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Modelling and Simulation of Electric Power Transmission Line Current as Wave

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Abstract

In this paper, a well-known mathematical model of electric power transmission line under steady state conditions is considered. From this model, the mathematical expressions that describe the current travelling and refracted waves along a power transmission line have been developed taking as starting point the end of the line.

We use the fore-mentioned mathematical expressions and the data of a typical electric transmission line to calculate how the current travelling and refracted waves vary. The results are also graphed in order to have an optical view of how the current travelling and refracted waves behave. Finally, the results are analysed and the relative conclusions are drawn.

Keywords

Current, wave, electric power transmission line, current travelling wave, current refracted wave, modelling, simulation, current refraction co-efficient

List of Symbols

R = long-wise omhic resistance of power transmission line (under sinusoidal voltage) per unit length of line (Ω/km)

L = long-wise inductance of power transmission line (under sinusoidal voltage) per unit length of line (H/km)

C = transversal capacitance of power transmission line (under sinusoidal voltage) per unit length of line (F/km)

G = transversal conductance of power transmission line (under sinusoidal voltage) per unit length of line (S/km)

l = length of power transmission line (km)

z = R+jωL = long-wise complex impedance of power transmission line per unit length of line (Ω/km)

y = G+jωC = transversal complex conductance of power transmission line per unit length of line (S/km)

Z = z.l = total long-wise complex impedance of power transmission line (Ω)

Y = y.l = total transversal complex conductance of power transmission line (S)

$\mathbf{V_S}$ = complex line to earth voltage at the beginning of power transmission line, Sending voltage (V)

$\mathbf{V_R}$ = complex line to earth voltage at the end of power transmission line, Receiving voltage (V)

$\mathbf{I_S}$ = complex phase current at the beginning of power transmission line, Sending current (A)

$\mathbf{I_R}$ = complex phase current at the end of power transmission line, Receiving current (A)

$\boldsymbol{\gamma} = \sqrt{zy}$ = α+jβ = transmission co-efficient of power transmission line (km^{-1})

α = reduction co-efficient of power transmission line (neper/km)

β = phase co-efficient of power transmission line (rad/km)

$\mathbf{z_C} = \sqrt{\frac{z}{y}}$ = characteristic impedance of power transmission line (Ω)

$\mathbf{e^{j\varphi}}$ = cosφ +jsinφ = Euler's equation

$\boldsymbol{\lambda} = \frac{2\pi}{\beta}$ = wave length of power transmission line (km)

υ = wave transmission velocity along the power transmission line (km/sec)

τ = wave travelling time in order to cover the length of power transmission line (sec)

Δ = electric phase (angle) of power transmission line (rad)

$\frac{\Delta}{l}$ = electric phase (angle) of power transmission line per unit length of line (rad/km)

$\mathbf{I_{trav}(x)}$ = current travelling wave as a function of distance x (A)

$\mathbf{I_{refr}(x)}$ = current refracted wave as a function of distance x (A)

$\boldsymbol{\rho_I(x)} = \frac{I_{refr}(x)}{I_{trav}(x)}$ = current refraction co-efficient as a function of distance x

$\varphi(x)$ = electric phase(angle) of respective complex quantity as function of distance x **(°)**

1. Introduction

Most people think of the voltage as an element that when they put it on, it is applied immediately. They cannot imagine that the voltage and the respective current that is generated are waves (electromagnetic waves) that travel and refract with almost the speed of light. This understanding is due to the length of line and the inability that people have to perceive the very small time intervals (psecs, μsecs, msecs depending on the line length) that the waves need to cover these distances.

In this paper, the length under consideration is that of a power transmission line of an electric power system [1-14], a length of some hundred kilometers. The equivalent electric circuit of an electric power transmission line under steady state conditions is drawn and the respective differential equations are extracted from it using as independent variable the distance x from either the rears of the line. The above mathematical model already exists in the literature and can easily be found [1-6].

Solving the differential equations, the mathematical expressions describing the current travelling and refracted waves are obtained (section 2). The proof that the above currents are the travelling and refracted wave respectively is the mathematical expressions themselves. They are the mathematical expressions of a travelling and refracted wave respectively.

As far as I know and search in the literature, I could not find calculation and graphical representation of the current travelling and refracted waves along an electric power transmission line. Thus, in this paper, the above mathematical expressions are tested on a typical electric power transmission line and the results are presented in section 3. Furthermore, in section 3, the above results are graphed in order to have an optical image of how the current travelling and refracted waves along the line behave. Finally, in section 4, a discussion is developed, the results are studied, analysed and in section 5, the respective conclusions are drawn.

2. Development and analysis of the mathematical modelling of current travelling and refracted waves

In figure 1, the electric equivalent representation of power transmission line under steady state conditions and using divided elements has been drawn.

where z dx = the infinitesimal long-wise complex impedance of dx

y dx = the infinitesimal transversal complex conductance of dx

From the infinitesimal element dx, the following equations are drawn :

1st law of Kirchhoff : [I(x)+dI(x)] = I(x) + dI(x)

2nd law of Kirchhoff : [V(x)+dV(x)] = V(x) + dV(x)

Voltage drop on element z dx : dV(x) = [I(x)+dI(x)] zdx =

$$\cong I(x)\, zdx \rightarrow \frac{dV(x)}{dx} = I(x)\, z \qquad (1)$$

Voltage drop on element y dx : $dI(x) = V(x)\, ydx \rightarrow \frac{dI(x)}{dx} = V(x)\, y$ (2)

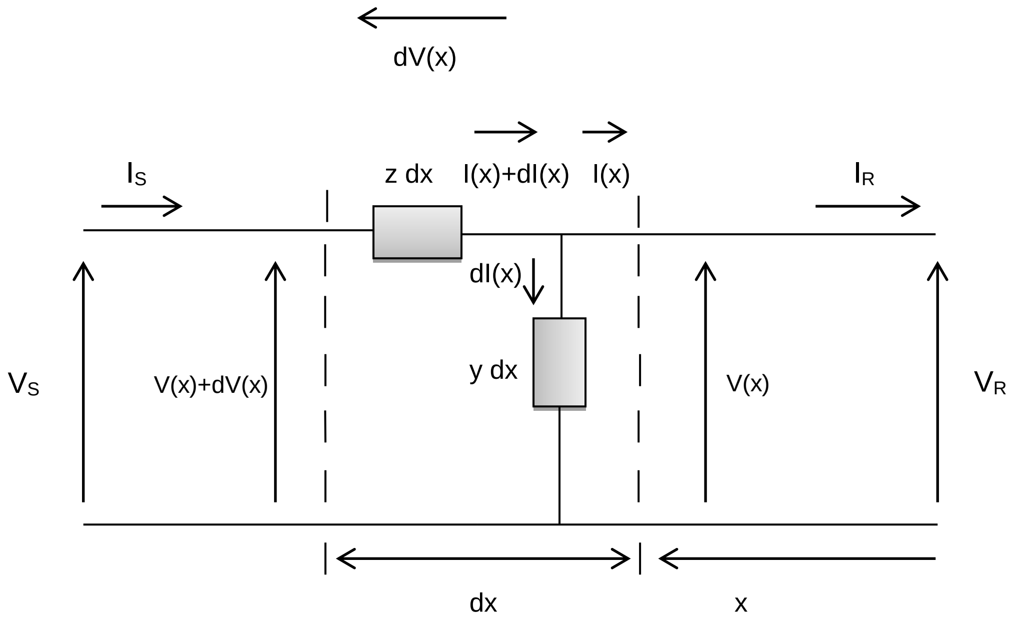


Figure 1. Electric equivalent representation of electric power transmission line

Differentiating eqn 1 and replacing it into eqn 2, we get :

$$\frac{d^2V(x)}{dx^2} = yz\, V(x) \qquad (3)$$

Differentiating eqn 2 and replacing it into eqn 1, we also get :

$$\frac{d^2I(x)}{dx^2} = yz\, I(x) \qquad (4)$$

From equations (3) and (4), V(x) and I(x) are described by the same differential equations. The above implies that V(x) and I(x) are described by similar mathematical functions.

We take as initial conditions :

$$V(x{=}0) = V_R \qquad (5)$$

$$\text{and} \quad I(x{=}0) = I_R \qquad (6)$$

i.e. we take as x=0 the end of electric power transmission line

Then, from equations (3), (4), (5) and (6), we extract the following mathematical expressions of current travelling and refracted wave respectively :

$$I_{trav}(x) = \frac{\frac{V_R}{z_C}+I_R}{2}\, e^{\gamma x} \qquad (7)$$

$$I_{refr}(x) = -\frac{\frac{V_R}{z_C}-I_R}{2}\, e^{-\gamma x} \qquad (8)$$

The above equations (7) and (8) are nothing else but the mathematical expressions of a wave.

Then, the current refraction co-efficient $\rho_I(x)$ can be set as a function of distance x. The voltage refraction co-efficient is defined as : $\rho_I(x) = \frac{I_{refr}(x)}{I_{trav}(x)}$ (9)

3. Simulation, calculation and graphical presentation of current travelling and refracted waves

We consider a typical electric power transmission line with the following parameters :

$R = 0.107\ \Omega/km$	$L = 1.362\ mH/km$
$G = 0\ S/km$	$C = 0.0085\ \mu F/km$
$f = 50\ Hz$	$l = 360\ km$
$V_R = 115470 \angle 0^\circ\ V$	$I_R = 360.844 \angle 0^\circ\ A$

Then using the list of symbols and the analysis of section 2, we can calculate the other complex parameters of the above line in polar and/or cartesian form :

$$\gamma = 1.085 x 10^{-3} \angle 82.98^\circ\ km^{-1} = (0.1326x10^{-3} + j\ 1.07687x10^{-3})\ km^{-1}$$

$$\frac{\frac{V_R}{z_C}+I_R}{2} = 321.886 \angle 3.092^\circ\ A$$

$$-\frac{\frac{V_R}{z_C}-I_R}{2} = 43.079 \angle -23.767^\circ\ A$$

$$z_C = 406.41 \angle -7.02^\circ\ \Omega$$

$$\gamma = 1.085 x 10^{-3} \angle 82.98^\circ\ km^{-1} = (0.1326x10^{-3} + j\ 1.07687x10^{-3})\ km^{-1}$$

$\alpha = 0.1326x10^{-3}$ neper/km $\qquad \beta = 1.07687x10^{-3}$ rad/km

λ = 5834.674 km

υ = 291733.696 km/sec τ = 1.234 msecs

Δ = 22.212° Δ/l = 0.0617°/km

Then, equations (7), (8) and (9) using the above parameters become :

$$I_{trav}(x) = 321.886 < 3.092^{\circ}\ e^{(0.1326x10-3 + j\,1.07687x10-3)x} \quad A \tag{10}$$

$$I_{refr}(x) = 43.079 < -23.767^{\circ}\ e^{-(0.1326x10-3 + j\,1.07687x10-3)x} \quad A \tag{11}$$

$$\rho_I(x) = \frac{43.079 < -23.767^{\circ}\ e-(0.1326x10-3 + j\,1.07687x10-3)x}{321.886 < 3.092^{\circ}\ e(0.1326x10-3 + j\,1.07687x10-3)x} \tag{12}$$

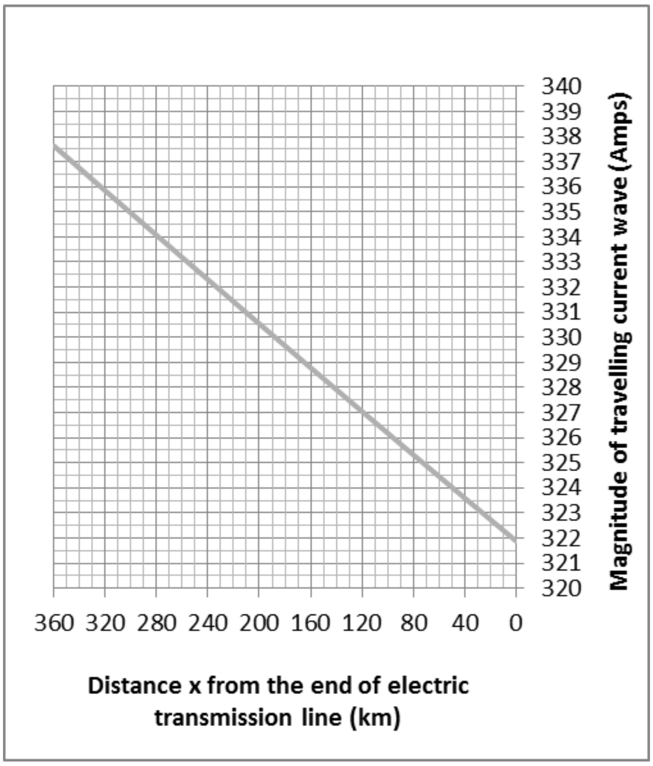
Using equations (10), (11) and (12) and taking step Δx=10km, we calculate the values of current travelling and refracted waves as well as current refraction co-efficient and the results are presented in table 1. Since the currents are vectors, the results are complex numbers and are given in polar form ie. in current magnitude(Amps) and current phase(°) representation. The current refraction co-efficient $\rho_I(x)$ is a pure complex number since is derived from the division of the current waves and is also given in table 1 in polar form ie. in magnitude(pure real number) and phase(°) form.

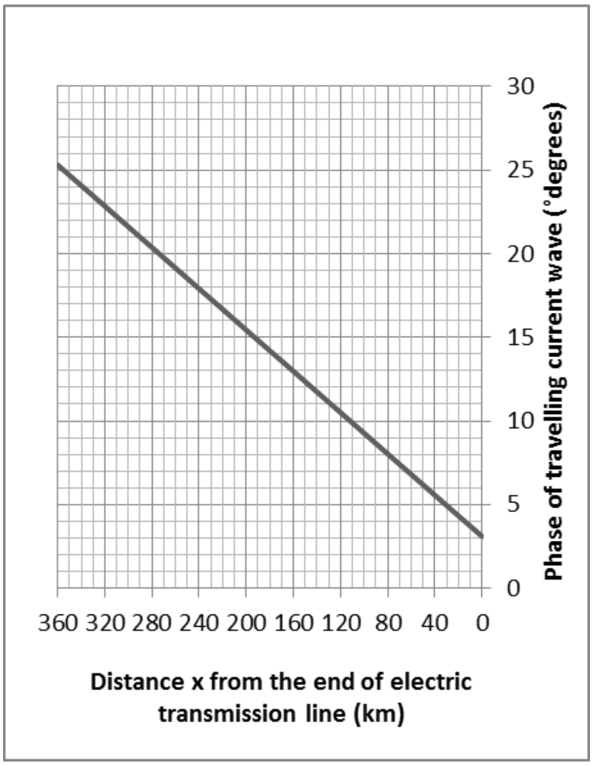
α/α	**x (km)**	**$I_{trav}(x)$ (Amps)**	**$\varphi_{Itrav}(x)$ (°)**	**$I_{refr}(x)$ (Amps)**	**$\varphi_{Irefr}(x)$ (°)**	**$\rho_I(x)$**	**$\varphi\rho_I(x)$ (°)**
1	0	321.8865	3.091922	43.07951	-23.7671	0.1338345	-26.8591
2	10	322.3137	3.709069	43.02241	-24.3843	0.1334799	-28.0934
3	20	322.7415	4.326216	42.96538	-25.0014	0.1331263	-29.3277
4	30	323.1699	4.943362	42.90843	-25.6186	0.1327736	-30.5619
5	40	323.5988	5.560509	42.85156	-26.2357	0.1324219	-31.7962
6	50	324.0283	6.177655	42.79476	-26.8529	0.1320711	-33.0305
7	60	324.4583	6.794802	42.73804	-27.4700	0.1317212	-34.2648
8	70	324.8890	7.411948	42.68139	-28.0872	0.1313722	-35.4991
9	80	325.3202	8.029095	42.62482	-28.7043	0.1310242	-36.7334
10	90	325.7519	8.646242	42.56832	-29.3215	0.1306771	-37.9677
11	100	326.1843	9.263388	42.51190	-29.9386	0.1303309	-39.2020

12	110	326.6172	9.880535	42.45555	-30.5558	0.1299856	-40.4363
13	120	327.0507	10.49768	42.39928	-31.1729	0.1296413	-41.6706
14	130	327.4848	11.11483	42.34308	-31.7900	0.1292979	-42.9049
α/α	**x (km)**	**$I_{trav}(x)$ (Amps)**	**$\varphi_{Itrav}(x)$ (°)**	**$I_{refr}(x)$ (Amps)**	**$\varphi_{Irefr}(x)$ (°)**	**$\rho_I(x)$**	**$\varphi\rho_I(x)$ (°)**
14	130	327.4848	11.11483	42.34308	-31.7900	0.1292979	-42.9049
15	140	327.9194	11.73197	42.28695	-32.4072	0.1289553	-44.1392
16	150	328.3546	12.34912	42.23090	-33.0243	0.1286137	-45.3735
17	160	328.7904	12.96627	42.17493	-33.6415	0.1282730	-46.6078
18	170	329.2268	13.58341	42.11903	-34.2586	0.1279332	-47.8421
19	180	329.6638	14.20056	42.06320	-34.8758	0.1275942	-49.0763
20	190	330.1013	14.81771	42.00745	-35.4929	0.1272562	-50.3106
21	200	330.5394	15.43485	41.95177	-36.1101	0.1269191	-51.5449
22	210	330.9781	16.05200	41.89616	-36.7272	0.1265829	-52.7792
23	220	331.4174	16.66915	41.84063	-37.3444	0.1262475	-54.0135
24	230	331.8573	17.28629	41.78517	-37.9615	0.1259131	-55.2478
25	240	332.2977	17.90344	41.72979	-38.5787	0.1255795	-56.4821
26	250	332.7388	18.52059	41.67447	-39.1958	0.1252468	-57.7164
27	260	333.1804	19.13773	41.61924	-39.8130	0.1249150	-58.9507
28	270	333.6226	19.75488	41.56407	-40.4301	0.1245841	-60.1850
29	280	334.0654	20.37203	41.50898	-41.0472	0.1242541	-61.4193
30	290	334.5088	20.98917	41.45396	-41.6644	0.1239249	-62.6536
31	300	334.9527	21.60632	41.39901	-42.2815	0.1235966	-63.8879
32	310	335.3973	22.22347	41.34414	-42.8987	0.1232692	-65.1222
33	320	335.8424	22.84061	41.28934	-43.5158	0.1229426	-66.3564
34	330	336.2882	23.45776	41.23461	-44.1330	0.1226169	-67.5907
35	340	336.7345	24.07491	41.17996	-44.7501	0.1222921	-68.8250
36	350	337.1814	24.69205	41.12538	-45.3673	0.1219681	-70.0593
37	360	337.6290	25.30920	41.07086	-45.9844	0.1216450	-71.2936

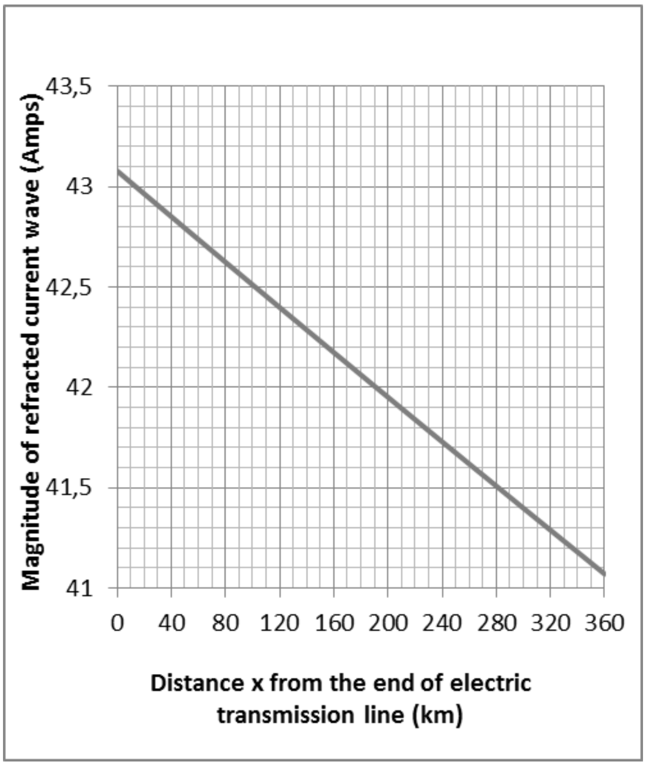
Table 1. Calculation results of current travelling and refracted waves

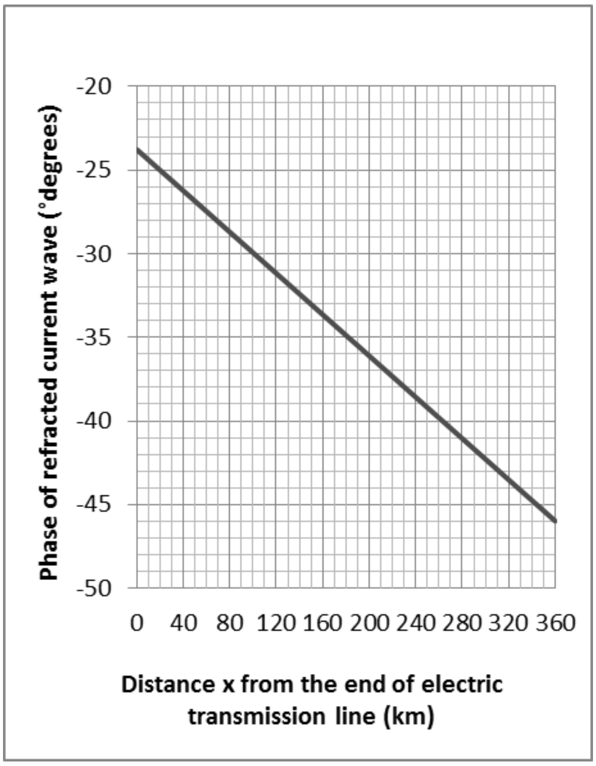
The graphical presentations of results obtained in table 1 are given in graphs 1 to 3.



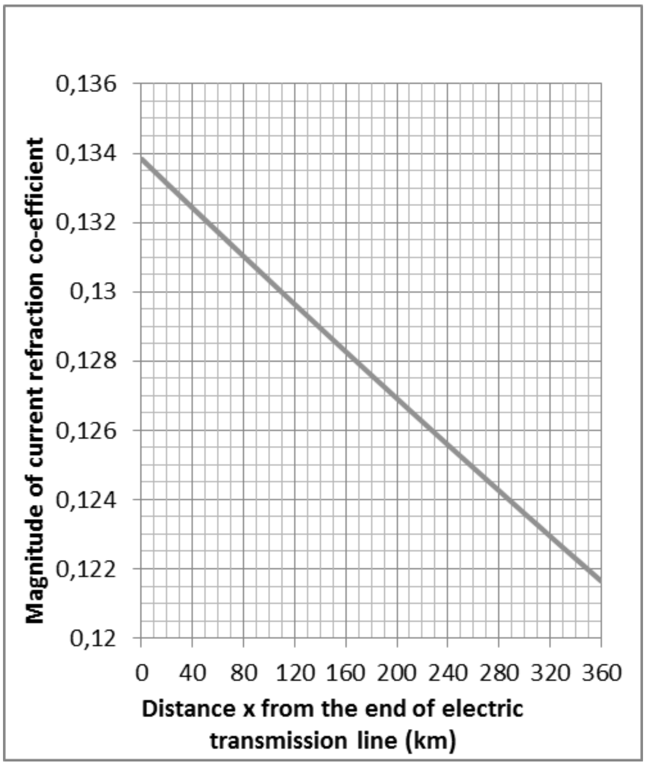


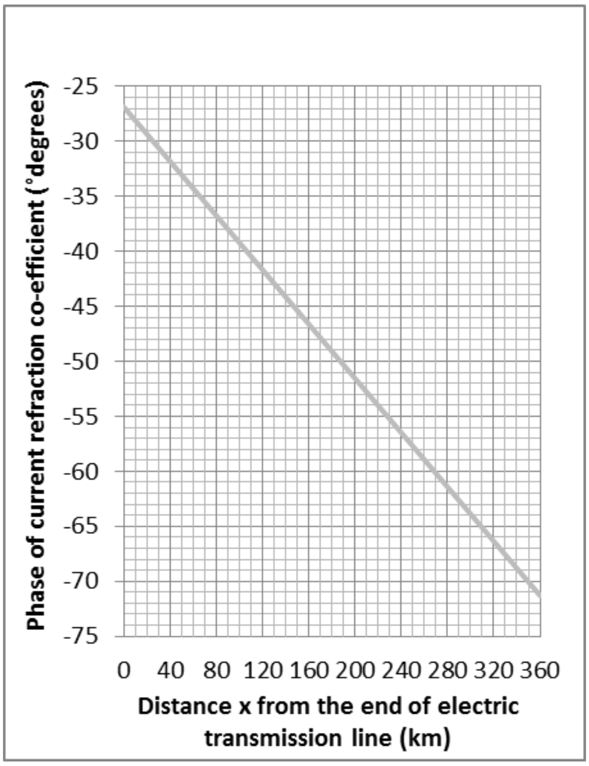
Graph 1. Magnitude (intensity) and phase (angle) of current travelling wave from the beginning towards the end of line ie. the direction the travelling wave moves (direction right to left of electric power transmission line of figure 1)





Graph 2. Magnitude (intensity) and phase (angle) of current refracted wave from the end towards the beginning of line ie. the direction the refracted wave moves (direction opposite to that of graph 1 ie. left to right of electric power transmission line of figure 1)





Graph 3. Magnitude and phase (angle) of current refraction co-efficient from the end where the refraction occurs towards the beginning of line (direction opposite to that of graph 1 ie. left to right of electric power transmission line of figure 1)

4. Discussion

The curves of graphs 1, 2 and 3 may appear common but they are not. Some of them may look straight lines or almost straight lines but they are not. The above quantities have an exponential behaviour as someone can verify from the respective equations in section 2. Their graphical representations depend on the values of their exponential constant factors (α and β). If their values are small and as variable x increases, the values αx and βx do not change enough in order their exponential behaviour to appear on the graphs. This is the reason they seem to be straight or almost straight lines.

The above explanation is given regarding their form. Regarding now their variation, the following reasoning is developed.

On one hand, the terms (V_R/z_C+I_R) and (V_R/z_C-I_R) are constant complex numbers since V_R, I_R and z_C are constant complex numbers. That implies that they have a constant magnitude and a constant phase.

On the other hand, the terms $e^{\gamma x}$ and $e^{-\gamma x}$ vary with distance x from the end of power transmission line.

The term $e^{\gamma x}$ can be written as $e^{(\alpha+j\beta)x} = e^{\alpha x}\, e^{j\beta x} = e^{\alpha x}[\cos(\beta x) + j \sin(\beta x)]$

The values of α and β are real positive numbers for a typical real power transmission line. This will be understood from the following analysis.

The $e^{\alpha x}$ is the magnitude of the above term while the $e^{j\beta x}$ is the phase (angle) of the above term.

The term $e^{\alpha x}$ increases as x increases i.e. the magnitude of current travelling wave increases as we approach the beginning of line. In other words, the magnitude (intensity) of current travelling wave (eqn 7) diminishes as the wave travels from the beginning of line (where the voltage is applied and the current travelling wave starts) to the end of line as one expects in real world (the intensity of signal diminishes as it moves away from source).

The term βx similarly increases as x increases. With similar as above reasoning, the term βx i.e. the phase of current travelling wave (eqn 7) diminishes as the wave travels from the beginning of line and moves to the end of line.

Similarly, the term $e^{-\gamma x}$ can be written as $e^{-(\alpha+j\beta)x} = e^{-\alpha x}\, e^{-j\beta x} = e^{-\alpha x}[\cos(-\beta x) + j \sin(-\beta x)]$

With similar as above reasoning, the term $e^{-\alpha x}$ decreases as x increases. In other words, the magnitude (intensity) of current refracted wave (eqn 8) decreases as the wave moves from the end where the refraction occurs towards the beginning of line as one expects. It is really the part of current travelling wave that arrives at the end of line and refracts travelling in the opposite direction of line. This is implied by the negative value of -γx. The opposite flow of current is indicated by the symbol minus (-) of equation (8).

Additionally, the term -βx decreases as x increases i.e. the phase (angle) of current refracted wave (eqn 8) decreases as the wave moves from the end towards the beginning of line.

Using similar thinking, the term $e^{-2\gamma x}$ of equation (9) regarding the current refraction co-efficient can be written as follows :

$$e^{-2(\alpha+j\beta)x} = e^{-2\alpha x}\, e^{-j2\beta x} = e^{-2\alpha x}[\cos(-2\beta x) + j \sin(-2\beta x)]$$

Thus, using similar as above reasoning, both the magnitude and the phase angle of the current refraction co-efficient decrease as we move from the end (where the refraction occurs) towards the beginning of line as one expects.

5. Conclusions

Studying and analyzing the results presented in table 1 and their graphs 1 to 3 of section 3, we can observe and conclude the following:

1) the magnitude of current travelling wave decreases as the wave travels from the beginning towards the end of line ie. along the direction the current travelling wave moves
2) the phase (angle) of current travelling wave decreases as the wave travels from the beginning towards the end of line ie. along the direction the current travelling wave moves
3) the magnitude of current refracted wave decreases as the wave moves from the end towards the beginning of line ie. along the direction the current refracted wave moves
4) the phase (angle) of current refracted wave decreases as the wave moves from the end towards the beginning of line ie. along the direction the current refracted wave moves
5) the magnitude of current refraction co-efficient decreases from the end of line where the refraction occurs towards the beginning of line
6) the phase (angle) of current refraction co-efficient decreases from the end of line where the refraction occurs towards the beginning of line

Then, we can conclude that the above observations verify the analysis and discussion developed in section 4 of the paper. For better understanding of electric power transmission line current as a wave, we propose to study it using cartesian co-ordinates. This will be the subject of a future paper.

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