Greek symbols

δ	difference	
ϕ	volume fraction	
μ	dynamic viscosity	Pascal.s
ρ	density	kg/m^3

Abbreviations

C O P	coefficient of performance
LCD	liquid-crystal display
PID	proportional, integral and derivative
POE	polyester oil
TiO_2	titanium dioxide

APPENDICES

Appendix A: Error analysis

To estimate the uncertainties of the *derived* quantities, δz, δϕ and δh, etc, we first recall the uncertainties of the *participating* quantities, which are;

Length is measured using a vernier caliper with uncertainty ± 0.02 mm

Temperature: the resolution of the digital indicator is ± 0.1 ^{o}C.

The Nano-refrigerant *volume* flow rate, $\overset{*}{V}$, is measured with an uncertainty of 0.05 L/min

Mass is weighed with the readability of 0.01 gm.

Then, we can estimate the uncertainties in the derived quantities as follows;

$$\delta z = \left(\frac{m_{np}}{m_{np}+m_f}\right)\sqrt{\left(\frac{\delta m_{np}}{m_{np}}\right)^2+\left(\frac{\delta m_{np}^{\,2}+\delta m_f^{\,2}}{(m_{np}+m_f)^2}\right)} \quad (A.1)$$

$$\delta\phi = \frac{z\rho_f}{\rho_{np}(1-z)+z\rho_f}\sqrt{\left(\frac{\rho_f\,\delta z}{\rho_f\, z}\right)^2+\frac{(\rho_{np}\,\delta z)^2+(\rho_f\,\delta z)^2}{(\rho_{np}(1-z)+z\rho_f)^2}} \quad (A.2)$$

$$\delta\rho_{nf} = \sqrt{(\delta\phi\,\rho_{np})^2+(\delta\phi\,\rho_f)^2} \quad (A.3)$$

$$\delta\overset{*}{m} = (\overset{*}{V}\rho_{nf})\sqrt{\left(\frac{\delta\overset{*}{V}}{\overset{*}{V}}\right)^2+\left(\frac{\delta\rho_{nf}}{\rho_{nf}}\right)^2} \quad (A.4)$$

$$\delta Cp_{nf} = \frac{\phi\rho_{np}Cp_{np}+(1-\phi)\rho_f Cp_f}{\rho_{nf}}\sqrt{\frac{(\rho_{np}Cp_{np}\,\delta\phi)^2+(\rho_f Cp_f\,\delta\phi)^2}{(\phi\rho_{np}Cp_{np}+(1-\phi)\rho_f Cp_f)^2}+\left(\frac{\delta\rho_{nf}}{\rho_{nf}}\right)^2} \quad (A.5)$$

$$\delta\mu_{nf} = \mu_f(1-\phi)^{2.5}\sqrt{\left(\frac{\delta\mu_f}{\mu_f}\right)^2+2.5\left(\frac{\delta\phi}{(1-\phi)^{3.5}}\right)} \quad (A.6)$$

$$\delta k_{nf} = \sqrt{(k\rho_{np})^2\left[\left(\frac{\delta k_f}{k_f}\right)^2+\left(\frac{\delta\rho_f}{\rho_f}\right)^2\right]+\left[(1-\phi)k_f\right]^2\left[\left(\frac{\delta\phi}{\phi}\right)^2+\left(\frac{\delta k_f}{k_f}\right)^2\right]} \quad (A.7)$$

$$\delta\mathrm{Re} = \frac{\overset{*}{m}d}{A\mu}\sqrt{\left(\frac{\delta\overset{*}{m}}{\overset{*}{m}}\right)^2+3\left(\frac{\delta D}{D}\right)^2+\left(\frac{\delta\mu}{\mu}\right)^2} \quad (A.8)$$

$$\delta h = \frac{[\overset{*}{m}Cp(T_o-T_i)]_{nf}}{\pi d_i L(T_{s,av}-T_{nf,av})}\sqrt{\left(\frac{\delta\overset{*}{m}}{\overset{*}{m}}\right)^2+\left(\frac{\delta Cp}{Cp}\right)^2+\left(\frac{\delta T^2+\delta T^2}{(T_O-T_i)^2}\right)^2+\left(\frac{\delta d_i}{d_i}\right)^2+\left(\frac{\delta L}{L}\right)^2+\left(\frac{\delta T^2+\delta T^2}{(T_{s,av}-T_{f,avi})^2}\right)^2} \quad (A.9)$$

$$\delta Nu = \frac{hd}{k}\sqrt{\left(\frac{\delta\overset{*}{h}}{h}\right)^2+\left(\frac{\delta d}{d}\right)^2+\left(\frac{\delta k}{k}\right)^2} \quad (A.10)$$

$$\delta(C.O.P) = \frac{[Cp(T_o-T_i)]_{nf,e}}{[Cp(T_o-T_i)]_{nf,c}}\left[\left(\frac{2\delta Cp}{Cp}\right)_e^2+\frac{4\delta T^2}{(T_O-T_i)_c^2}\right] \quad (A.11)$$

According to the above formulas, the worst relative errors in the measured quantities are;

Relative error in measuring the weight raTiO$_2$ = 0.0017.

Relative error in measuring the volume fraction = 0.0016.

Relative error in measuring the nano-fluid density = 1.6 e-005.

Relative error in measuring the mass flow rate = 0.027.

Relative error in measuring the nano-fluid specific heat = 2.01 e-5.

Relative error in measuring the heat flux = 0.025.

Relative error in measuring the Nusselt number = 0.0123819.

Relative error in measuring the COP = 0.0294.

Appendix B: Tests of the suggested Correlation

COEFFICIENTS OF EQUATION (15)

0.18 0.02 1.1 -0.36

Correlation	Experimental	Percentage
512.60	505	1.5
631.09	655	-3.6
743.38	744	-0.08
846.72	811	4.4
539.22	522	3.3
665.51	688	-3.2
780.91	788	-0.8
882.80	860	2.6
549.02	545	0.7
686.88	722	-4.8
794.03	831	-4.4
898.82	910	-1.2
564.08	537	5.0
693.70	708	-2.0
813.17	811	0.2
919.31	877	4.8