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MHD Falkner-skan Flow through Porous Medium Over Permeable Surface

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Abstract

In this paper, we study the magnetohydrodynamic Falkner-Skan flow of an electrically conducting viscous fluid over a permeable plate embedded in a porous medium with uniform porous matrix. The approximate solutions for the boundary layer equations of the flow are presented by two methods (i) DTM-Pade for obtaining analytical solution, (ii) Runge-Kutta Method for numerical solution. The results are compared with earlier works (i) without magnetic field, (ii) without magnetic field and porous medium. It is observed that the results of the present study are in good agreement with previous works in some particular cases establishing the generality of the present study as well as consistency and accuracy of the methods applied to Darcian MHD boundary layer wedge flow. From the flow analysis it is suggested that application of magnetic field and embedding the wedge surface in a porous matrix contribute to stream lined flow. Further, the progressive DTM-Pade i.e. higher order Pade yields better result and therefore an effective method for solution of non-linear boundary value problem.

Key words

Falkner-skan, DTM-pade, Runge-kutta, MHD, Wedge flow, Suction/injection.

1. Introduction

There are some fluids which do not obey the Newton's law of viscosity; these fluiods are characterized as the Non-Newtonian fluids. Study of Non-Newtonian fluids has numerous scopes in industrial, engineering and biological applications. Rajagopal et al. [1] has studied the Falkner-

Skan flows of a homogeneous incompressible fluid f second grade past a wedge placed symmetrically with respect to the flow direction. Their study includes the flow past a flat plate and the flow near a stagnation point as special cases. In recent years a large number of investigations dealing with the Falkner-Skan problem under various aspects have been discussed extensively for viscous fluid [2,3,4]. Olagunju [5] considered the flow problem for viscoelastic fluid. Massoudi and Ramezan [6] discussed the effect of injection or suction on the Falkner-Skan flows of second grade fluids. Yao [7] also presented approximate analytical solution to the Falkner-Skan wedge flow with the permeable wall of uniform suction.

Moreover, Abbasbandy and Hayat [8] suggested a series solution for the Falkner-Skan wedge flow. They solved the nonlinear governing equations by homotopy analysis method. Ishak et al. [9] considered the steady MHD boundary layer flow of a conducting fluid subject to a variable transverse magnetic field along a moving wedge. More recently Robert and Vajravelue [10], Parand et al. [11] and Abbasbandy et al. [12] considered the Falkner-Skan MHD flow of Oldroyd-B fluid. The above-mentioned authors have not considered the flow through porous medium. Kim [13] investigated the flow of power-law fluid through porous medium.

Now coming to method of solution, the Differential Transform Method (DTM) is one of the effective and reliable numerical method for handling both linear and nonlinear differential equations. Zhao [14] first introduced an iterative procedure for analytical solution of ordinary or partial differential equations in the form of a polynomial. The advantages of DTM are high accuracy and minimal calculations. It can be applied directly to nonlinear differential equation in physics and mathematics without requiring linearization. DTM has some drawbacks, it is valid in a small region but in an unbounded domain it is not valid. This is because by using DTM, we obtain a series solution which is divergent when the variable of the problem goes to infinity. For this reason a combination of the DTM and the Pade approximation [15-18] has been used to obtain approximated solutions.

The objective of the present study is two-fold. The study of wedge flow through porous media subjected to magnetic field leads to a modified Falkner-Skan equation. Secondly to examine the suitability of the methods (i) Numerical method (R-K method) (ii) DTM-Pade approximants analytical method. In the course of discussion, the effects of magnetic field, permeability of the medium, suction/injection at the plate and wedge angle are shown and the comparison of the works of previous authors [19] are made.

2. Governing Equations

In this section we consider a steady MHD boundary layer flow of a viscous incompressible electrically conducting fluid over a permeable wall through a uniform porous medium. The Darcy model has been applied to account for the permeability of the medium. The magnetic field acts transversely to the flow. The magnetic Reynolds number is considered to be small so that the induced Magnetic field is negligible. The strength of the electric field, due to polarization of the electric charges is to be negligibly small. $U(x)=ax^m$ is the velocity of the exterior flow over the wedge, $K'_p(x)=k'_p x^{1-m}$ is the porosity parameter, $B(x)=B_0 x^{(m-1)/2}$ is the magnetic field. Using these assumptions, the boundary layer equations are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(x)}{\rho}(u-U)-\frac{\nu(u-U)}{K_p'} \tag{2}$$

The corresponding boundary conditions are given by

$$\left.\begin{array}{l} u(x,0)=0 \\ v(x,0)=v_w \\ u(x,\infty)=U(x) \end{array}\right\} \tag{3}$$

where $v_w=v_o x^{(m-1)/2}$ is the suction or blowing velocity across the surface of the boundary wall as $v_w<0$ for suction and $v_w>0$ for blowing. To examine the boundary layer flow adjacent to the wall, the following transformations are used.

$$\eta=y\left[\frac{(m+1)U}{2\nu x}\right]^{1/2} \quad \psi=f(\eta)\left(\frac{2\nu x U}{m+1}\right)^{1/2} \tag{4}$$

The velocity components

$$u=\partial\psi\big/\partial y, \quad v=\partial\psi\big/\partial x \tag{5}$$

and hence

$$u = f'(\eta)U \quad , \qquad v = -\left[f(\eta)\left(\frac{\upsilon(m+1)U}{2x}\right)^{1/2} + f'(\eta)y\left(\frac{m-1}{2x}\right)U \right] \tag{6}$$

With the help of equation (6), equation (2) and the boundary conditions (3) are transformed to

$$\begin{aligned} & f'''(\eta) + f(\eta)f''(\eta) + \beta\left(1 - f'^{2}(\eta)\right) \\ & -\left(M^{2} + 1/K_{p}\right)\left(f'(\eta) - 1\right) = 0 \end{aligned} \tag{7}$$

and $f'(0) = 0,\ f(0) = -C,\ f'(\infty) = 1$ (8)

where

$$M^{2} = \frac{2\sigma B_{0}^{2}}{\rho a(1+m)}, \quad \frac{1}{K_{p}} = \frac{2\upsilon}{K_{p}{}'' a(1+m)},$$
$$C = v_{0}\left(\frac{2}{(m+1)\upsilon a}\right)^{1/2} \quad and \quad \beta = \frac{2m}{1+m}.$$

C is positive for blowing and negative for suction. The flow is divergent for $\beta_{>0}$ and is convergent for $\beta_{<0}$.

3. Approximate Solution by Using Dtm-pade

Differential transformation method (DTM) based on Taylor expansion. This method tries to find coefficients of series expansion of unknown function by using the initial data on the problem. Basic definitions and operations of differential transformation are introduced as follows.

Definition 1.

The one dimensional differential transform of a function *y(x)* at the point $x=x_o$ is defined as follows:

$$Y(k) = \frac{1}{k!}\left[\frac{d^{k}}{dx^{k}} y(x)\right]_{x=x_0} \tag{9}$$

where *y(x)* is the original function and *Y(k)* is the transformed function.

Definition 2.

The differential inverse transform of $Y(k)$ is defined as follows:

$$y(x)=\sum_{k=0}^{\infty}Y(k)(x-x_0)^k \approx \sum_{k=0}^{n}Y(k)(x-x_0)^k \tag{10}$$

The differential transform of functions are shown in the following table.

Original Functions	**Transformed functions**
$f(x)=\alpha g(x)\pm\beta h(x)$	$F(k)=\alpha G(k)\pm\beta H(k).$
$f(x)=\dfrac{d^n g(x)}{dx^n}$	$F(k)=\dfrac{(k+m)!}{k!}G(k+m)=(k+1)(k+2).....(k+n)G(k+m)$
$f(x)=g(x)h(x)$	$F(k)=G(k)\otimes H(k)=\sum_{k_1=0}^{k}G(k_1)H(k-k_1)$
$f(x)=g(x)\dfrac{d^2h(x)}{dx^2}$	$F(k)=G(k)\otimes P(k)=\sum_{r=0}^{k}(k-r+1)(k-r+2)F(r)P(k-r+2)$
$f(x)=(x-x_0)^n$	$F(k)=\delta(k-n)=\begin{cases}1, & k=n\\ 0, & k\neq n.\end{cases}$

The pade approximant is based on the notion of rational approximation for functions. The function *f(x)* is to be approximated over a small portion of its domain [*a*,*b*]. Then this can be used to compute for any value of x that lies outside the interval [*a*,*b*].

A rational approximation to *f(x)* on the interval [*a*,*b*] is the quotient of two polynomials $P_N(x)$ and $Q_M(x)$ of degrees *N* and *M* respectively. We use the notation $R_{N,M}(x)$ to denote this quotient:

$$R_{N,M}(x)=\frac{P_N(x)}{Q_M(x)} \quad for \quad a\le x\le b \tag{11}$$

The method of Padé requires that *f(x)* and its derivatives are continuous at $x=0$. There are two reasons for the arbitrary choice of $x=0$. First, it makes the manipulations simpler. Second, a change of variable can be used to shift the calculations over to an interval that contains zero. The polynomials used in (11) are

$$P_N(x) = p_0 + p_1 x^1 + p_2 x^2 + \cdots + p_N x^N \tag{12}$$

$$Q_M(x) = 1 + q_1 x^1 + q_2 x^2 + \cdots + q_M x^M \tag{13}$$

The polynomials (12) and (13) are constructed so that $f(x)$ and $R_{N,M}(x)$ agree at $x=0$ and their derivatives up to $N+M$ agree at x=0. In the case $Q_0(x)$=1, the approximation is just the Maclaurin expansion for $f(x)$. For a fixed value of $N+M$ the error is smallest when $P_N(x)$ and $Q_M(x)$ have the same degree or when $P_N(x)$ has degree one higher than $Q_M(x)$ [20].

We should implement the differential transform for equation (7) and the following iterative formula can be obtained using the fundamental operation of the DTM

$$\begin{aligned}F(k+3) &= \frac{1}{(k+1)(k+2)(k+3)}((M^2+\frac{1}{K_p})(k+1)F(k+1)\\ &-(\beta+M^2+\frac{1}{K_p})\delta(k)-\sum_{i=0}^{k}((k+1-i)(k+2-i)F(i)F(k+2-i)\\ &-\beta(i+1)(k+1-i)F(i+1)F(k+1-i))).\end{aligned} \tag{14}$$

where $\delta(k)$ is the Kroneker delta.

Here, $f'(0)$ is not known. In order to find the numerical value of $f'(0)$, which is the skin friction coefficient, the series obtained by the DTM and the diagonal Pade approximants of different degrees are to be combined. Let $f'(0)=2\alpha$. Now equation (7) subjected to the following initial conditions is solved by using DTM

$$f'(0)=0, \quad f(0)=-C, \quad f''(0)=2\alpha \tag{15}$$

The DTM of (15) is as follows

$$F(0)=-C, \quad F(1)=0, \quad F(2)=\alpha. \tag{16}$$

Substituting equation (16) into the iterative formula (14) and taking different values of k, we have

$$F(3) = \frac{C\alpha}{3} - \frac{1}{6}\left(M^2 + \frac{1}{K_p} + \beta \right),$$

$$F(4) = \frac{\alpha}{12}\left(M^2 + \frac{1}{K_p} + C^2 \right) - \frac{C}{24}\left(M^2 + \beta \right),$$

$$F(5) = \frac{\alpha^2}{15}\left(\beta - \frac{1}{2} \right) + \frac{\alpha}{30}\left(CM^2 + \frac{C^3}{2} \right)$$
$$- \frac{1}{120}(M^4 + M^2\beta + M^2C^2 + C^2\beta)$$

Using all the terms of *F(x)* we can get the solution to the initial value problems (7) and (15) in power series form,

$$f(\eta) = \sum_{k=0}^{\infty} F(k)\eta^k \approx \sum_{k=0}^{n} F(k)\eta^k \tag{17}$$

and

$$f'(\eta) = \sum_{i=0}^{2N} (i+1)F(i+1)\eta^i = \frac{\sum_{i=0}^{N} p_i\eta^i}{1 + \sum_{i=0}^{N} q_i\eta^i} \tag{18}$$

Now, taking N=2 and using the power series and the Pade approximant of $f(\eta)$, we get the solution of equations (7) with boundary conditions (8). After obtaining $f'(0)$ we compute $f(\eta)$ and $f'(\eta)$ for various values of the parameters β, C, M and K_P. For example, the Pade approximants to $f(\eta)$ and $f'(\eta)$ for β=4/3, C=1/5, M=3 and K_P =0.5 are as follows:

$$f(\eta) = \frac{-0.2 - 0.2127\eta + 1.7654\eta^2}{1 + 1.0634\eta + 0.2674\eta^2} \tag{19}$$

$$f'(\eta) = \frac{3.6378\eta + 1.4569\eta^2}{1 + 1.9957\eta + 1.4564\eta^2} \tag{20}$$

4. Results and Discussion

The effects of the emerging parameter characterizing the wedge flow in the presence of magnetic field and porous matrix are detailed in the following lines. From governing equation (7) we can discuss the following cases

(i) $M = 0, K_p \to \infty$ (Falkner-Skan equation with modified boundary conditions).

(ii) $M \neq 0, K_p \to \infty$ [19].

Case (i) represents the wedge flow of a non-conducting viscous flow in the absence of magnetic field and without porous medium. Case (ii) represents the case of conducting fluid flow without porous medium. The equation (7) with the boundary conditions (8) has been solved by an approximate analytical method DTM-Pade and Numerical method, R-K Method with Shooting, a self corrective procedure. The consistency of the solution of the governing equation (7) by DTM-Pade is evident from (19) and (20) as the expressions satisfy the boundary conditions (8).

The dimensionless stream functions *f(η)* and velocity *f(η)* are presented in the Figures 1 to 9. It is interesting to note that Figsures 1(a) and (b) (curve-IV*) without porous medium coincides with Figure 1 of [19]. Thus, the present result agrees with [19] in a particular case. Moreover, the results of DTM-Pade and Numerical methods in the present study are found to be in good agreement.

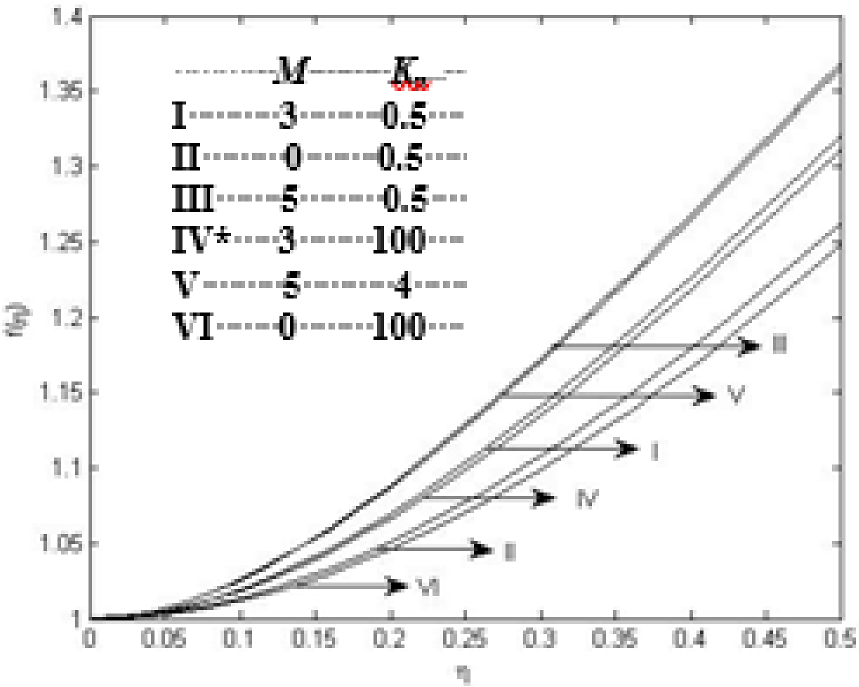

Fig.1(a). Stream Function for β=*4/3* and *C=-1* (DTM-Pade)

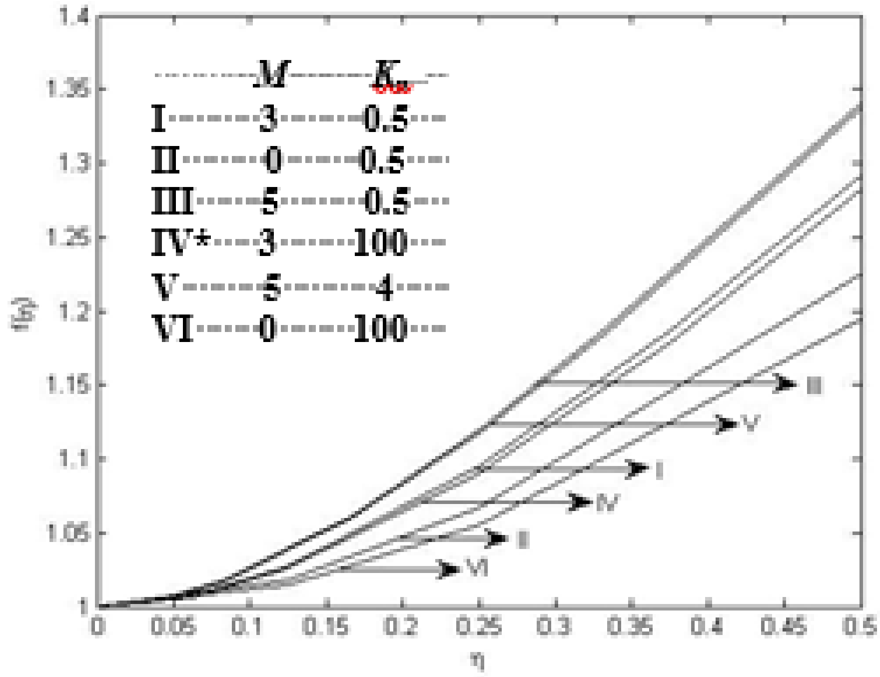

Fig.1(b). Stream Function for β=*4/3* and *C=-1* (R-K Method)

We now proceed to discuss the flow characteristics of various physical parameters such as magnetic field strength, permeability of the medium and wedge angle. Figures 1(a), 2, 3 and 4 exhibit the graphical representations of *f*, the stream function, obtained by DTM-Pade. Their counter parts obtained by R-K method. But in order to save space we have presented Figure 1(b) only. It is seen that the values of *f* computed by both the methods for various values of the magnetic parameter *M*, wedge angle *β*, permeability parameter K_P and suction/blowing parameter *C* are in good agreement. Further, it is seen that *f* increases with *M* but decreases slightly in the presence of suction/blowing. Thus, thinning of boundary layer occurs due to suction which is a standard practice for controlling boundary layer.

Figures 5(a) to 7(a) show the velocity distribution for different values of *M, β,* K_P and *C*. Their counter parts by R-K Method are exhibited through Figures 5(b) to 7(b). From Figures 5(a) and (b), it is seen that the velocity increases with an increase in magnetic parameter but the presence of porous matrix reduces it at all points in the presence of injection at the surface. On careful study of velocity profiles by DTM-Pade and R-K method, it is revealed that convergence of DTM-Pade is delayed in comparisons with R-K method, as the fluid flow attains ambient state latter in case of DTM-Pade.

Figures 6(a) and (b) are devoted to the case of blowing (C>0) as well as converging flow (β<0). It is seen that absence of magnetic field accelerates the attainment of ambient state irrespective of the presence/absence of porous matrix. This is true for both the methods (DTM-

Pade and R-K method). Figures 6(a) and (b) show that for negative value of β the velocity shoots up near the plate in the presence of blowing. From the R-K method (Figure 6(b)) it is clear that slightly flow instability is marked near the plate in the absence of magnetic field (M=0), i.e. a point of inflexion appears at $\eta \approx 0.2$.

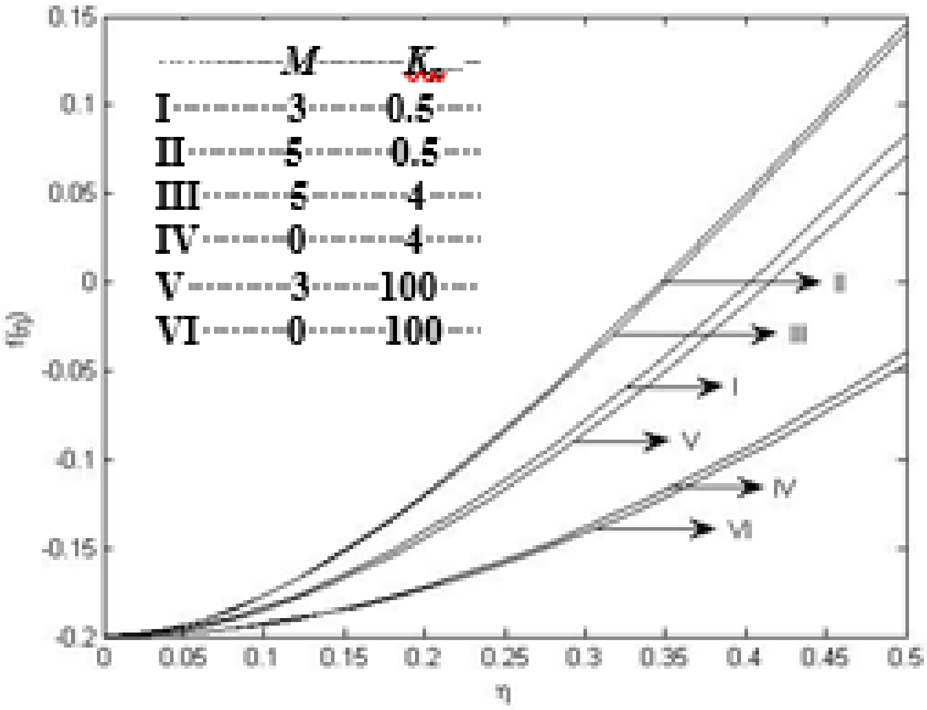

Fig.2. Stream Function for β=*4/3* and *C=1/5* (DTM-Pade)

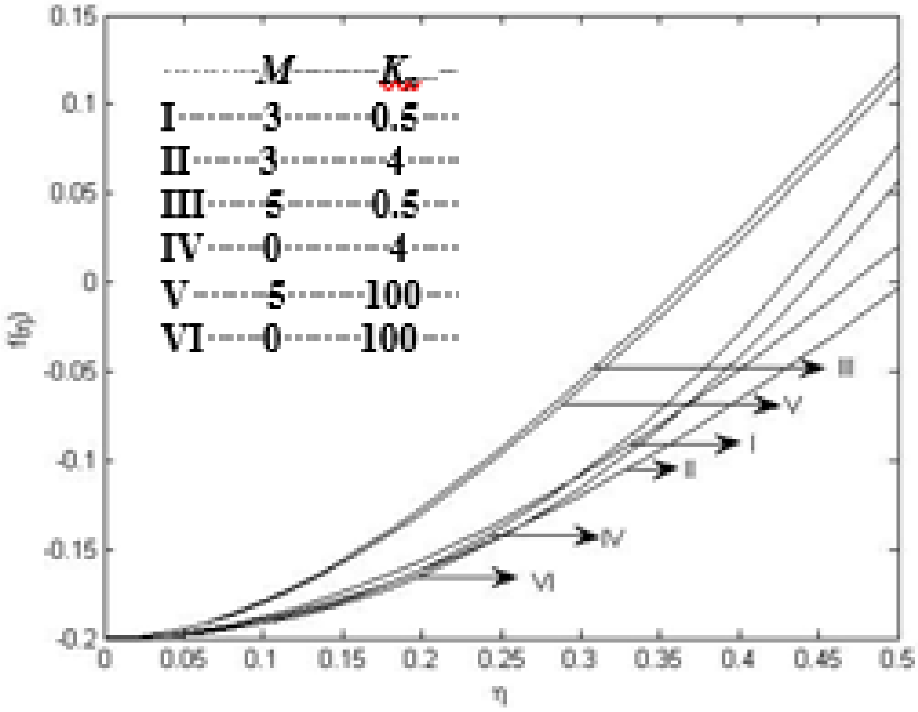

Fig.3. Stream Function for β=*-3* and *C=1/5* (DTM-Pade)

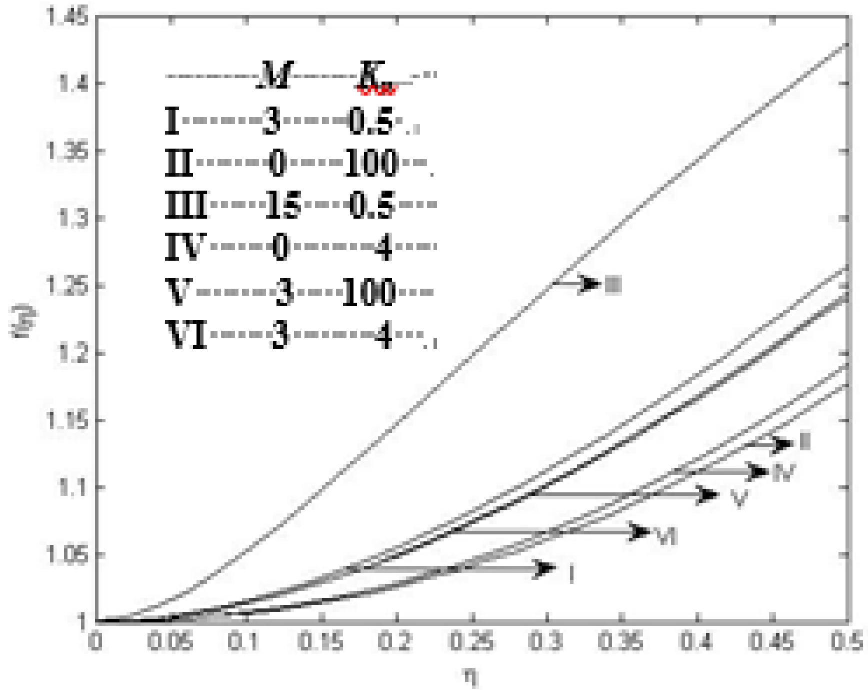

Fig.4. Stream Function for β=*-3* and *C=-1* (DTM-Pade)

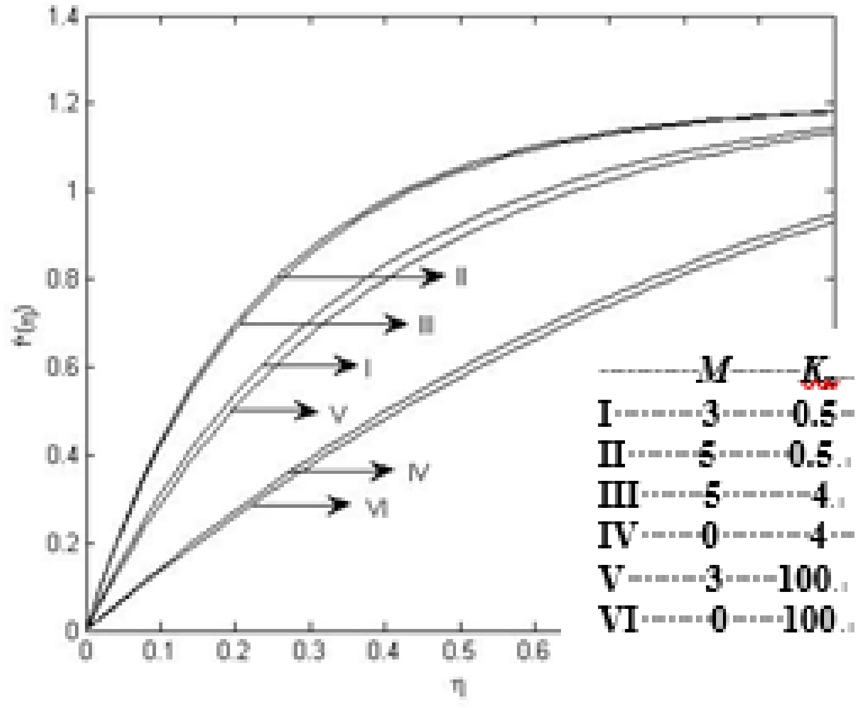

Fig.5(a). Velocity profile for β*=4/3* and *C=1/5* (DTM-Pade)

Figures 7(a) and (b) show the similar velocity profiles for suction at the plate. On careful analysis it is remarked that for negative wedge angle (converging flow) velocity distribution is not that smooth as in case of positive wedge angle (diverging flow).

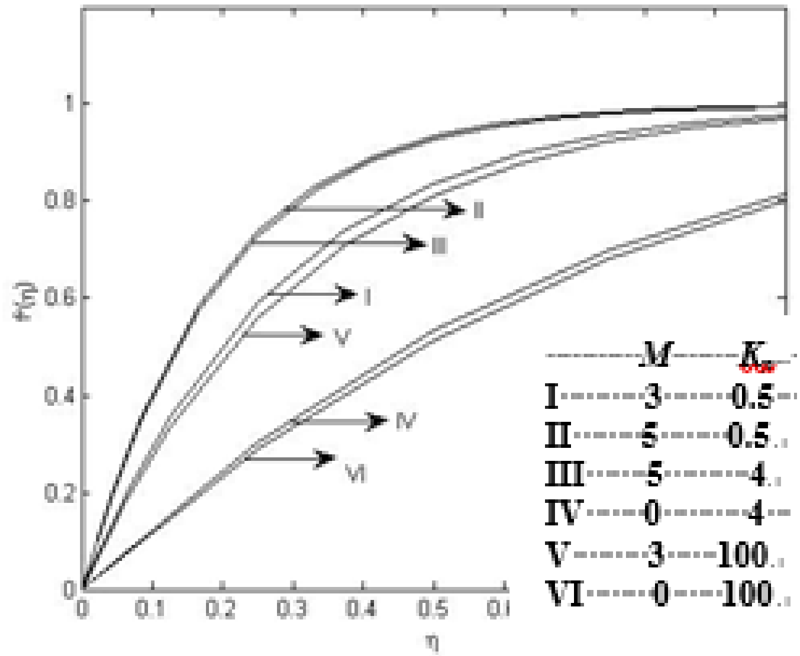

Fig.5(b). Velocity profile for β=*4/3* and *C=1/5* (R-K Method)

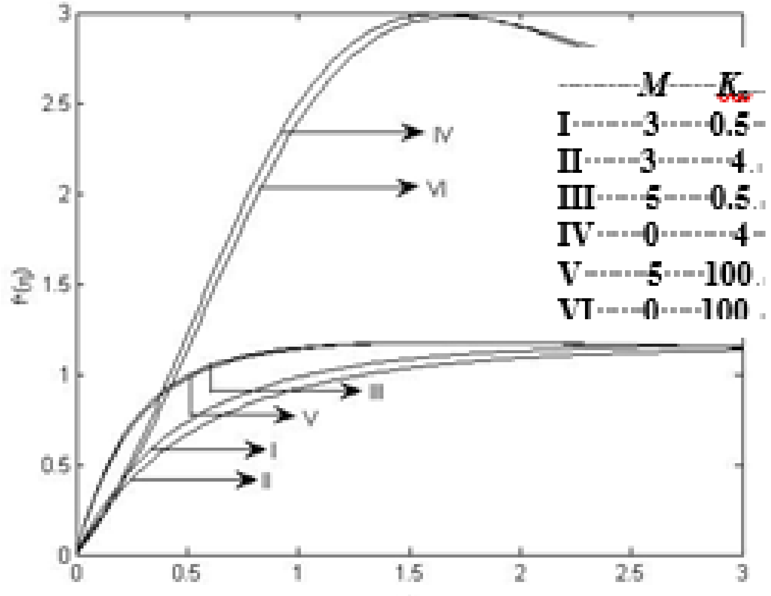

Fig.6(a). Velocity Profile for β=-*3* and *C=1/5* (DTM-Pade)

One striking feature of the velocity profiles IV and VI in Figure 6(a) (case of blowing); II and IV in Figure 7(a) (case of suction); is that in the absence of magnetic field i.e. M=0 the velocity increases sharply in the layers close to the plate and attains the ambient state far off.

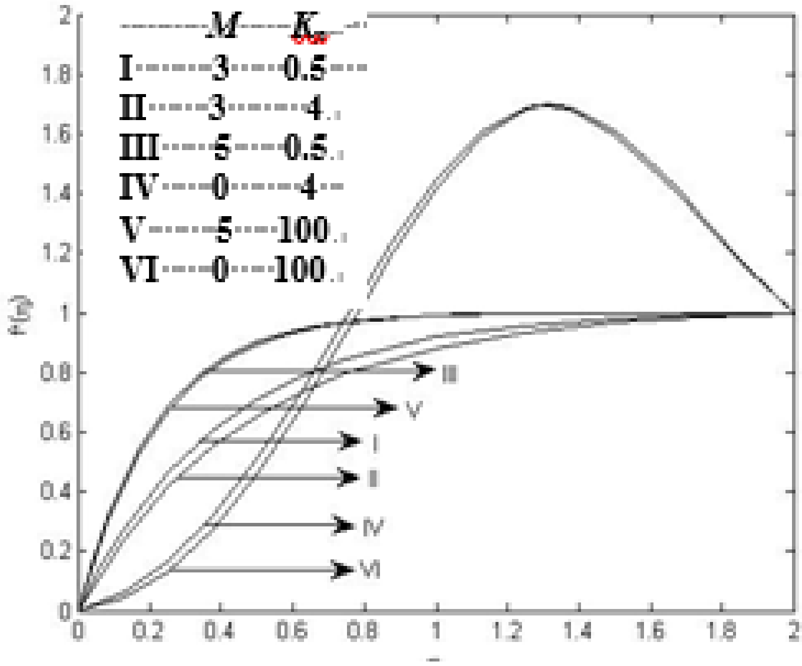

Fig.6(b). Velocity Profile for β=-*3* and *C=1/5* (R=K Method)

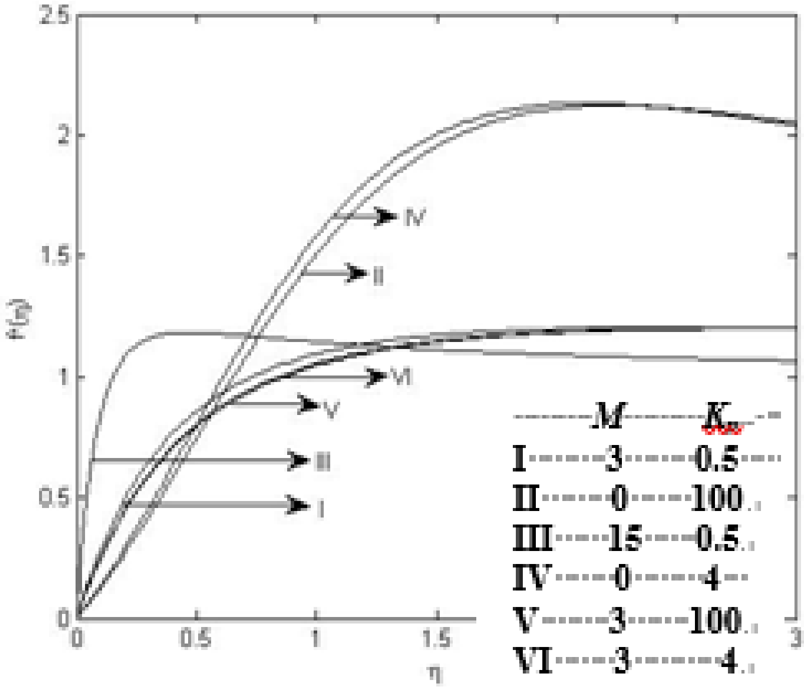

Fig.7(a). Velocity Profile for β=-*3* and *C=-1* (DTM-Pade)

Thus, it is concluded that the presence of magnetic field imbibes the resistive electromagnetic force which controls the sharp rise and imposes the stability of flow in both converging and diverging flow.

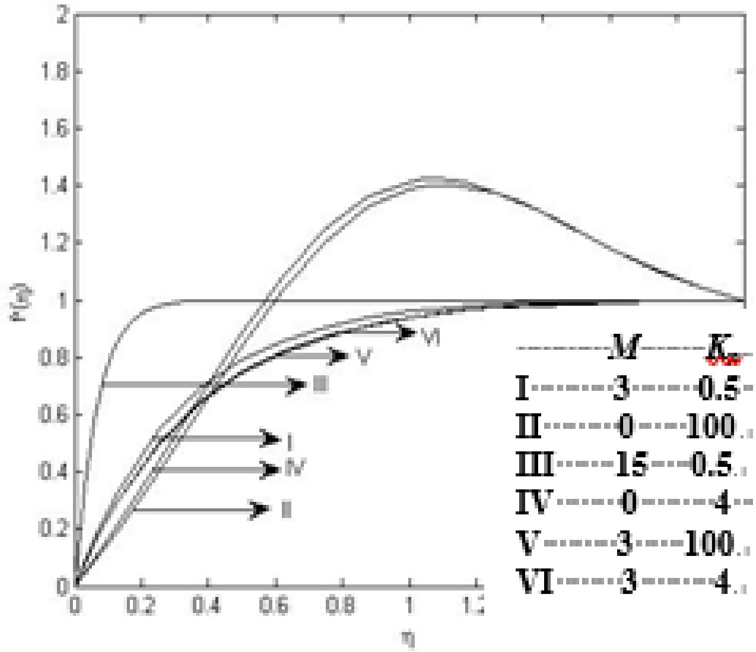

Fig.7(b). Velocity profile for β=*-3* and *C=-1* (R-K Method)

Conclusions

(1) The presence of magnetic field embedding the resistive electromagnetic force which controls the sharp rise and imposes the stability of flow in both converging and diverging flow. Hence presence of magnetic field and embedding the wedge surface in a porous medium contribute to stream lined flow.

(2) DTM associated with Pade approximant is suitable for the solution of non-linear boundary value problem.

In this paper [2/2] Pade approximant has been used to approximate the solution generated by DTM. The higher order may yield better result.

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Nomenclature

B	magnetic field
B_0	magnetic field strength
C	suction or blowing parameter
f	dimensionless stream function
f'	dimensionless tangential velocity
K_p'	porosity parameter
K_p	permiability parameter
M	magnetic field parameter
m	constant parameter
U	stream velocity
u	velocity of the fluid along x-axis
v	velocity of the fluid along y-axis
v_w	suction or blowing velocity
x	axis along the wall
y	axis perpendicular to the wall

Greek Symbols

β	wedge parameter
η	similarity variable
α	positive constant
ν	kinematic viscosity
ρ	fluid density
σ	electric conductivity
ψ	stream function