alpha_6) + M_{32}(\alpha_6+\alpha_1) + M_{33}(\alpha_6+\alpha_2) + M_{34}(\alpha_1+\alpha_2)\}$$

$$Nu_2 = (C_3\alpha_3 + C_4\alpha_4) + Ec\{C_{11}\alpha_3 + C_{12}\alpha_4 + 2M_{15}\alpha_8 + 2M_{16}\alpha_9 + 2M_{17}\alpha_3 + 2M_{18}\alpha_4 + M_{19}(\alpha_8+\alpha_9) + M_{20}(\alpha_3+\alpha_8) + M_{21}(\alpha_4+\alpha_8) + M_{22}(\alpha_3+\alpha_9) + M_{23}(\alpha_4+\alpha_9) + M_{24}(\alpha_3+\alpha_4)\}$$

Influences of the non-dimensional governing parameters namely visco-elasticity (e_1 & e_2), Magnetic parameter (M_1 & M_2), Reynolds number (Re_1 & Re_2) have been calculated numerically on shearing stresses and rate of heat transfers at the two surfaces. The objective of the present study is to minimize the shearing stress at the surfaces and also to check the parameters responsible for abnormal rate of heat transfer.

Table 1a. Numerical values of shear stresses and Nusselt numbers at the lower surface for M_1=2, M_2=3, Re_1=0.2, Re_2=0.4, Gr_1=10, Gr_2=8, Pr_1=10, Pr_2=12, a=2, m=2, h=5, Q=4, k=0.3, Ec_1=Ec_2=0.01

Cases	σ_1*10^{-5}	Nu_1
e_1=0.1, e_2=0.2	1.94	14.889
e_1=0.5, e_2=0.2	0.206	8.8038
e_1=0.6, e_2=0.2	0.1467	7.8918
e_1=0.7, e_2=0.2	0.1043	6.7895
e_1=0.8, e_2=0.2	0.0742	5.7015

Table 1b. Numerical values of shear stresses and Nusselt numbers at the upper surface for M_1=2, M_2=3, Re_1=0.2, Re_2=0.4, Gr_1=10, Gr_2=8, Pr_1=10, Pr_2=12, a=2, m=2, h=5, Q=4, k=0.3, Ec1=Ec_2=0.01

Cases	σ_2	Nu_2*10^{-3}
e_1=0.1, e_2 =0.2	85.94	1.679
e_1=0.1, e_2=0.3	93.79	2.095
e_1=0.1, e_2=0.5	143.8	2.425
e_1=0.1, e_2=0.6	347.4	2.508
e_1=0.1, e_2=0.8	920	2.606

Tables (1a and 1b) show the influences of visco-elastic parameters on shearing stresses and Nusselt number at the two surfaces of the channel. It can be concluded that shearing experiences a decreasing trend during the enhancement of visco-elasticity of region-I but it experiences an opposite trend during the growth of visco-elasticity of region-II. There is decrease in rate of heat of heat transfer at the surface of the region-I but reverse effect is seen at the upper surface.

Table 2a. Numerical values of shear stresses and Nusselt numbers at the lower surface for e_1=0.1, e_2=0.2, Re_1=0.2, Re_2=0.4, Gr_1=10, Gr_2=8, Pr_1=10, Pr_2=12, a=2, m=2, h=5, Q=4, k=0.3, Ec_1=Ec_2=0.01

Cases	σ_1*10^{-5}	Nu_1
M_1= 0.5, M_2= 3	1.249	11.33
M_1= 1.5, M_2= 3	1.88	14.59
M_1= 2, M_2= 3	1.94	14.88

Table 2b. Numerical values of shear stresses and Nusselt numbers at the upper surface for e_1=0.1, e_2=0.2, Re_1=0.2, Re_2=0.4, Gr_1=10, Gr_2=8, Pr_1=10, Pr_2=12, a=2, m=2, h=5, Q=4, k=0.3, Ec_1=Ec_2=0.01

Cases	Nu_2*10^{-3}	σ_2*10^{-4}
M_1= 2, M_2= 0.5	2.052	2.0504
M_1= 2, M_2= 2	1.696	0.014836
M_1= 2, M_2= 3	1.679	0.0085

Application of transverse magnetic field generates Lorentz force and its effects are exhibited through the non-dimensional parameters M_1 and M_2 respectively. During the enhancement of magnetic parameter, there is an increment of shearing stress or viscous drag at the surface of region I but it is seen that shearing at the region-II experiences a declined trend. During the application of magnetic field, a part of magnetic energy is converted into heat and this affects the rate of heat transfer. It can be concluded from table 2a and 2b that Nusselt number increases with magnetic parameter in the surface of region I but an opposite trend is experienced in the surface of the region-II

Table 3a. Numerical values of shear stresses and Nusselt numbers at the lower surface for e_1=0.1, e_2=0.2, M_1=2, M_2=3, Gr_1=10, Gr_2=8, Pr_1=10, Pr_2=12, a=2, m=2, h=5, Q=4, k=0.3, Ec_1=Ec_2=0.01

Cases	σ_1*10^{-6}	Nu_1
Re_1=0.1, Re_2=0.4	2.0315	34.1081
Re_1=0.2, Re_2=0.4	0.194	14.8849
Re_1=0.5, Re_2=0.4	0.003481	2.1163

Table 3a and 3b state that Increase in the values of the Reynolds number of the fluid in the region-I has a negative impact on the shearing stress and Nusselt number of the two fluids but an opposite effect is seen in case of the increase in the values of Reynolds number in the region-II.

Table 3b. Numerical values of shear stresses and Nusselt numbers at the upper surface for e_1=0.1, e_2=0.2, M_1=2, M_2=3, Gr_1=10, Gr_2=8, Pr_1=10, Pr_2=12, a=2, m=2, h=5, Q=4, k=0.3, Ec_1=Ec_2=0.01

Cases	σ_2	Nu_2*10^{-3}
Re_1=0.2, Re_2=0.2	6.0495	0.083775
Re_1=0.2, Re_2=0.4	85.9442	1.6787
Re_1=0.2, Re_2=0.5	615.1308	7.5091

5. CONCLUSIONS

Hydromagnetic convective flows of two immiscible non-Newtonian fluids through an inclined channel have been studied. Following points are concluded from the above work.

- To avoid damage or wear and tear at the surfaces, shearing stress should be minimized. To do that, visco-elasticity and magnetic parameter should be controlled.
- Visco-elasticity and magnetic parameter enhance the rate of heat transfer.
- Reynolds number helps to reduce the viscous drag at the surface.
- Impacts of visco-elasticity, magnetic parameter and Reynolds number are different at the surfaces of two regions

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NOMENCLATURE

u_1', u_2'	Dimensional velocities
T_1, T_2	Dimensional tempeartures
g	gravitational acceleration, m.s^{-2}
B_0	strength of magnetic field
Nu	local Nusselt number along the heat source
k_{0i}	Visco-elasticities
c_{pi}	Specific heat capacities
k_i	Thermal conductivities
Q_i	External heat agents
M_i	Hartmann number
Re_i	Reynolds number
Ec_i	Eckert number
Pr_i	Prandtl number
Q_{Hi}	Dimensionless external heat agent
e_i	Dimensionless visco-elasticities
Gr_i	Grashof number

Greek symbols

α	Angle of inclination of the surface with vertical
σ_i	electrical conductivities
β_i	Co-efficient of volume expansion
ρ_i	densities
μ	dynamic viscosity, kg. m^{-1}.s^{-1}

Subscripts

i	1, 2