Hall Current Effects on Binary Mixture Flow of Oldroyd-B Fluid through a Porous Channel

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ABSTRACT

Free convective binary mixture flow of visco-elastic fluid through a porous channel in presence of Hall effects, radiation and first order chemical reaction has been considered. A magnetic field of uniform strength has been applied along the transverse direction to the channel. The visco-elastic fluid flow is governed by Oldroyd fluid-B model. Two important rheological parameters involved in the constitutive equation are relaxation parameter and retardation parameter. In the mixture, one of the components is assumed to be rarer lighter. The temperature and concentration of the fluid at one surface is assumed to be constant but temperature and concentration of the fluid at the second surface are assumed to be oscillating about a constant temperature and concentration respectively. The governing equations of momentum, temperature and concentration are solved analytically by using separation of variable technique. Results are discussed graphically for various values of flow parameters involved in the solution.

Keywords: Relaxation and retardation, Oldroyd-B fluid model, Free convection, Separation of variable, Shearing stress.

1. INTRODUCTION

Mechanism of relaxation and retardation parameters in the Oldroyd fluid model has attracted many researchers as it can study the visco-elastic fluid motion in more generalized way. The constitutive equation of Oldroyd fluid model (Oldroyd [1], [2]) is given by

\[ \sigma_{ij} = -p \delta_{ij} + \tau_{ij} \& \left( 1 + \lambda_1 \frac{d}{dt} \right) \tau_{ij} = 2\mu \left( 1 + \lambda_2 \frac{d}{dt} \right) \dot{e}_{ij} \]

(1.1)

Dey[3] and Dey and Khound[4] have investigated the effects of relaxation and retardation of visco-elasticity on governing fluid motion. Nigam and Singh [5] have derived the asymptotic solutions of the energy equation of heat-transfer problem of fluid flow between two infinite parallel plates. Soundalgekar and Bhat [6] have investigated oscillatory channel flow and heat transfer. Hydro-magnetic Couette flow with heat transfer and Hall effects has been investigated by Soundalgekar et al. [7], Saoundalgekar and Uplekar [8] and Hossain and Rashid [9]. Rapits et al. [10], Rapitis and Perdkis [11], Hassanien and Mansour [12] and Aldoss et al. [13] have analysed hydro-magnetic flow problem through a porous medium. Vajravelu [14] have obtained an exact periodic solution of a hydromagnetic flow in a horizontal channel considering hydro-magnetic and hydrodynamic cases. Attia and Kotb [15] have discussed numerically the problem of steady flow bounded by two parallel infinite insulated horizontal plates and the heat transfer. Hall current effects of unsteady Hartmann flow between two parallel porous plate of Newtonian and visco-elastic fluid have been analysed by Attia[16] & [17]). Closed form solution of heat and mass transfer problem in elastico-viscous fluid past an impulsively started infinite vertical plate with Hall effect has been obtained by Choudhury and Jha[18]. Effect of oscillatory motion of visco-elastic fluid over an infinite stretching sheetthrough porous media in the presence of magnetic field with applied suction has been studied by Rajagopal et al. [19]. Radiative heat transfer in MHD oscillatory flow through porous medium bounded by two vertical porous plates has been analysed by Singh and Garg [20]. Singh [21] have investigated visco-elastic mixed convective MHD oscillatory flow through a porous medium filled in a vertical channel. Exact solution of MHD mixed convection periodic flow in a rotating vertical channel with heat radiation has been derived by Singh [22]. Singh and Pathak [23] have analysed the Effects of slip conditions and Hall current on an oscillatory convective flow in a rotating vertical porous channel with thermal radiation. Seth et al. ([24], [25]) have studied the effects of Hall current in presence of ramped temperature. Ahmed et al. [26] have investigated the effects of Hall current on MHD mass transfer flow in a rotating system.

In various science and technology problems, separation process of binary mixture components has been used.
chemical industry, problem of visco-elastic fluid flow with heat and mass transfer is often used to study polymer solution mixed with various organic compounds. Examples of multiple component electrically conducting fluids are molten fluid in earth’s crust, crude oil in petroleum etc [27]. In our study, we have assumed a binary mixture of visco-elastic fluid, where one of component is present in extremely small proportion.

The objective of this problem is to investigate the problem of oscillatory binary mixture flow of visco-elastic fluid with relaxation and retardation through a porous channel. Singh et al [28] have studied the hall current effects of visco-elastic fluid flow governed by second order fluid model though a porous medium with radiative heat transfer. The other physical properties like free convection (heat and mass transfer due to density differences) and first order chemical reaction also have been considered in this flow problem.

2. MATHEMATICAL FORMULATION

An unsteady binary mixture flow of visco-elastic fluid characterized by Oldroyd model past a porous channel has been investigated. The channel is bounded by two infinite vertical plates separated by distance \(d\). Here x-axis is taken along the length of the plates and z-axis is taken perpendicular to it. To stable the system, a magnetic field is applied along the transverse direction to the surface. Application of transverse magnetic field generates Lorentz force and Hall current. In the mixture, let \(C_1\) and \(C_2\) be the concentrations of lighter and heavier components respectively and \(C_1= 1 - C_2\). The motion of binary mixture is similar to normal fluid flow with velocity \(u' = \frac{u_1 + u_2}{\rho_1 + \rho_2} \frac{\rho_1 + \rho_2}{\rho_1 + \rho_2} \) and density \(\rho = \rho_1 + \rho_2\), where \(\rho_1, \rho_2, u_1\) and \(u_2\) are densities and velocities of rarer and heavier components respectively.

**Figure 1. Geometry of the problem**

**Momentum equation:**

\[
\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \frac{\partial u'}{\partial t} = -\rho \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left[\mu \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \frac{\partial u'}{\partial x} + \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \sigma B_0 \left(\frac{u'}{v U} - \frac{\nu v'}{k_p}\right) + g \beta(T' - T_0) + g \beta'(C' - C_0)\right] \quad (2.1)
\]

\[
\left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \frac{\partial v'}{\partial t} = -\rho \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \frac{\partial p}{\partial y} + \frac{\partial}{\partial y} \left[\mu \left(1 + \lambda_2 \frac{\partial}{\partial t}\right) \frac{\partial v'}{\partial y} + \left(1 + \lambda_1 \frac{\partial}{\partial t}\right) \sigma B_0 \left(\frac{v'}{v U} - \frac{\nu v'}{k_p}\right) + g \beta(T' - T_0) + g \beta'(C' - C_0)\right] \quad (2.2)
\]

**Energy equation:**

\[
\rho C_p \frac{\partial T'}{\partial t} = \frac{k}{\alpha} \frac{\partial^2 T'}{\partial x^2} - \frac{\alpha}{\alpha} \frac{\partial T'}{\partial x} \quad (2.3)
\]

For a optically thin fluid, the radiative heat transfer (Cogley et al. [29]) is given by, \(\frac{\partial T'}{\partial x} = 4\sigma_2 (T' - T_0)\), where, \(\sigma_2 = \int_0^\infty K_{a2} \frac{\partial T'}{\partial x} d\lambda'\), \(K_{a2}\) is absorption coefficient and \(e_{\lambda h}\) is Plank’s function.

Neglecting the pressure diffusion co-efficient and thermal diffusion coefficient, we get the energy equation for species concentration \((C_1 = C'\) in presence of first order chemical reaction (rate of reaction is proportional to concentration) as follows [22]:

\[
\frac{\partial C'}{\partial t} = D \frac{\partial^2 C'}{\partial x^2} - k_1 (C' - C_0) \quad (2.4)
\]

Here, \(p = p' - p_s\) pressure difference, \(p'\) fluid pressure and \(p_s\) fluid pressure at static case.

The corresponding boundary conditions are as follows:

\[
u' = 0; v' = 0; T' = T_0; C' = C_0; z' = -\frac{d}{2},
\]

\[
u' = 0; v' = 0; T' = T_0 + (T_w - T_0) e^{\omega t'},
\]

\[
(C' = C_0 + (C_w - C_0) e^{\omega t'}; z' = \frac{d}{2} \quad (2.5)
\]

3. METHOD OF SOLUTION

The following non-dimensional quantities have been used into the equations from (2.1) to (2.4) to make them dimensionless.

\[
x = \frac{x'}{d}; y = \frac{y'}{d}; z = \frac{z'}{d}; u = \frac{u'}{U}; v = \frac{v'}{U}; t = \frac{t'}{U};
\]

\[
p = \frac{p^*}{\rho U^2}; \omega = \frac{\omega d}{U}; T = \frac{T' - T_0}{T_w - T_0}; \quad C = \frac{C' - C_0}{C_w - C_0}; \quad \alpha = \frac{\lambda_1 U}{d};
\]

\[
b = \frac{d}{d^2}; M = \frac{\sigma B_0^2 d^2}{v^2 U^2};
\]

\[
G_r = \frac{g \beta d^2 (T - T_0)}{\rho U^2}; \quad G_m = \frac{g \beta d^2 (C - C_0)}{\rho U^2}; \quad h = \frac{\partial P}{\partial x};
\]

\[
\frac{P e}{\rho C_p d U^2 k} = \frac{2ad}{\sqrt{k}}; h = \frac{d^2 k}{D}; \quad k_p = \frac{k'_p}{d}; \quad m = \omega_{\lambda e}
\]

The dimensionless equations are as follows:

\[
Re \left(1 + \frac{\partial u}{\partial x}\right) = -Re \left[1 + a \frac{\partial}{\partial x}\left(\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + b \frac{\partial^2 u}{\partial x^2} + \frac{1}{1 + m^2} \right)\right] + \frac{1}{1 + m^2} \left[M \left(\frac{m u + v}{1 + m^2}\right) - \frac{\nu v'}{k_p}\right] \quad (3.1)
\]

\[
Re \left(1 + \frac{\partial v}{\partial y}\right) = -Re \left[1 + a \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial y^2} + b \frac{\partial^2 v}{\partial y^2} + \frac{1}{1 + m^2} \right)\right] + \frac{1}{1 + m^2} \left[M \left(\frac{m u + v}{1 + m^2}\right) - \frac{\nu v'}{k_p}\right] \quad (3.2)
\]

\[
Pe \frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial x^2} - N^2 T \quad (3.3)
\]

\[
Sc \frac{\partial C}{\partial x} = \frac{\partial^2 C}{\partial x^2} - hC \quad (3.4)
\]

Equations (3.1) and (3.2) can be combined into a single differential equation by assuming \(F = u + iv\) and we get,
\[ Re \left( 1 + a \frac{d}{dt} \right) \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial \Psi}{\partial y} + i \frac{\partial \Phi}{\partial y} \right) + b \frac{\partial^3 \Phi}{\partial x^2 \partial y} + \left( 1 + a \frac{d}{dt} \right) \left[ \frac{-MF(1+i\omega)}{1+m^2} - \frac{F}{k} + G,T + G_m C \right] \]  

(3.5)

Following ([23][28]), the pressure gradient of the oscillatory flow is taken as \( \frac{d\Phi}{dx} = -A e^{-i\omega t} \) \( \frac{d\Psi}{dy} = 0 \)

To solve the above equations from (3.3) to (3.5) the following boundary conditions are used:

\[ T = C = F = 0; z = -\frac{1}{2} \quad \& \quad T = C = e^{i\omega t}, F = 0; z = \frac{1}{2} \]  

(3.6)

To solve the above equations separation of variable method is used and the complex form of the solution corresponding to the boundary are taken as

\[ T = T_1(z)e^{i\omega t}, C = C_1(z)e^{i\omega t} \& F = f_0(z)e^{i\omega t} \]  

(3.7)

Using (3.7) into the equations and equating the like terms, we get

\[ T''_1 - (N^2 + i\omega Pe)T_1 = 0 \]  

(3.8)

\[ C''_1 - (h + i\omega Sc)C_1 = 0 \]  

(3.9)

\[ f''_0 + (A_7 + iA_9)f_0 = -(A_9 + iA_{10})[G_m(B_3e^{(A_5z+iA_2)} + B_2e^{(-A_5z+iA_2)})] + G_m(B_3e^{(A_5z+iA_2)} + B_2e^{(-A_5z+iA_2)}) + (A_{11} + iA_{12})Re \]  

(3.10)

The relevant boundary conditions are as follows:

\[ T_1 = C_1 = f_0 = 0; z = -\frac{1}{2} \quad \& \quad T_1 = C_1 = 1, f_0 = 0; z = \frac{1}{2} \]  

(3.11)

Solution of the equations (3.8) to (3.10) corresponding to boundary conditions (3.11) are obtained but not presented here for the sake of brevity.

4. RESULTS AND DISCUSSIONS

Figure 2 to 8 represent the velocity profiles against the displacement variable for primary and secondary flows for various values of flow parameters involved in the solution. The figures enable the fact that the effect of flow parameters is more prominent in the centre of the channel. It is seen that during the growth of relaxation parameter, fluid flows (both primary and secondary) have experience an increasing speed as it can be interpreted that during the growth of relaxation parameter (figure 2), stress relaxes more rapidly, and more energy can be stored, as a consequence speed increases. Retardation parameter (figure 3) is connected with creeping motion of visco-elastic fluid and increasing values of retardation parameter accelerates the fluid motion, as physically it is interpreted as the growth of retardation parameter reduces the creepiness of fluid flows (primary and secondary).

Application of transverse magnetic field generates a force field known as Lorentz force, the combination of Lorentz force and viscosity makes the system thicker and as a result speed slows down. This physical phenomenon is clearly seen in our result [figure 4]. During the growth of M (magnetic parameter) by 27.27% (from M=2.2 to 2.8), there is a fall in magnitude of velocity by 91% (approximately) at the central portion of the channel. The Hall current parameter also has a negative impact on the motion (figure 5) as it decelerates the fluid motion by 99.36% (approximately) at the central portion of channel during the enhancement of m by 60% only. Thus the influence of m (Hall current parameter) on fluid motion is greater than M (magnetic parameter).

Another force which guides the fluid motion is the pressure gradient and its effect on fluid motion is shown by figure 6 and 7 and it is seen that as magnitude of pressure gradient increases by 300%, maximum increment of speed of primary flow is observed as 62.5% (approximately) and speed of secondary fluid motion is increased by 66.67%.

Formation of viscous drag of governing fluid motion at the surface is very useful in aerodynamics as the wings of aeroplane are constructed in such a manner that shearing stress should be less. In our study we have shown the effects (figure 8 to 12) of visco-elastic parameters, magnetic parameter, Hall current parameter, pressure gradient and chemical reaction on shearing stress or viscous drag at z=-1/2 in the time period [0, 20]. The oscillating nature of shearing stress with respect to time is clearly seen from the figures. Figures 8 and 9 depict the nature of shearing stress formed by primary and secondary fluid motion during Newtonian fluid (a=0, b=0) and visco-elastic fluid (a =1.35, b=3.5). It is noticed that during the motion of visco-elastic fluid, the amplitude of variation of shearing stress is increased by 39% (approximately) in compare Newtonian fluid. It is due to the presence creepiness of friction and resistance forces in visco-elastic fluid motion. During the oscillatory nature of shearing stress, phase difference is seen between Newtonian fluid and visco-elastic fluid.

Effects of pressure gradient on shearing stresses are represented by figure 10 and it tells that during the growth in magnitude of pressure gradient (A) by 300% there is a fall by 19% (approximately) in shearing stress it shows the power of pressure gradient is dominant over frictional force. A fall in shearing stress by 8% and 25% respectively have been noticed during the growth of magnetic parameter by 27.27% (figure 11) and Hall current parameter by 60% (figure 12).

5. CONCLUSION

Some of the important points from the above study are highlighted as below:

• To control the fluid motion, magnetic parameter or Hall current parameter may be increased.
• Speed of fluid motion experiences an increasing trend with pressure gradient.
• Growth of pressure gradient, magnetic parameter, Hall current parameter and chemical reaction parameter helps to reduce the strength of shearing stress at the surface.
• Amplitude of the shearing stress of visco-elastic fluid motion is higher than the shearing stress formed due to the motion of Newtonian fluid.
6. GRAPHS

Figure 2. Primary velocity against z for ω=0.01, M=2, Re=0.3, Pe=8, N=8, $G_m=4$, $G_r=7$, h=0.5, Sc=2, $k_p=0.2$, m=1, t=0.1

Figure 3. Secondary velocity against z for ω=0.01, M=2, Re=0.3, Pe=8, N=8, $G_m=4$, $G_r=7$, h=0.5, Sc=2, $k_p=0.2$, m=1, t=0.1

Figure 4. Primary velocity against z for a=1.35, b=3.5, ω=0.01, $A= - 0.1$, Re=0.3, Pe=8, N=8, $G_m=4$, $G_r=7$, h=0.5, Sc=2, $k_p=0.2$, m=1, t=0.1

Figure 5. Primary velocity against z for a=1.35, b=3.5, ω=0.01, $A= - 0.5$, M=2, Re=0.3, Pe=8, N=8, $G_m=4$, $G_r=7$, h=0.5, Sc=2, $k_p=0.2$, t=0.1

Figure 6. Primary velocity against z for a=1.35, b=3.5, ω=0.01, M=2, Re=0.3, Pe=8, N=8, $G_m=4$, $G_r=7$, h=0.5, Sc=2, $k_p=0.2$, m=1, t=0.1.

Figure 7. Secondary velocity against z for ω=0.01, M=2, Re=0.3, Pe=8, N=8, $G_m=4$, $G_r=7$, h=0.5, Sc=2, $k_p=0.2$, m=1, t=0.1
Figure 8. Shearing stress due to primary flow at $z = -\frac{1}{2}$ for $\omega=0.01, M=2, Re=0.3, Pe=8, N=8, G_m=4, G_r=7, h=0.5, Sc=2, k_p=0.2, m=1$ against $t$

Figure 9. Shearing stress due to secondary flow $z = -\frac{1}{2}$ for $\omega=0.01, M=2, Re=0.3, Pe=8, N=8, G_m=4, G_r=7, h=0.5, Sc=2, k_p=0.2, m=1$ against $t$

Figure 10. Shearing stress for primary and secondary flows at $z = -\frac{1}{2}$ for $a=1.35, b=3.5, \omega=0.01, M=2, Re=0.3, Pe=8, N=8, G_m=4, G_r=7, h=0.5, Sc=2, k_p=0.2, m=1$ against $t$

Figure 11. Shearing stress for primary and secondary flows at $z = -\frac{1}{2}$ for $\omega=0.01, a=1.35, b=3.5, Re=0.3, Pe=8, N=8, G_m=4, G_r=7, h=0.5, Sc=2, k_p=0.2, m=1$ against $t$

Figure 12. Shearing stress for primary and secondary flows $z = -\frac{1}{2}$ for $a=1.35, b=3.5, \omega=0.01, A=0.5, M=2, Re=0.3, Pe=8, N=8, G_m=4, G_r=7, h=0.5, Sc=2, k_p=0.2$

REFERENCES


**NOMENCLATURE**

\( u \) & \( v \) are velocity of fluid (LT\(^{-1}\)), \( x', y' \) & \( z' \) are displacement variable (L), \( t' \) be time (T), \( p' \) fluid pressure (ML\(^{-1}\)T\(^{-2}\)), \( p \) fluid pressure at static case (ML\(^{-1}\)T\(^{-2}\)), \( p \) dimensionless fluid pressure difference, \( d \) distance between two plates (L), \( T' \) temperature of fluid (K), \( C' \) concentration of rarer component (MoL\(^{-3}\)), \( C_s \) specific heat of fluid at constant pressure (L\(^2\)T\(^{-1}\)K\(^{-1}\)), \( k \) thermal conductivity (ML\(^{-1}\)T\(^{-1}\)).
$B_0$ magnetic field strength ($MT^{-1}$), $x$, $y$ and $z$ are dimensionless displacement variable, $u$ & $v$ dimensionless velocity of fluid, $t$ dimensionless time, $T$ dimensionless temperature of fluid, $C$ dimensionless concentration, $Re$ Reynolds number, $M$ magnetic parameter, $Gr$ Grashof number for heat transfer, $Gm$ Grashoff number for mass transfer, $N$ radiation parameter, $Sh$ dimensionless shearing stress, $T_0$ temperature of fluid at static case (K), $T_w$ mean temperature of fluid at $z = -0.5$ (K), $C_0$ concentration of rarer component at static case (MolL$^{-3}$), $C_w$ mean concentration of rarer component at $z= -0.5$ (MolL$^{-3}$), $A$ amplitude of pressure gradient, $a$ dimensionless relaxation time, $b$ dimensionless retardation time, $m$ Hall current parameter, $k_1$ chemical reaction parameter ($T^{-1}$), $h$ dimensionless reaction parameter, $k_p$ dimensionless permeability parameter, $D$ molecular diffusivity ($L^2T^{-1}$), $k_p'$ permeability parameter ($L^2$), $\rho$ density of fluid, ($ML^{-3}$), $\nu$ Kinematic viscosity ($L^2T^{-1}$), $\sigma$ electrical conductivity ($L^{-3}M^{-1}T^{2}$), $\lambda_1$ relaxation time parameter (T), $\lambda_2$ retardation time parameter (T), $\omega'$ frequency of oscillation, ($T^{-1}$), $\omega$ dimensionless frequency, $\beta$ co-efficient of volume expansion due to heat ($K^{-1}$), $\beta^*$ co-efficient of volume expansion due to concentration, Mol$^{-1}$L$^3$, $\tau_0$ viscous stress ($ML^{-2}T^{-2}$), $\eta_0$ dynamic viscosity ($ML^{-1}T^{-1}$), $U$ mean flow velocity($LT^{-1}$), $g$ acceleration due to gravity($LT^{-2}$), $Pe$ peclent number, $\omega_e$ electron frequency($T^{-1}$), $\tau_e$ electron collision time(T).