

## Heat transfer of the TiO<sub>2</sub>/water nanofluid in an annulus of the finite rotating cylinders

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### ABSTRACT

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**Keywords:**

*nanofluid, finite rotating annulus, co-rotating, counter rotating*

Study on the heat transfer of TiO<sub>2</sub>/water nanofluid flows inside annulus of the finite rotating cylinders was done numerically. Inner shaft and outer tube were rotated in co-rotating and counter-rotating direction. The k-epsilon turbulent model and the Mixture-multiphase model were used to treats the turbulence flow and the multiphase flows of the TiO<sub>2</sub>/water nanofluid, respectively. Results of the current work are in agreement with published work. Results showed that increased in Reynolds number the Nusselt number increases. The distribution of the Nusselt number at the specific location along the heated inner shaft for co-rotating and counter-rotating cases shows a different distribution profile. The counter-rotating case was found to be more efficient in enhancing the heat transfer rate in comparison to the co-rotating case. This observation is suggested because of the boundary layers disturbances that originate from the additional vortices produced by the competing rotational speed between inner shaft and outer tube.

### 1. INTRODUCTION

Flow in an annulus of two concentric rotating cylinders is known as the Taylor-Couette-Poiseuille flow. This type of flow is important for the study of rotating shaft involving heat transfer. Heat transfer in rotating shaft is a crucial issue since the lubrication oil needs a certain working temperature to make it work at optimum level. Thus, by understanding the heat transfer behaviour in an annulus, it will help to lubricate rotating shaft efficiently. An experimental work to elucidate the heat transfer behaviour of an annulus is difficult due to a narrow gap of the annulus. Hence, a numerical simulation turns out to be a powerful tool to elucidate the heat transfer behaviour of flow in an annulus.

Water and air were used as a working fluid in previous heat transfer studies of an annulus and found that the Reynolds number [1] and the flow structure [2] are significantly influencing the heat transfer characteristics. Water and air exhibit low thermal properties, whereby for a good heat transfer process, the working fluids must possess high thermal conductivity values. The thermal conductivity of the fluid can be enhanced using nanoparticles [3]. A significant heat transfer enhancement was observed when particles at nanometres size were added into conventional fluids (water, ethylene glycol and oil), which were eventually called nanofluids [3]. Many studies on nanofluids were done to investigate the effect of particle sizes, types, volume percentages, weight percentages, type of based fluid and its application in heat exchanger [4-10]. A study using Al<sub>2</sub>O<sub>3</sub>-water nanofluid shows that heat transfer increased by 25.5% when 2.5 vol % of Al<sub>2</sub>O<sub>3</sub> nanoparticles was added into the pure water [7].

Study on the heat transfer characteristics using nanofluid as a working fluid inside an enclosure of the infinite rotating

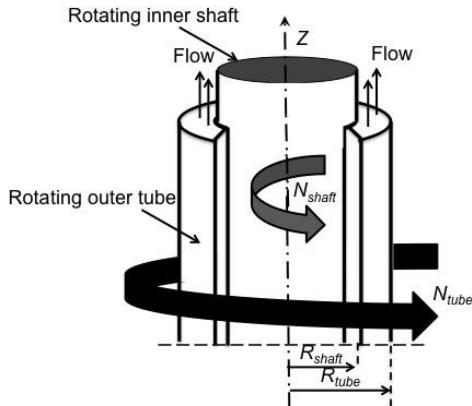
cylinder found that the heat transfer enhancement and flow circulation were strongly depend upon the rotation speed of an ordinary shaft size placed at the center of an enclosure and also the nanoparticle concentration [11]. Moreover, increase in an average Nusselt number as increases in the cylinder size of the mixed convection case for various Richardson number was observed [12]. Furthermore, the nanoparticle volume fraction [13]-[16], cylinder rotation speed [16]-[18] and the Reynolds number [17]-[19] were found significantly influenced the heat transfer rate of rotating cylinder. In spite of the enhancement in the heat transfer rate, increases in the thickness of thermal boundary layer was also observed as the effect of the increment in the nanoparticle volume/weight fraction for infinite single rotating tube [19]-[21]. The Nusselt number is decrease as increases in drag due to the rotation of the tube [22].

Previously, studies on the heat transfer of rotating cylinders using nanofluid was focused on forced, natural and mixed convection of the infinite cylinder length. In the authors' knowledge, no study on heat transfer has been conducted in an annulus of finite length with independently rotating cylinders using nanofluid as a working fluid. Therefore, the objective of this study is to investigate numerically the heat transfer characteristiss of TiO<sub>2</sub>/water nanofluid inside an annulus of finite length independently rotating cylinders.

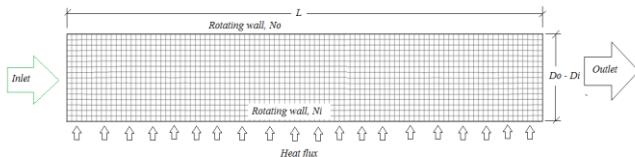
### 2. NUMERICAL WORKS

Figures 1 and 2 show the geometry of the cylinders with a  $L/D_H = 60$ . A uniform velocity is assigned at the inlet and is used to calculate the Reynolds number. A fully developed flow is assumed at the outlet with all derivatives are zero.

Walls of the inner shaft and outer tube are rotating with non-slip condition and the inner shaft is heated with a constant heat flux. This simulation is conducted in a turbulent region and the gravity was neglected.



**Figure 1.** Geometry of the study



**Figure 2.** Numerical domain with respective boundary condition

The conservation equations were solved using Ansys FLUENT 15.0. Uniform size grid of  $1 \times 10^{-5} \text{ m} \times 1 \times 10^{-5} \text{ m}$  was used. The mixture multiphase model [21]-[23] is employed to treat the nanoparticle and water. In the mixture multiphase model, the conservation equations are solved independently. The secondary phase is solved using volume fraction equation and the relative velocity of the particle to the base fluid is calculated using algebraic expression. The steady state continuity equation is given as;

$$\nabla \cdot \left( \phi_p \rho_p + \phi_f \rho_f \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \right) = 0 \quad (1)$$

where  $\vec{v}_p$  is the particle velocity,  $\vec{v}_f$  is the base fluid velocity.

The steady state momentum equation is given as;

$$\begin{aligned} (\phi_p \rho_p + \phi_f \rho_f) & \left( \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \cdot \nabla \left( \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \right) \right) \\ &= -\nabla P \\ &+ \mu_f \left[ \nabla \left( \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \right) \right. \\ &\quad \left. + \left( \nabla \left( \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \right) \right)^T \right] \\ &+ \nabla \cdot \left( \phi_p \rho_p \left( \vec{v}_p - \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \right) \right. \\ &\quad \left. + \phi_f \rho_f \left( \vec{v}_f - \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \right) \right) \end{aligned} \quad (2)$$

The steady state energy equation is given as;

$$\nabla \cdot (\phi_p \vec{v}_p \rho_p h_p + \phi_f \vec{v}_f \rho_f h_f) = \nabla \cdot (k_{eff} \nabla T) \quad (3)$$

where  $h_p$  and  $h_i$  are the enthalpy of particles and base fluid respectively. The nanofluid effective thermal conductivity is  $k_{eff}$ .

The volume fraction equation is given as;

$$\begin{aligned} \nabla \cdot & \left( \phi_p \rho_p \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \right) \\ &= -\nabla \cdot \left( \phi_p \rho_p \left( \vec{v}_p - \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \right) \right) \end{aligned} \quad (4)$$

The slip velocity  $\vec{v}_{f,p}$  represents the velocity of the particles relative to base fluid and determined by Manninen *et al.* through Schiller and Naumann drag formulation as below;

$$\vec{v}_{f,p} = \vec{v}_p - \vec{v}_f = \frac{d_p^2 (\rho_p - \phi_p \rho_p + \phi_f \rho_f)}{18 \mu_f f_d \rho_p} \vec{a} \quad (5)$$

where  $\vec{a}$  (particle's acceleration) and  $f_d$  are given as;

$$\vec{a} = \left( \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \cdot \nabla \right) \frac{\phi_p \rho_p \vec{v}_p + \phi_f \rho_f \vec{v}_f}{\phi_p \rho_p + \phi_f \rho_f} \quad (6)$$

$$f_d = 0.0183 Re_p \quad Re_p \geq 1000 \quad (7)$$

The particle Reynolds number is defined as;

$$Re_p = \frac{U_m d_p}{\mu_f} (\phi_p \rho_p + \phi_f \rho_f) \quad (8)$$

This study was performed using the realizable  $k-\varepsilon$  turbulence model with enhanced wall treatment. Relation  $\mu_t = \rho C_\mu k^2 / \varepsilon$  is used to combine the turbulent kinetic energy and turbulent dissipation rate. The turbulent kinetic energy and the turbulent dissipation rate at the tube inlet were obtained from Eqs. (9)-(10) and the initial assumption were computed using Eq. (11).

$$k = \frac{3}{2} (u \cdot I)^2 \quad (9)$$

$$\varepsilon = C_\mu^{3/4} \frac{k^{3/2}}{L} \quad (10)$$

$$I = \frac{0.16}{Re^{1/8}} \quad (11)$$

In this study  $TiO_2$ /water nanofluid is used and thermal properties relation are shown in Table 1.

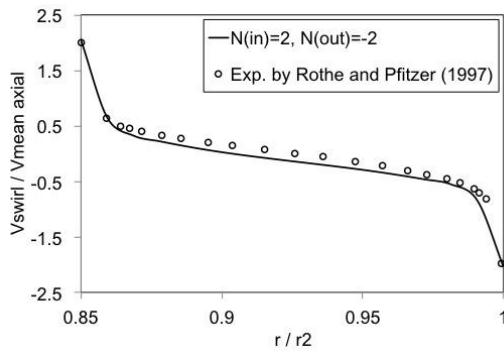
**Table 1.** Thermo-physical properties of  $TiO_2$  /water nanofluid

Thermo-physical property	Relation
Density ( $\text{kg/m}^3$ )	Mixture rule [1] $\rho_{eff} = (1 - \phi_p) \rho_f + \phi_p \rho_p$

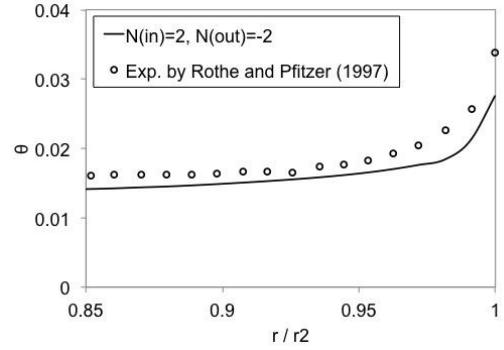
Specific heat (J/kg·K)	Thermal equilibrium [1] $c_{p,\text{eff}} = \frac{(1 - \phi_p)\rho_f c_{p,f} + \phi_p \rho_p c_{p,p}}{\rho_{\text{eff}}}$
Viscosity (m Pa·s)	Einstein's model [26] $\mu_{\text{eff}} = \mu_f(1 + 2.5\phi_p)$
Thermal conductivity (W/m·K)	Maxwell Garnett model [27] $k_{\text{eff}} = \frac{k_p + 2k_f - 2\phi_p(k_f - k_p)}{k_p + 2k_f + \phi_p(k_f - k_p)}$

### 3. RESULTS AND DISCUSSION

A numerical scheme was validated with the published experimental data [28] as shown in Figures 3 and 4. Figures 3 and 4 show the swirl velocity and the profile of dimensionless temperature for the counter rotating case at  $Re=10000$  respectively. It is clearly seen that the current simulation setup are in close agreement with the published experimental work [28].

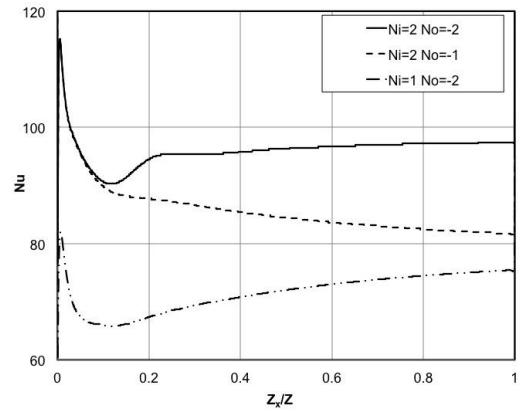


**Figure 3.** Profile velocity of swirl for counter-rotating case at  $Re=10000$



**Figure 4.** Profile of dimensionless temperature for counter-rotating case at  $Re=10000$

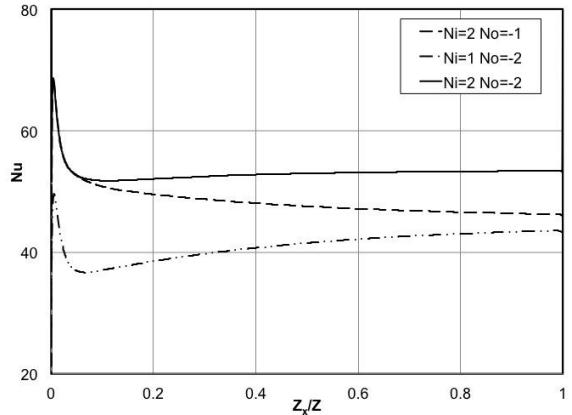
Figures 5-7 shows the local Nusselt number distribution along the inner shaft for the counter rotating case. In Figure 5, the local Nusselt number for the 0.1 vol% TiO<sub>2</sub>/water nanofluid at  $Re = 10000$  is plotted.



**Figure 5.** Local Nusselt number distribution along the inner shaft wall for the 0.1 vol% TiO<sub>2</sub>/water nanofluid counter rotating case at  $Re = 10000$

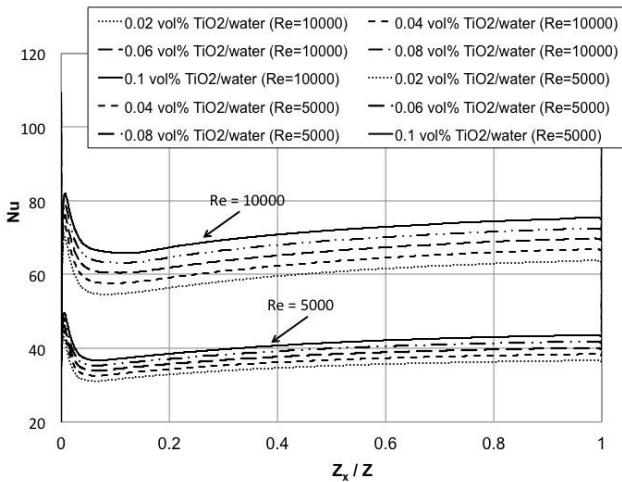
Figure 5 shows that the highest local Nusselt number is located at the tube inlet. The local Nusselt number gradually decrease towards the tube outlet. Later, at 10 percent of the total tube length from the inlet, behaviour of the local Nusselt number distribution is changed. Increase in the local Nusselt number is seen for the case of rotational speed of inner shaft is lower/equal than/outer tube. While the local Nusselt number is decreased for the case of rotational speed of inner shaft is higher than outer tube.

At  $Re=5000$  with the 0.1 vol% TiO<sub>2</sub>/water nanofluid the local Nusselt number distribution for the varies rotational speed of the inner shaft and outer tube is shown in Figure 6.



**Figure 6.** Local Nusselt number at the inner shaft wall for the 0.1 vol% TiO<sub>2</sub>/water nanofluid counter rotating case at  $Re = 5000$

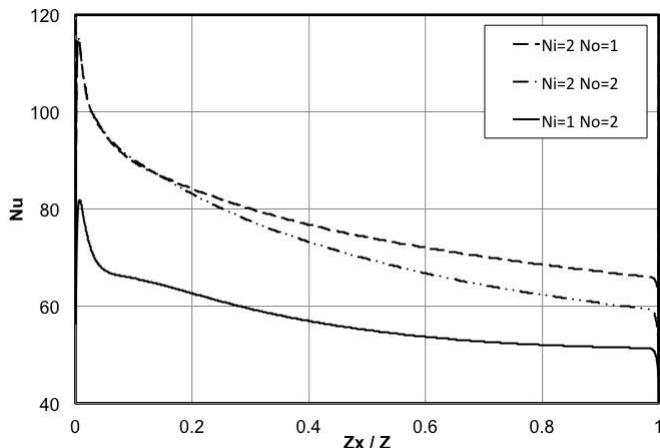
Similar behaviour of the local Nusselt number distribution in Figure 5 is seen in Figure 6. The highest local Nusselt number for all cases is observed at the inlet. But, in overall the highest Nusselt number in Figure 6 is lower than Figure 5, this result is expected because of the Reynolds number in Figure 6 is less than the Reynolds number in Figure 5.



**Figure 7.** Local Nusselt number at the inner shaft wall for various volume percentages of  $\text{TiO}_2/\text{water}$  nanofluid at  $Re=10000$  and  $Re=5000$  (counter rotating case,  $Ni=1$   $No=-2$ )

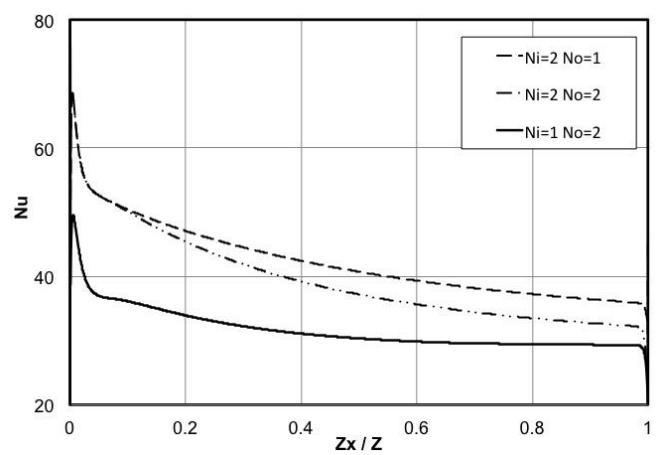
Figure 7, show the local Nusselt number along the inner shaft for all cases at the Reynolds number of  $Re=5000$  and  $Re=10000$ . The local Nusselt number profile along the tube is identical for both cases, where the highest local Nusselt number is observed at the inlet. Moreover, a significant effect of the nanoparticle concentration in the nanofluid is observed for  $Re=10000$  case compared to  $Re=5000$  case.

Figures 8-10 shows the local Nusselt number along the inner shaft wall for the co-rotating case.



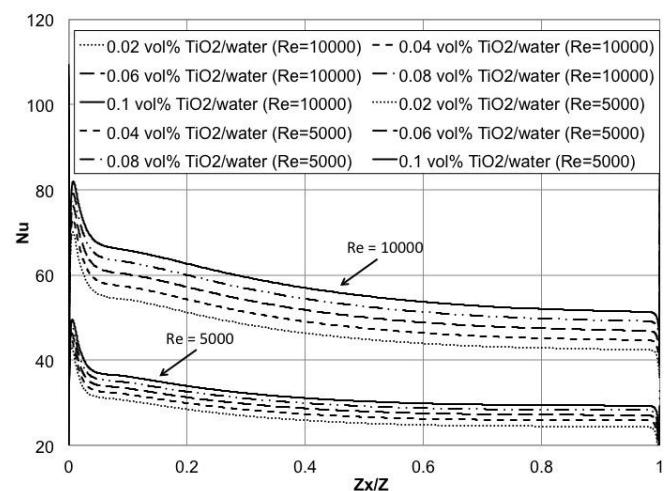
**Figure 8.** Local Nusselt number along the inner shaft wall for the 0.1 vol%  $\text{TiO}_2/\text{water}$  nanofluid co-rotating case at  $Re=10000$

It can be seen in Figure 8, that the maximum local Nusselt number is observed at the inlet and decreased gradually along the tube towards the outlet for all co-rotating cases. The lowest average Nusselt number is observed for the case of inner shaft rotate at the rotational speed lower than the outer tube. On the other hand, when the inner shaft rotated faster than outer tube the average Nusselt number is highest.



**Figure 9.** Local Nusselt number at the inner shaft wall for the 0.1 vol%  $\text{TiO}_2/\text{water}$  nanofluid co-rotating case at  $Re=5000$

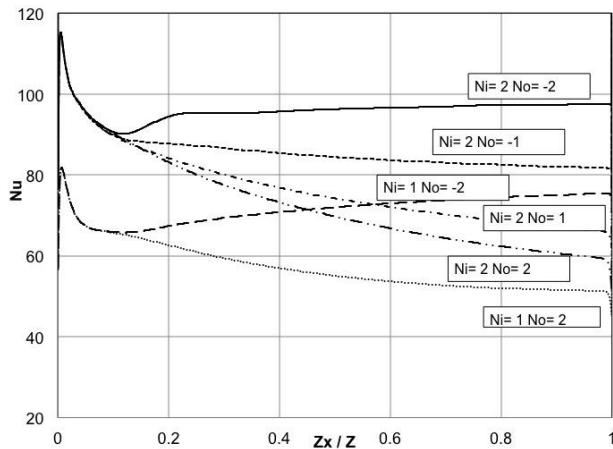
In Figure 9, similar behaviour of the local Nusselt number along the inner shaft in Figure 8 is observed. Moreover, the average Nusselt number in Figure 9 is less than the average Nusselt number in Figure 8. These finding is similar to counter rotating cases in Figures 5 and 6 where the Reynolds number controlling the average Nusselt number.



**Figure 10.** Local Nusselt number at the inner shaft wall for the various volume percentage of  $\text{TiO}_2/\text{water}$  nanofluid at  $Re=10000$  and  $Re=5000$  (co-rotating case,  $Ni=1$   $No=2$ )

An effect of the volume percentage of  $\text{TiO}_2/\text{water}$  nanofluid in co-rotating case at  $Re=10000$  and  $Re=5000$  is shown in Figure 10. At the Reynolds number of  $Re=10000$ , a significant effect of the particle concentration in the nanofluid is observed in comparison to  $Re=5000$ . Thus, in spite of the inner shaft and outer tube rotational speed, the effect of the volume percentage of nanofluid on the Nusselt number is significantly due to the Reynolds number.

Figure 11, shows a local Nusselt number along the inner shaft for the co-rotating and counter rotating cases with various inner shaft and outer tube rotation speed at  $Re=10000$ .



**Figure 11.** Local Nusselt number at the inner shaft wall for various inner shaft and outer tube rotation speed at  $Re = 10000$

It is clearly seen that the highest average Nusselt number is produced by the counter rotating case with a same rotational speed of the inner and outer tubes. While the lowest average Nusselt number is observed for the co-rotating case with a rotational speed of inner shaft is lower than the outer tube. In overall, the counter rotating case produces higher average Nusselt number in comparison to co-rotating case. Moreover, effect of the inner shaft rotation speed is significant, increase in the rotational speeds of inner shaft increases the average Nusselt number. This observation is suppose due to the extra turbulences generated by competing rotational speeds between the inner shaft and outer tube. Furthermore, a sharp drop of the local Nusselt number at 10 percent entrance length for all cases was also observed. Beyond the 10 percent of the entrance length, a significant effect of the rotation direction between the inner shaft and outer tube and the rotational speed is seen. This is thought, to be caused by the boundary layers disturbances that source form the additional vortices produced by the inner shaft and outer tube rotation. The rotation of the inner shaft enhanced the turbulent kinetic energy and lead to a considerable increment of momentum and heat transfer.

## 7. CONCLUSIONS

The heat transfer behaviour of the  $TiO_2$ /water nanofluid flows inside annulus of finite rotating cylinders was successfully done. The distribution of local Nusselt number for  $Re=5000$  and  $10000$  cases showed similarity at the tube entrance regions for both co-rotating and counter-rotating cases. It is also seen that an increase in vol% of the  $TiO_2$  nanoparticle, increases the Nusselt number. The counter-rotating case with the rotational speed of outer tube is higher than the inner shaft was found to be more efficient in enhancing the heat transfer rate compared to the co-rotating case. While, the lowest average Nusselt number was observed for the co-rotating case with the rotational speed of outer tube is higher than the inner shaft. This observation is suggested due to the boundary layers disturbances and enhancement of the turbulent kinetic energy originated from the competing rotational speed between inner shaft and outer tube in the turbulent flow region.

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## NOMENCLATURE

- $\vec{a}$  particle's acceleration  
 $c_p$  specific heat, J/kg K  
 $d_H$  hydraulic diameter, m  
 $h$  convective heat transfer, W/m<sup>2</sup>K  
 $h_k$  sensible enthalphy, kJ/kg  
 $k$  thermal conductivity, W/mK  
 $L$  tube length, m  
 $q$  heat flux  
 $Re$  Reynolds number  
 $p$  pressure, Pa  
 $T$  temperature, K  
 $v$  velocity, m/s  
 $v_{dr}$  drift velocity, m/s

## Greek symbols

- $\phi$  volume fraction  
 $\mu$  dynamics viscosity, kg/ms  
 $\rho$  density, kg/m<sup>3</sup>  
 $\eta$  kinematic viscosity, m<sup>2</sup>/s

## Subscripts

- $bf$  base fluid  
 $eff$  effective  
 $f$  base fluid  
 $p$  particle  
 vol% volume percentage