Effect of variable properties on the flow over an exponentially stretching sheet with convective thermal conditions

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ABSTRACT

This article emphasizes on the study of the flow, heat and mass transfer from an exponentially stretching surface in a viscous fluid with heat source. The viscosity and thermal conductivity are assumed to vary as a linear function of temperature. The equations of the flow are converted into ordinary differential equations by utilizing the similarity transformations. The resulting non-linear system is solved applying the Successive linearization method along with the Chebyshev collocation method. The physical quantities of the flow problem are computationally analyzed and exhibited via graphs. It is noticed that the rate of heat transfer increased with raise in Biot and decreased with increase in the value of thermal conductivity and heat source parameters. While, the rate of mass transfer increased with increase in the values of Biot, thermal conductivity and heat source parameters and skin-friction is increasing with viscosity parameter and decreasing with thermal conductivity and heat source parameters.

1. INTRODUCTION

The analysis of flow, heat and mass transfers over a sheet stretching exponentially in a viscous fluid is an important research area due to its applications in industry and technological processes, such as, the boundary layer along a liquid film condensation process, the aerodynamic extrusion of plastic sheets, crystal growth, glass and polymer industries. After the pioneering works of Sakiadis [1, 2], Crane [3] studied the stretched flow problem considering the velocity is proportional to the distance from the slit. Kumaran and Ramaaniah [4] reported the incompressible viscous fluid flow over a stretching sheet subjected to a linear mass flux, considering the stretching velocity as a second-degree polynomial of the distance between the sheet and the slit. Magyari and Keller [5] addressed the heat and mass transfer analysis of boundary layer flow on an exponentially stretching continuous surface. In continuation, several researchers, to mention a few Sajid and Hayat [6], Malvandi et al. [7], Rohni et al. [8], Hussain and Ahmed [9], Ur-rehman et al. [10], Emam and Elmaboud [11], Kumar et al. [12], Aleng et al. [13], Srinivasacharya and Jagadeeshwar [14], Hayat and Nadeem [15] etc. investigated the heat and mass transfer characteristics of an exponentially stretching sheet via diverse physical situations.

In most of the previous studies on the heat and mass transport, the thermo physical properties of fluid were assumed to be constant. However, it is known especially that the fluid viscosity and thermal conductivity may change with temperature. Considering the variation of viscosity is necessary to predict the heat transfer rate accurately. Even though the variation of the physical properties is important in many engineering applications, it has received rather little attention. Lai and Kulacki [16] described the effect of variations in the viscosity on the mixed convection along a vertical plate in a porous medium. Chiam [17-18] investigated this flow problem for stretching sheet assuming linear variations of the thermal conductivity with temperature. Khan et al. [19] analysed the flow and heat transfer in a laminar fluid film on a horizontal shrinking/stretching sheet by considering the variable viscosity and thermal conductivity. Rahman [20] reported the impact of variable viscosity and thermal conductivity on an unsteady laminar incompressible and electrically conducting non-Newtonian fluid flow and heat transfer over a non-isothermal stretching sheet under the external transverse magnetic field in a porous medium. Siddheshwar et al. [21] examined the flow performance and heat exchange of a Newtonian fluid past an exponentially stretching sheet in presence of variable viscosity. Hayat et al. [22] studied the influence of chemical reaction and cross-diffusion effects on the three dimensional flow of a stretching surface with heat generation. Megahed [23] reported the flow and heat transfer analysis of Powell-Eyring fluid due to a sheet stretching exponentially, with heat flux and variable thermal conductivity. Hazarika and Phukan [24] addressed the effects of temperature dependent viscosity and thermal conductivity on MHD flow and heat transfer of a micropolar fluid through a horizontal channel, lower being a stretching sheet and upper being a permeable plate bounded by porous medium. Recently, Srinivasacharya and Jagadeeshwar [25] studied the effect of variable viscosity and thermal conductivity on the flow over an exponentially stretching sheet in presence of Hall effect with thermal convective boundary conditions and heat source.

Hence, motivated by the aforesaid investigations, here we made an effort to analyze the convective boundary layer flow considering the variable fluid properties, heat source over an exponential stretching surface subjected to suction or injection.
2. MATHEMATICAL FORMULATION

Consider a stretching sheet in a laminar slip flow of viscous incompressible fluid with a temperature $T_r$ and concentration $C_r$. The Cartesian framework is selected by taking positive $x$-axis is along the sheet and $y$-axis is orthogonal to the sheet. The stretching velocity of the sheet is assumed as $U(\tilde{x}) = U_0 e^{\gamma \tilde{x}}$, where $\tilde{x}$ the distance from the slit. Assume that the sheet is either cooled or heated convectively through a fluid with temperature $T_f$ and which induces a heat transfer coefficient $h_{r}$, where $h_{r} = h \sqrt{\frac{U_0}{2L}} e^{\gamma L}$ . $(\tilde{u}_x, \tilde{u}_y)$ is the velocity vector, $\tilde{C}$ is the concentration and $\tilde{T}$ is the temperature. The suction/injection velocity of the fluid through the sheet is $V_r(\tilde{x}) = V_0 e^{\gamma L}$ , where $V_0$ is the strength of suction/injection. Further, the heat source is assumed as $Q(\tilde{x}) = Q_0 e^{\gamma L}$, where $Q_0$ is the constant. Hence, the following are the equations which governs the present flow.

\[
\frac{\partial \tilde{u}_x}{\partial \tilde{x}} + \frac{\partial \tilde{u}_y}{\partial \tilde{y}} = 0
\]

\[
\tilde{u}_x \frac{\partial \tilde{u}_y}{\partial \tilde{x}} + \tilde{u}_y \frac{\partial \tilde{u}_y}{\partial \tilde{y}} = \frac{1}{\rho} \frac{\partial}{\partial \tilde{x}} \left( \mu \frac{\partial \tilde{u}_x}{\partial \tilde{y}} \right)
\]

\[
\tilde{u}_x \frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{u}_y \frac{\partial \tilde{T}}{\partial \tilde{y}} = \frac{1}{\rho c_p} \frac{\partial}{\partial \tilde{x}} \left( \kappa \frac{\partial \tilde{T}}{\partial \tilde{y}} \right) + \frac{Q}{\rho c_p} (\tilde{T} - T_{\infty})
\]

\[
\tilde{u}_x \frac{\partial \tilde{C}}{\partial \tilde{x}} + \tilde{u}_y \frac{\partial \tilde{C}}{\partial \tilde{y}} = D \frac{\partial^2 \tilde{C}}{\partial \tilde{y}^2}
\]

\[
\kappa (\tilde{T}) \text{ fluctuates as the linear function of temperature [17] such as}
\]

\[
\frac{1}{\mu} = b(\tilde{T} - T_{\infty}), \kappa = \kappa_{s}(1+\varepsilon) \left( \frac{T - T_{\infty}}{T_{w} - T_{\infty}} \right)
\]

where $T_r = T_{\infty} - 1/\delta$, $b = \delta/\mu$, $b_r$, and $T_r$ are the constants and their values depend on the reference state, $\delta$ is the thermal property of the fluid, $\varepsilon$ is the variable conductivity parameter and $\kappa_{s}$ is the conductivity of the fluid far away from the sheet. The conditions on the surface of the stretching sheet are

\[
\tilde{u}_x = U_r, \tilde{u}_y = -V_r(\tilde{x}), -\kappa \frac{\partial \tilde{T}}{\partial \tilde{y}} = h_r(T_f - \tilde{T}), \tilde{C} = C_r \text{ at } \tilde{y} = 0
\]

\[
\tilde{u}_x \to 0, \tilde{T} \to T_{\infty}, \tilde{C} \to C_{\infty} \text{ as } \tilde{y} \to \infty
\]

Presenting the stream functions through $\tilde{u}_x = \frac{\partial \psi}{\partial \tilde{y}}$ and $\tilde{u}_y = -\frac{\partial \psi}{\partial \tilde{x}}$ and then the following dimensionless variables

\[
y = \tilde{y} \sqrt{\frac{U_0}{2V_r L}}, \quad \psi = \sqrt{2V_r U_0 e^{2\gamma L}} F(x, y),
\]

\[
\tilde{T} = T_{\infty} + (T_f - T_{\infty})T(x, y), \quad \tilde{C} = C_r + (C_r - C_{\infty})C(x, y)
\]

into the Eqs. (1) – (4), we obtain

\[
\left(1 - \frac{T}{\theta_r}\right) F'' + \frac{1}{\theta_r} T F' + \left(1 - \frac{T}{\theta_r}\right)\left( FF'' - 2 F'^2\right) = 0
\]

\[
(1+\varepsilon T)T'' + \varepsilon T^2 + Pr \left(1 - \frac{T}{\theta_r}\right)(1+\varepsilon T)(FT' + q_r T) = 0
\]

\[
C'' + Sc FC' = 0
\]

The corresponding conditions on the boundary are

\[
F(0) = S, \quad F'(0) = 1, \quad T'(0) = -B_i(1 - T(0)) \text{, } C(0) = 1 \text{ at } y = 0 \quad F(\infty) \to 0, \quad T(\infty) \to 0, \quad C(\infty) \to 0 \text{ as } y \to \infty
\]

where $v_{sc}$ is the kinematic viscosity of the fluid in the free stream, the prime denotes derivative with respect to $y$, $\theta_r = -1/\delta(T_f - T_{\infty})$ is the fluid viscosity parameter, $B_i = \frac{h}{\kappa} \sqrt{V_r}$ is the Biot number, $S = V_0 \sqrt{\frac{2L}{V_r U_0}}$ is the suction/injection parameter according as $S > 0$ or $S < 0$ respectively, $Sc = v_{sc}/D$ is the Schmidt number, $q_r = 2LQ_0/\rho c_p U_0$ is the internal heat source/sink and $Pr = V/\alpha$ is the Prandtl number.

\[
\text{Figure 1. Physical model and coordinate system}
\]
The non-dimensional skin friction \( \overline{C_f} = \frac{2\tau_w}{\rho U_\infty^2} \), the local Nusselt number \( \overline{Nu}_l = \frac{\overline{q}_w}{\kappa (T_l - T_\infty)} \), and local Sherwood number \( \overline{Sh}_l = \frac{\overline{q}_w}{\kappa (C_l - C_\infty)} \), are given by

\[
\overline{Nu}_l = \sqrt{\frac{\kappa}{2\ell}} \overline{C_f} C \left( \frac{\overline{q}_w}{\kappa (T_l - T_\infty)} \right), \quad \overline{Sh}_l = \sqrt{\frac{\kappa}{2\ell}} \overline{C_f} C \left( \frac{\overline{q}_w}{\kappa (C_l - C_\infty)} \right)
\]

where \( \overline{q}_w = \frac{\kappa}{2\ell} \overline{C_f} C \) is the skin friction coefficient.

3. NUMERICAL SOLUTION

The system of Eqs. (8) to (10) is linearized using successive linearisation method (SLM) ([26, 27]). In this method, the functions \( F(y), T(y) \) and \( C(y) \) are expressed as

\[
F(y) = F_i(y) + \sum_{i=0}^{\infty} F_i(y), T(y) = T_i(y) + \sum_{i=0}^{\infty} T_i(y), C(y) = C_i(y) + \sum_{i=0}^{\infty} C_i(y)
\]

where \( F_i(y), T_i(y) \) and \( C_i(y) \) \((r = 1, 2, 3, \ldots)\) are functions, which are not known and \( F_i(y), T_i(y) \) and \( C_i(y) \) \((r > 1)\) are approximations. Substituting Eq. (13) in Eqs. (8) to (10) and taking the linear part, we get

\[
\chi_{11,j-1} F_i'' + \chi_{12,j-1} F_i' + \chi_{13,j-1} F_i + \chi_{14,j-1} T_i'' + \chi_{15,j-1} T_i' + \chi_{16,j-1} T_i = \zeta_{1,j-1}
\]

\[
\chi_{21,j-1} F_i'' + \chi_{22,j-1} T_i'' + \chi_{23,j-1} T_i' + \chi_{24,j-1} T_i = \zeta_{2,j-1}
\]

\[
\chi_{31,j-1} F_i' + \chi_{32,j-1} C_i' = \zeta_{3,j-1}
\]

where the coefficients \( \chi_{ik,r+1} \) and \( \zeta_{k,j-1} \), \((l = 1, 2, 3 \text{ and } k = 1, 2, 3, 4, 5, 6)\) are in terms of the approximations \( F_i, T_i \) and \( C_i \), \((i = 1, 2, 3, \ldots, r - 1)\) and their derivatives.

The boundary associated conditions are

\[
F_i(0) = F_i'(0) = F_i'(\infty) = T_i'(0) = T_i'(\infty) = C_i(0) = C_i(\infty) = 0
\]

Choosing the initial approximation \( F_0(y), T_0(y) \) and \( C_0(y) \) satisfy the conditions (11) and solving Eqs. (14) to (16) recursively, we get the solutions for \( F_i(y), T_i(y) \) and \( C_i(y) \) \((r > 1)\) and hence, \( F_i(y), T_i(y) \) and \( C_i(y) \). To solve Eqs. (14) to (16) along with the boundary conditions (17), Chebychev spectral method is used (see for reference [28]).

4. RESULTS AND DISCUSSIONS

To elucidate the significance of relevant parameters, the numerical calculations are carried out by taking \( S = 0.5, q_1 = 0.1, \ e = 0.1, \ Sc = 0.22, \ Pr = 1.0, \ q_0 = 3.0, \ Bi = 1.0, \ N = 100 \) and \( L = 20 \) unless otherwise mentioned.
The variation of velocity profile for diverse values of $\theta_r$, $q_1$, $Bi$ and $S$ is presented in the Figs. (2a) – (2d). It is obvious that fluid velocity decreasing with enhancement in the value of $\theta_r$ as shown in the Fig. (2a). This is due to the fact that, viscosity of the fluid thickens the boundary layer and hence, reduction in fluid velocity. Heat source parameter $q_1$ has almost negligible influence on velocity. Figure (2b) depicts that the velocity is increasing, but negligibly, as the value of $q_1$ is increasing. Biot number $Bi$ has considerable effect on fluid velocity as shown in the Fig. (2c). It is seen that velocity is enhances with raise in the value of $Bi$. From Fig. (2d), it is clear that velocity rising with injection ($S < 0$) and reducing with suction ($S > 0$). This is due to the fact that wall suction has the tendency to reduce the momentum boundary.

The effect of the parameters $\varepsilon$, $q_1$, $Bi$ and $S$ on temperature is shown in the Figs. (3a) – (3d). It is observed that temperature increases as the value of thermal conductivity parameter $\varepsilon$ increases as shown in the Fig. (3a). Heat source $(q_1 > 0)$ in the boundary layer generates energy which causes the temperature to increase, while the presence of heat sink $(q_1 < 0)$ in the boundary layer absorbs the energy which causes the temperature to decrease. Therefore, increase in the values of heat source parameter increases the temperature as shown in the Fig. (3b). On the other hand, Biot number increases the temperature as shown in the Fig. (3c). Further, for large value of Biot number, the convective thermal condition from (11) transforms to $T(0) \rightarrow 1$, which signifies the constant wall condition. i.e., the stronger convection leads to the higher surface temperatures which appreciably increases the temperature and the thermal boundary layer thickness. Reduction in temperature is observed with increase in suction and enhanced with blowing as shown in the Fig. (3d). This is due to the fact that the wall suction, reduces thermal boundary layer thickness.

The behaviour of concentration profile for various values of the parameters $\theta_r$, $q_1$, $Bi$ and $S$ is depicted in the Figs. (4a) – (4d). Raising the value of $\theta_r$, concentration of the fluid is increasing as shown in the Fig. (4a). It is noticed from the Figs. (4b) and (4c) that increase in the values of $q_1$ and $Bi$, increases the concentration. It is also noticed that the effect of heat source parameter $q_1$ on concentration is almost negligible. On the other hand, from the Fig. (4d) increase in the value of suction/injection parameter $S$ reduces the concentration of the fluid. This is because of the fact that the wall suction, reduces the concentration boundary layer thickness. Therefore, the concentration of the fluid decreases with suction and increases with injection as depicted in the Fig. (4d).

**Figure 3.** Influence of (a) $\varepsilon$, (b) $q_1$, (c) $Bi$ and (d) $S$ on $T$
Figure 4. Influence of (a) $\theta_r$, (b) $q_1$, (c) $Bi$ and (d) $S$ on $C$

Figure 5. Influence of (a) $\theta_r$, (b) $Bi$, (c) $q_1$ and (d) $\varepsilon$ on $(\frac{\theta_r}{\theta_r - \theta(0)}) F''(0)$

The variation of skin-friction coefficient with varying values of $\theta_r$, $Bi$, $q_1$ and $\varepsilon$ against $S$ is presented in the Figs. (5a) - (5d). It is evident from the Fig. (5a) that increase in the value of viscosity parameter increases the skin-friction. Hence, decrease in the fluid velocity. Increase in the value of $Bi$, diminishing the skin-friction and increases the fluid velocity in the boundary layer as shown in the Fig. (5b). While, there is negligible effect of $q_1$ and $\varepsilon$ on skin-friction as depicted in the Figs. (5c) and (5d). It is obvious from these figures that skin-friction reducing very slightly with increase in the value of heat source and thermal conductivity parameters. Further, due to wall suction the momentum boundary reduces and hence, skin-friction is reducing with increasing in the value of $S$.

Behaviour of rate of heat transfer for several values of $\theta_r$, $Bi$, $q_1$ and $\varepsilon$ is shown in the Fig. (5a) - (5d). It is evident from the Fig. (5a) that increase in the value of viscosity parameter increases the skin-friction. Hence, decrease in the fluid velocity. Increase in the value of $Bi$, diminishing the skin-friction and increases the fluid velocity in the boundary layer as shown in the Fig. (5b). While, there is negligible effect of $q_1$ and $\varepsilon$ on skin-friction as depicted in the Figs. (5c) and (5d). It is obvious from these figures that skin-friction reducing very slightly with increase in the value of heat source and thermal conductivity parameters. Further, due to wall suction the momentum boundary reduces and hence, skin-friction is reducing with increasing in the value of $S$.
Bi, \( q_1 \) and \( \varepsilon \) against \( S \) is portrayed in the Figs. (6a) – (6d). The rate of heat transfer from the sheet to the fluid is diminishing with raise in \( \theta_r \) as shown in the Fig. (6a). Further, it is noticed that the trend is reversed. i.e., heat transfer from the sheet to the fluid is increasing with increase in the values of \( S \) and \( \theta_r \). Figure (6b) narrates that the raise in Bi enhances the rate of heat transfer from the sheet to the fluid. Increase in the value of the heat generation parameter leads to a decrease in the local Nusselt number. This is because the heat generation mechanism will increase the fluid temperature near the surface and thus temperature gradient at the surface decreases, thereby decreasing the heat transfer at the sheet as shown in the Fig. (6c). On the other hand, rate of heat transfer reduced with raise in the value of \( \varepsilon \) as shown in the Fig. (6d). While, it is clear from the figures that the rate of heat transfer increasing with wall suction. Further, the wall suction has the tendency to cut down the thermal boundary, and hence, maximum heat transfer at the surface of the boundary as shown in the Figs. (6a) – (6d).

For distinct values \( \theta_r \), Bi, \( q_1 \) and \( \varepsilon \), the fluctuation of rate of mass transfer is graphitised against \( S \) through the Figs. (7a) – (7d). Increasing the value of viscosity parameter \( \theta_r \), the rate of mass transfer is diminishing as shown in the Fig. (7a). Whereas, from the Fig. (7b), it is obvious that the rate of mass transfer is increasing with raise in Bi. On the other hand, there's mild effect of heat source and thermal conductivity parameters on the rate of mass transfer as depicted in the Figs. (7c) and (7d). It is obvious from these figures that the rate of mass transfer is slightly enhanced with raise in \( q_1 \) and \( \varepsilon \). Further, it is noticed that the wall suction increases the rate of mass transfer as it reduces the concentration boundary layer thickness.
The significance of variable viscosity and thermal conductivity on the flow over a sheet stretching exponentially, in presence of heat source is studied. Successive linearization procedure along with the Chebyshev spectral method is used to solve the governing equations.

- The velocity is increasing with increase in the values of heat source parameter and Biot number. But, decreasing with variable viscosity and suction parameters.
- Temperature of the fluid enhances, raising the values of thermal conductivity, heat source parameters and Biot number and decreases with suction parameter.
- Concentration of the fluid increases with increase in viscosity parameter and decreasing with heat source, suction parameters and Biot number.
- Skin-friction is increasing with viscosity parameter and decreasing with thermal conductivity, heat source, suction parameters and Biot number.
- The rate of heat transfer increased with a raise in $Bi$ and decreased with increase in the value of thermal conductivity and heat source parameters. But, dual nature is observed with increase in the value of viscosity parameter.

The rate of mass transfer is reduced with raise in the value of viscosity parameter and increased with increase in the values of $Bi$, thermal conductivity and heat source parameters.

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