Water holdup in no-slip oil-water two-phase stratified flow

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ABSTRACT

This paper develops a water holdup calculation model based on flow characteristics for no-slip oil-water two-phase flow. The model is constructed in consideration of two flow patterns: the stratified flow (ST) and the stratified flow with some mixing at the interface (ST&MI). The predicted values obtained by the model are compared with the test data on white oil-water flow. The results show that the model can accurately predict the water holdup of no-slip oil-water two-phase flow, which is of great significance for the determination of the working parameters in oilfields.

Keywords: Oil-water Two-phase Flow, No-slip Water Holdup, Inlet Water Fraction, Stratified Flow Model, Three-Phase Segregated Flow Model

1. INTRODUCTION

The research of oil-water flow has direct bearing on the economic design and operation in oilfields [1] as 1/3 of the total investment on surface engineering goes to oil pipelines and 40% of the total energy consumption of production takes place during transportation.

The oil-water two-phase flow is a common phenomenon in the oil industry. When the oil is finally delivered, the oil and water are immiscible with each other and have significant differences in density. During the movement in well bores or pipelines, the oil and water slip past each other owing to the velocity difference. The smaller the slippage velocity between oil and water, the lower the pressure loss of the oil-water two-phase flow. Thus, the research of water holdup in no-slip oil-water two-phase flow can laya theoretical basis for identifying the working parameters in oil gathering and transmission systems.


2. THEORETICAL ANALYSIS

2.1 The ST model

The momentum balance in the oil layer and the water layer of the ST is expressed as follows.

In the oil layer:

\[-\frac{dp}{dx} + \tau_o S_o + \tau_i S_i + A_o \rho_o g \sin \alpha = 0\]  \hspace{1cm} (1)

In the water layer:
\[- \frac{dp}{dx} A_w - \tau_w S_w + \tau_i S_i = A_w \rho_w g \sin \alpha = 0 \]  
\hspace{1cm} \text{(2)}

Assume at the two layers share the same pressure drop, and cancel out the pressure drop terms in (1) and (2).

\[ \tau_o = \rho \left( \frac{U_o - U_i}{2} \right)^2 \]  
\hspace{1cm} \text{(6)}

where the friction factors are evaluated by the Brauner’s approach [28] (1989):

\[ f_o = C_o \left( \frac{D U_o \rho_o}{\mu_o} \right)^{n_o} \]  
\hspace{1cm} \text{(7)}

\[ f_w = C_w \left( \frac{D U_w \rho_w}{\mu_w} \right)^{n_w} \]  
\hspace{1cm} \text{(8)}

\[ f_i = B f_o \left( U_o > U_o \right) \]  
\hspace{1cm} \text{(9)}

\[ f_i = B f_o \left( U_o < U_o \right) \]  
\hspace{1cm} \text{(10)}

\[ f_m = C_m \left( \frac{D U_m \rho_m}{\mu_m} \right)^{n_m} \]  
\hspace{1cm} \text{(11)}

If the slippage velocity is 0, then \( U_o = U_w \).

The shear stress of the two layers are evaluated by a Blasius-type equation (Taitel and Dukler, 1976) [27]:

\[ \tau_o = f_o \rho_o \frac{U_o^2}{2} \]  
\hspace{1cm} \text{(4)}

\[ \tau_w = f_w \rho_w \frac{U_w^2}{2} \]  
\hspace{1cm} \text{(5)}

The shear stress of phase interface is obtained by the following equation:

\[ \tau_i = f_i \rho \left( \frac{U_i - U_o}{2} \right)^2 \]  
\hspace{1cm} \text{(6)}

In a horizontal pipe, the inclination angle of the pipe \( \alpha = 0 \phi \).

Substitute Equations (6)–(20) into (13), and we have:

\[ \left( \frac{\mu \rho_o}{\mu_w} \right)^{\frac{1}{2}} \left( \frac{S_o}{S_w} \right) \left( \frac{A_w}{A_o} \right) ^{\frac{1}{2}} = \frac{\beta}{\pi - \beta} = \frac{2(\pi - \beta) + \sin 2 \beta}{2\pi} \]  
\hspace{1cm} \text{(21)}

where \( \eta_o = \frac{2(\pi - \beta) + \sin 2 \beta}{2\pi} \) is the hydraulic equivalent diameter is expressed as:

\[ D_o = \frac{4A_o}{S_o + S_i} \] \[ D_w = \frac{4A_w}{S_w} \left( U_o > U_w \right) \]  
\hspace{1cm} \text{(14)}

\[ D_w = \frac{4A_o}{S_o} \] \[ D_w = \frac{4A_w}{S_w} \left( U_o \approx U_w \right) \]  
\hspace{1cm} \text{(15)}

\[ D_w = \frac{4A_o}{S_o} \] \[ D_w = \frac{4A_w}{S_w} \left( U_o < U_w \right) \]  
\hspace{1cm} \text{(16)}

\[ \eta_o = \frac{2(\pi - \beta) + \sin 2 \beta}{2\pi} \]  
\hspace{1cm} \text{(22)}

**2.2 The three-phase segregated flow model**

When it comes to the ST&M, Vedapuri (1997) [3] developed a three-layer flow model for predicting the water film in oil-water two-phase flow. The model treats the oil layer, water layer and mixed layer as three different phases with distinctive properties. Thus, the oil-water two-phase flow is converted to a three-phase stratified flow of oil, mixture and water. Figure 2 shows the sketch map of this model.
Equation 1.2

\[ \tau_{12} = \frac{f_{12}}{2} \rho(U_m - U_s) [U_m - U_s] \]

Substitute Equations (7) to (11), (14) to (16), (30) and (31) into (29):

\[ \mu_m = \left( \frac{\rho_o}{\rho_w} \right)^{n-1} (1 - h_{wm}) + \rho_w h_{wm} \]

Thus, the following equation applies to the mixing layer.

\[ \mu_m = \left( \frac{\rho_o}{\rho_w} \right)^{n-1} (1 - h_{wm}) + \rho_w h_{wm} \]

The mass balances between the three layers are expressed as:

\[ Q_{WT} = Q_{WP} + h_{WM} Q_M \]

\[ Q_{OT} = Q_{OP} + (1 - h_{WM}) Q_M \]

From Equations (34) and (35), the surface velocities can be derived as:

\[ U_{SWinlet} = U_{SW} + h_{WM} U_{SM} \]

\[ U_{SOinlet} = U_{SO} + (1 - h_{WM}) Q_M \]

When the slippage velocity is 1, the critical water holdup can be analyzed by the Vedapuri model (1997). Figure 3 is the sketch map of oil-water flow:

Figure 3. Sketch map of the wetted perimeter in the three-phase segregated oil-water flow

The water fraction of the mixture layer \( \eta_{wm} \) can be expressed as:

\[ \eta_{wm} = \frac{Q_{sw}}{Q_{sw} + Q_{so}} \times 100\% = \frac{\gamma - \sin \gamma}{\beta + \gamma - \sin \beta - \sin \gamma} \]

Under the no-slip condition, \( 1.2U_w = 1.2U_o = U_m \).

\[ S_o = \frac{D}{2} \beta \quad S_w = \frac{D}{2} \gamma \quad S_m = \frac{D}{2} (2\pi - \beta - \gamma) \]
\[
A_1 = \frac{D^3}{8} (\beta - \sin \beta) \\
A_2 = \frac{D^3}{8} (\gamma - \sin \gamma) \\
A_3 = \frac{D^3}{8} (2\pi - \beta - \gamma + \sin \beta + \sin \gamma)
\]  

(40)

Substitute equations (39) and (40) into (32):

\[
\left\{ \frac{1.44(2\pi - \beta - \gamma) + 0.08(\sin \frac{\beta}{2} + \sin \frac{\gamma}{2})}{2\pi - \beta - \gamma + \sin \beta + \sin \gamma} \right\} \left( \frac{\beta - \sin \beta}{\beta - \sin \beta} \right) \left( \frac{\gamma - \sin \gamma}{\gamma - \sin \gamma} \right) = \frac{2\pi - \beta - \gamma}{2\pi - \beta - \gamma + \sin \beta + \sin \gamma}
\]

\begin{equation}
= \frac{1}{2} \left[ \rho \frac{\mu_{\text{m}} + (\rho \mu_{\text{m}} - \rho_{\text{m}}) \eta_{\text{m}}}{\rho \mu_{\text{m}} + (\rho \mu_{\text{m}} - \rho_{\text{m}}) \eta_{\text{m}}} \right]^{\beta - \sin \beta} \left[ \rho \frac{\mu_{\text{m}} + (\rho \mu_{\text{m}} - \rho_{\text{m}}) \eta_{\text{m}}}{\rho \mu_{\text{m}} + (\rho \mu_{\text{m}} - \rho_{\text{m}}) \eta_{\text{m}}} \right]^{\gamma - \sin \gamma}
\end{equation}

(41)

where

\[
\eta_{\text{m}} = \eta_n = \frac{\beta - \sin \beta}{\beta - \sin \beta}
\]

(42)

Similarly, the following equation is established according to the momentum of the oil layer and the mixed layer:

\[
\left\{ \frac{1.44(2\pi - \beta - \gamma) + 0.08(\sin \frac{\beta}{2} + \sin \frac{\gamma}{2})}{2\pi - \beta - \gamma + \sin \beta + \sin \gamma} \right\} \left( \frac{\beta - \sin \beta}{\beta - \sin \beta} \right) \left( \frac{\gamma - \sin \gamma}{\gamma - \sin \gamma} \right) = \frac{2\pi - \beta - \gamma}{2\pi - \beta - \gamma + \sin \beta + \sin \gamma}
\]

\begin{equation}
= \frac{1}{2} \left[ \rho \frac{\mu_{\text{m}} + (\rho \mu_{\text{m}} - \rho_{\text{m}}) \eta_{\text{m}}}{\rho \mu_{\text{m}} + (\rho \mu_{\text{m}} - \rho_{\text{m}}) \eta_{\text{m}}} \right]^{\beta - \sin \beta} \left[ \rho \frac{\mu_{\text{m}} + (\rho \mu_{\text{m}} - \rho_{\text{m}}) \eta_{\text{m}}}{\rho \mu_{\text{m}} + (\rho \mu_{\text{m}} - \rho_{\text{m}}) \eta_{\text{m}}} \right]^{\gamma - \sin \gamma}
\end{equation}

(43)

3. TEST DESCRIPTION

3.1 Overview

The test was conducted on a multi-phase flow test equipment (Figure 4). The equipment consists of an oil tube, a water tube, and a gas tube. The oil tube and water tube pump working fluid to the test system. The test system is composed of a water inlet system, an oil inlet system, a test section and a measuring system.

*Figure 4. Multi-phase flow test equipment*

3.2 Test fluids

3.2.1 White oil viscosity

The oil viscosity was measured with a Brookfield DV3TLV digital viscometer and a Thermosel heater. Based on the results shown in Figure 5, the relationship between oil viscosity and temperature is obtained:

\[
\mu_o = 26.251 - 5.8032 \ln(T - 6.2104)
\]

*Figure 5. Viscosity-temperature curve of the white oil*

3.2.2 White oil density

The density of white oil varies little as temperature changes. Under the normal temperature (25°C), the white oil density is 0.857g/ml or 857kg/m³.

3.2.3 Water viscosity

Figure 6 shows the viscosity-temperature curve of the water. The relationship between water viscosity and temperature is obtained by the following formula.

\[
\mu_w = 2.5059 - 0.4876 \ln(T + 2.2805)
\]

*Figure 6. Viscosity-temperature curve of the water*

3.3 Test procedures

The test device for the oil-water two-phase flow is a closed loop. The oil and water are stored in separate tanks and pumped to the mixing tank. The oil-water ratio in the tank is adjusted to control the inlet water fraction, and to homogenize the mixture. The oil-water mixture is then pumped through the regulator and flow meters to the 9.2m-long transparent plexiglass test section. Finally, the mixture is pumped back to the mixing tank via the gas-liquid separator for further circulation. The central control system monitors the temperature, liquid level and stirring device in the mixing tank, as well as the pressure, temperature, pressure gradient and velocity of the liquid and gas in the test section. It also controls the closing valves in the test section.
All the tests are conducted under the horizontal flow conditions at normal temperature and pressure. Table 1 specifies the parametric ranges in the test.

**Table 1.** Parametric ranges in the test

<table>
<thead>
<tr>
<th>Working conditions</th>
<th>Volume velocity of the mixture flow</th>
<th>Inlet water fraction</th>
<th>Test tube diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2, 4, 6, 8, 10, 12, 14 (m³/h);</td>
<td>0, 10, 20, 30, 40, 50, 60, 80%;</td>
<td>40, 60 mm.</td>
</tr>
</tbody>
</table>

### 3.4 Results and discussions

For the ST model, the test data are substituted into the Equation (22) to obtain the variation of inlet water fraction with temperature (Figure 7). In the event of low velocity, the oil and water are completely dissociated. Without considering the effect of oil-water density, it is possible to obtain the relationship between critical inlet water fraction and temperature at the oil-water slippage velocity of 0 \((U_o=U_w)\). Figure 7 indicates that the inlet water fraction \(\eta_w\) is linearly correlated with temperature. When the oil and water share the same velocity, the \(\eta_w\) is about 35.5% at the normal temperature.

![Figure 7. Inlet water fraction variation with temperature](image)

The three-phase segregated flow model, \(n=0.2\) in turbulent flow. The variation in inlet water fraction \(\eta_w\) with temperature is obtained through the following steps: combine Equations (41) and (42), substitute \(n=0.2\) to the combined equation, and apply the constraint that \(1.2U_o=1.2U_w=U_m\). As shown in Figure 8, when \(1.2U_o=1.2U_w=U_m\), the temperature has little impact on the critical inlet water fraction. At 25° C \((\beta=2.8826, \gamma=3.1867)\), the critical inlet water fraction is about 55.17%.

![Figure 8. Inlet water fraction variation with temperature](image)

There are many possible flow patterns for the oil-water flow in a horizontal pipe. According to the flow pattern maps prepared by Arirachakam et al. (1989) and Angeli and Hewitt et al. (2000), the flow pattern (the ST) is independent of the inlet water fraction when the velocity of the mixture is less than 0.5 m/s; however, the flow pattern (the ST&MI) will become correlated with the inlet water fraction when the mixture velocity falls between 0.5 m/s and 1 m/s.

Figures 9 and 10 illustrate the variations of water holdup with inlet water fraction for the ST and the ST&MI, respectively. For the ST (Figure 9), the inlet water fraction grows slightly from 35.5% at the mixture velocity of 2 m/h to 39.5% at the mixture velocity of 4 m³/h, when the water holdup \(h_w\) is equal to inlet water fraction \(\eta_w\). For the ST&MI (Figure 10), the inlet water fraction is about 55% when the water holdup \(h_w\) is equal to inlet water fraction \(\eta_w\). The water holdup results obtained from the test are in good agreement with the predicted results. The two figures demonstrate that the water holdup \(h_w\) is approximately equal to the inlet water fraction \(\eta_w\) when the inlet water fraction reaches the critical inlet water fraction, indicating that the water holdup prediction of the no-slip oil-water flow water is well grounded.

![Figure 9. Waterholdup variation with water fraction in the ST](image)

![Figure 10. Waterholdup variation with water fraction in the ST&MI](image)

### 4. CONCLUSION

The oil-water two-phase flow patterns could be classified into two categories: the segregated flows (ST; ST&MI) and dispersed flows (Do/w&Ow; O/w; Dw/o&Dw/o; W/o). In the dispersed flows, the oil and water are expected to be fully mixed and the oil-water slippage velocity reaches zero. Therefore, only the ST and ST&MI are taken into account in this research.

The author puts forward a water holdup prediction model for the no-slip oil-water two-phase flow. The predicted results are proved to be true and accurate. Through further analysis of the test data, it is found that the critical inlet water fraction...
is about 35.5% in the ST model, and 55.17% in the three-phase segregated flow model, provided that $U=U_w$. The test results also indicate that the water holdup $h_w$ is approximately equal to the inlet water fraction $q_w$ when the inlet water fraction reaches the critical inlet water fraction.

REFERENCES


NOMENCLATURE

$A$ the cross-sectional area of the pipe, $m^2$;
$S$ the wetted perimeter, $m$;
$D$ the hydraulic diameter, $m$;
$\tau$ the shear stress, $N/m^2$;
$\alpha$ the inclination angle of the pipe;
$U$ the in-situ velocity, $m/s$;
$f$ the friction coefficient;
$\mu$ the dynamic viscosity, $cp$;
$R$ the pipe radius, $m$;
$\eta_w$ the water fraction, $\%$;
$\eta_{wm}$ the water fraction of the mixture layer, $\%$;
$Q$ the volume velocity, $m^3/s$;
$Q_T$ the volume velocity of the total flow, $m^3/s$.

$U_{SWinput}$ the input surface velocity of the water layer, $m/s$;
$U_{SOinput}$ the input surface velocity of the oil layer, $m/s$;
$U_{SW}$ the surface velocity of the water layer, $m/s$;
$U_{SO}$ the surface velocity of the oil layer, $m/s$;
$U_{SM}$ the surface velocity of the mixture layer, $m/s$;
$h_w$ the water holdup, $\%$.

Subscripts

o The oil layer
w The water layer
m The mixture layer
i The oil-water interface
i1 The water-mixture interface
i2 The mixture-oil interface