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LATTICE BOLTZMANN SIMULATION OF CONDUCTION-RADIATION HEAT TRANSFER IN A PLANAR MEDIUM

Imen Mejri[1], Ahmed Mahmoudi[1], Mohamed Ammar Abbassi[1] and Ahmed Omri[1]

[1] UR:Unité de Recherche Matériaux, Energie et Energies Renouvelables (MEER), Faculté des Sciences de Gafsa, B.P.19, Zarroug, Gafsa, 2112, Tunisie

ABSTRACT

In this paper, the 1-D conduction-radiation problem is solved by the lattice Boltzmann method. The effects of various parameters such as the scattering albedo, the conduction–radiation parameter, and the wall emissivity are studied. In order to check on the accuracy of the numerical technique employed for the solution of the considered problem, the present numerical code was validated with the published study. The found results are in good agreement with those published.

1. Introduction

Numerical modeling of the coupled transient radiative conductive heat transfer is an important field of research because of its relevance in various engineering applications such as the measurement of thermo-physical properties and the thermal control by ceramics and low density refractory material, heat transfer through the semitransparent, porous materials, multilayered insulations, glass fabrication, industrial furnaces, optical textile fiber processing, fibrous insulation [1–2]. Askri et al. [3] studied the accuracy and computational efficiency of the unstructured control volume finite element method (CVFEM) for computing coupled conduction, convection and radiation heat transfer in participating media, they found that the obtained results agree very well with other published works. Chaabane et al. [4] used the lattice boltzmann method to solve the energy equation in two dimensional enclosure of a problem involving a variety of boundary conditions, they found that the LBM results agree very well with the finite volume results. Ebrahimkhah et al. [5] solved one-dimensional inverse parabolic heat conduction problem by the conjugate gradient method (CGM) in estimating the unknown boundary heat flux based on the boundary temperature measurements. The obtained Results show that the inverse solutions can always be obtained with any arbitrary initial guesses of the boundary heat flux and the conjugate gradient method is an accurate and stable method to determine the unknown boundary heat flux in the inverse heat conduction problems. King et al. [6] studied numerically the total heat transfer in an air-filled enclosure, the effects of both laminar natural convection and surface radiation heat transfer are considered, the results show the significant effect of the interaction of surface radiation and natural convection on the total heat transfer. In recent years, use of the lattice Boltzmann method (LBM) as a potential computational fluid dynamics (CFD) tool for the solution of a large class of problems in science and engineering [7-9] has gained a momentum. As a different approach from the conventional CFD solvers, the LBM uses simple microscopic kinetic models to simulate complex transport phenomena. Its advantages include, among others, simple calculation procedure, simple and efficient implementation for parallel computation, easy and robust handling of complex geometries, and high computational performance with regard to stability and accuracy. Ho et al. [10, 11] solved a non-Fourier heat conduction problem in a planar medium using the LBM. Jiaung et al. [12] analyzed planar medium solidification using the LBM . Srinivasan et al. [13] analyzed microscale heat transfer in multilayered thin films using parallel computation of the Boltzmann transport equation. Guo and Zhao [14] solved a natural-convection problem and used temperature-dependent viscosity in the LBM formulation. Jamia et al. [15] used the LBM to solve natural-convection in a partitioned enclosure with inclined partitions attached to its hot wall. Chatterjee et al. [16] used the LBM to analyze solid–liquid phase transitions in the presence of thermal diffusion. Quite recently, its application has also been extended to solve energy equations of conduction, convection, and radiation heat transfer problems. Raj et al. [17] used the LBM to analyze the solidification of a semitransparent planer layer; they used the discrete transfer method (DTM) to compute the radiative information. Mishra et al. [18] used the LBM-FVM (finite volume method) to solve conduction–radiation problems in 1-D and 2-D geometries. Mondal et al. [19] used the lattice boltzmann method and the discrete ordinates method (DOM) for solving transient conduction and radiation heat transfer problems, they found that the LBM-DOM combination is in excellent agreement with the FDM-DOM combination, also the LBM-DOM was slightly faster than the FDM-DOM. Chaabane et al. [20] solved the conduction–radiation problems in enclosure using the lattice Boltzmann method and the control volume finite element method (CVFEM). Talukdar et al. [21] studied conduction–radiation problem using the collapsed dimension method (CDM) in one dimensional gray planar absorbing, emitting and anisotropically scattering medium. Mishra et al. [22] studied the performance of the collapsed dimension method (CDM) and the discrete transfer method (DTM) in terms of computational time and their abilities to provide accurate results in solving radiation and/or conduction mode problems in a 2-D rectangular enclosure. The CDM was found to be much more economical than the DTM.

In this paper, the 1-D conduction-radiation problem is solved by the lattice Boltzmann method. The effects of various parameters such as the scattering albedo, the conduction–radiation parameter, and the wall emissivity are studied. In order to check on the accuracy of the numerical technique employed for the solution of the considered problem, the present numerical code was validated with the published study.

2. Mathematical formulation

A one-dimensional planar conducting and radiating participating medium of length L is considered for the present study. The initial condition at time t = 0 for the temperature field T(x,t) is given by $T(x,0) = T_E$ and the boundary conditions at t > 0 by $T(0, t) = T_E$ and $T(L, t)= T_w > T_E$. The west and the east boundaries are diffuse gray with emissivities ε_W and ε_E, respectively. β, ω, and N are the extinction coefficient, the scattering albedo, and the conduction-radiation parameter, respectively.
For a homogeneous medium, the energy equation is given by:

$$\rho c_P \frac{\partial T}{\partial t} = k\nabla^2 T - \vec{\nabla}.\vec{q}_R \tag{1}$$

Where ρ is the density, c_P is the specific heat and k is the thermal conductivity. $\vec{q}_R$ is the radiative heat flux.

2.1 Energy equation

For a one-dimensional planar geometry, in the LBM with a D1Q2 lattice, the discrete Boltzmann equation with Bhatanagar- Gross-Krook (BGK) approximation is given by [9]:

$$\frac{\partial f_i(\vec{x},t)}{\partial t} + \vec{e}_i.\nabla f_i(\vec{x},t) = -\frac{1}{\tau}\left[f_i(\vec{x},t) - f^{eq}_i(\vec{x},t)\right] \quad i = 1 \text{ and } 2 \tag{2}$$

Where f_i is the particle distribution function denoting the number of particles at the lattice node $\vec{x}$ and time t moving in direction i with velocity $\vec{e}_i$ along the lattice $\Delta x = e_i \Delta t$ connecting the neighbors, τ is the relaxation time, and f^{eq}_i is the equilibrium distribution function. The relaxation time τ for the D1Q2 lattice is computed from:

$$\tau = \frac{\alpha}{|\vec{e}_i|^2} + \frac{\Delta t}{2} \tag{3}$$

Where α is the thermal diffusivity. For this lattice, the two velocities e_1 and e_2, and their corresponding weights w_1 and w_2, are given by:

$$e_1 = \frac{\Delta x}{\Delta t} \qquad e_2 = -\frac{\Delta x}{\Delta t} \tag{4}$$

$$w_1 = w_2 = \frac{1}{2} \tag{5}$$

After discretization, Eq. (2) is written as:

$$f_i(\vec{x} + \vec{e}_i\Delta t,\ t + \Delta t) = f_i(\vec{x},t) - \frac{1}{\tau}\left[f_i(\vec{x},t) - f^{eq}_i(\vec{x},t)\right] \tag{6}$$

The temperature is obtained after summing f_i over all direction:

$$T(\vec{x},t) = \sum_{i=1,2} f_i(\vec{x},t) \tag{7}$$

To process Eq. (6), an equilibrium distribution function is required, which for a conduction-radiation problem is given by:

$$f^{eq}_i(\vec{x},t) = w_i T(\vec{x},t) \tag{8}$$

To account for the volumetric radiation, the energy equation in the LBM formulation, Eq. (6) is modified to:

$$f_i(\vec{x} + \vec{e}_i\Delta t,\ t + \Delta t) = f_i(\vec{x},t) - \frac{1}{\tau}\left[f_i(\vec{x},t) - f^{eq}_i(\vec{x},t)\right] - \frac{\Delta t\, w_i}{\rho c_P}\frac{\partial q_R}{\partial x} \tag{9}$$

Where the divergence of radiative heat flux $\frac{\partial q_R}{\partial x}$ is given by:

$$\frac{\partial q_R}{\partial x} = \beta(1-\omega)(4\pi\frac{\sigma T^4}{\pi} - G) \tag{10}$$

G is the incident radiation.

2.2 radiative information

In the problem under consideration, the energy transport in any direction $\vec{s}$ is exclusively governed by the radiative transfer equation:

$$\frac{1}{c}\frac{\partial I(\vec{x},\vec{s},t)}{\partial t} + \frac{\partial I(\vec{x},\vec{s},t)}{\partial s} = -\beta I(\vec{x},\vec{s},t) + \beta(1-\omega)I_b(\vec{x},t) + \frac{\beta\omega}{4\pi}\int_{\Omega'=4\pi} I(\vec{x},\vec{s},t)p(\vec{s}\,'\rightarrow\vec{s})d\Omega' \tag{11}$$

Where c is the speed of light in the medium, $\vec{s}$ is the energy transport direction, $I_b = \sigma T^4/\pi$ is the Planck's black body intensity, $d\Omega$ is the solid angle and $p(\vec{s}\,'\rightarrow\vec{s})$ is the anisotropic scattering phase function. Eq. (11) can be recast as:

$$\frac{1}{c}\frac{\partial I(\vec{x},\vec{s},t)}{\partial t} + \vec{s}.\nabla I(\vec{x},\vec{s},t) = -\beta I(\vec{x},\vec{s},t) + \beta S(\vec{x},\vec{s},t) \tag{12}$$

Where S is the radiative source term given as:

$$S(\vec{x},\vec{s},t) = (1-\omega)I_b(\vec{x},t) + \frac{\omega}{4\pi}\int_{\Omega'=4\pi} I(\vec{x},\vec{s},t)p(\vec{s}\,'\rightarrow\vec{s})d\Omega' \tag{13}$$

The radiative boundary condition for Eq. (11), when the wall bounding the physical domain is assumed grey and emits and reflects diffusely, can be expressed as

$$I(\vec{x}_E,\vec{s}) = \varepsilon_E I_b(\vec{x}_E) + \frac{(1-\varepsilon_E)}{\pi}\int_{\Omega'=2\pi}\left[|I(\vec{x}_E,\vec{s})|\vec{n}.\vec{s}\right]_{\vec{n}.\vec{s}>0}d\Omega' \tag{14}$$

$$I(\vec{x}_W,\vec{s}) = \varepsilon_W I_b(\vec{x}_W) + \frac{(1-\varepsilon_W)}{\pi}\int_{\Omega'=2\pi}\left[|I(\vec{x}_W,\vec{s})|\vec{n}.\vec{s}\right]_{\vec{n}.\vec{s}<0}d\Omega' \tag{15}$$

If anisotropic scattering is approximated by the linear anisotropic phase function $p = 1 + a\cos\gamma\cos\gamma'$, where a is the anisotropy factor $(-1 < a < 1)$,the Eq.(13) for the source term can be written as :

$$S(\vec{x},\vec{s},t) = (1-\omega)I_b(\vec{x},t) + \frac{\omega}{4\pi}\int_{\Omega'=4\pi} I(\vec{x},\vec{s},t)(1 + a\cos\gamma\cos\gamma')d\Omega' \tag{16}$$

$$S(\vec{x},\vec{s},t) = (1-\omega)I_b(\vec{x},t) + \frac{\omega}{4\pi}G(\vec{x},t) + a\cos\gamma\, q_R(\vec{x},t) \tag{17}$$

γ is the polar angle.
Multiplying Eq.(12) throughout by the speed of light c, the radiative transfer equation along any lattice link designated by the index i can be written as:

$$\frac{DI_i}{Dt}(\vec{x},\vec{s},t) = \frac{\partial I_i}{\partial t} + \vec{c}.\nabla I_i = -c\beta(I_i - S_i) \quad i=1,\dots,M \tag{18}$$

Let $\vec{e}_i$ be the velocity of propagation along the ith lattice link of the D1QM lattice structure. If the velocity of light $\vec{c}$ is fictitiously made equal to the velocity of particle propagation in the LBM , $\vec{c}=\vec{e}$ a convenient tool would be obtained to solve the radiative transfer equation using the LBM approach

$$\frac{\partial I_i}{\partial t}+\vec{e}_i.\nabla I_i=e_i\beta(S_i-I_i) \qquad i=1,...,M \tag{19}$$

Discretizing Eq. (19), we obtain:

$$I_i(\vec{x}+\vec{e}_i\Delta t,\ t+\Delta t)=I_i(\vec{x},t)+\Delta t e_i\beta(S_i-I_i) \quad i=1,...,M \tag{20}$$

Clearly in Eq. (20),the term on the right hand side can be seen as the collision term in the LBM, where I i is the intensity particle distribution function. Using the standard LBM terminology, Eq.(11)can be written as:

$$I_i(\vec{x}+\vec{e}_i\Delta t,\ t+\Delta t)=I_i(\vec{x},t)+\frac{\Delta t}{\tau_R}\left[I_i^{eq}(\vec{x},t)-I_i(\vec{x},t)\right] \tag{21}$$

Where τ_R is the relaxation time for the collision process and I_i^{eq} is the equilibrium particle distribution function.

$$\tau_R=\frac{1}{e_i\beta} \quad \text{and} \quad I_i^{eq}=S_i \tag{22}$$

In Eq.(17), G is the irradiation and $\vec{q}_R$ is the heat flux due to diffuse radiation, are computed from the following:

$$G=4\pi\sum_{i=1,M} I_i\sin\gamma_i\sin(\frac{\Delta\gamma_i}{2}) \tag{23}$$

$$q_R=2\pi\sum_{i=1,M} I_i\sin\gamma_i\cos\gamma_i\sin(\Delta\gamma_i) \tag{24}$$

3. Results and discussion

In this paper, the energy equation of a 1-D transient conduction- radiation problem is solved with LBM. Initially the medium is at temperature T_E. For $t>0$, the west boundary temperature is maintained at $T_W=2T_E$. The medium is absorbing, emitting and isotropically scattering. The non dimensional time step $\Delta\xi=10^{-4}$ $(\xi=\alpha\beta^2 t)$ was considered and steady state condition was assumed to have been achieved when the maximum variation in temperature at any location between two consecutive time levels did not exceed 10^{-5}. First the effect of the grid size to the non-dimensional temperature results (T/T_w) is studied by comparing the steady state (SS) results at three locations in the medium for several grid sizes for β=1.0, N=0.1, T_E=0.0, T_w=1.0, ω=0.5 and ε_w = ε_E = 1.0. The results are listed in **table.1** and show that the non-dimensional temperature is stable and practically independent of the grid size. In **table.2**, for ξ = 0.05 β=1.0, N=0.1, T_E=0.0, T_w=1.0, ω=0.5, ε_w =1.0 and ε_E = 1.0 or 0.0, the non-dimensional temperature results (T/T_w) are compared with those reported in the literature [23,24] at three locations in the medium, It is observed that the LBM results are in good agreements with the published results.

Fig.1a shows the effect of the extinction coefficient by comparing the LBM results (T/T_w) and those published for N=0.1, ω=0.0 and ε_w = ε_E = 1.0. The LBM results are in good agreements with the published results.

Table.1 Effect of grid size on non-dimensional temperature steady state for $\beta=1.0$, $T_E=0.0$, $T_W=1.0$ $\omega=0.5$, $N=0.1$ and $\varepsilon_W=\varepsilon_E=1.0$

		x/L = 0.25	x/L = 0.50	x/L = 0.75
N_x=20	M=4	0.8265	0.6076	0.3339
	M=8	0.8356	0.6204	0.3438
	M=16	0.8389	0.6254	0.3479
	M=32	0.8400	0.6270	0.3492
M=32	N_x=20	0.8400	0.6270	0.3492
	N_x=30	0.8438	0.6210	0.3365
	N_x=40	0.8269	0.6181	0.3441
	N_x=60	0.8227	0.6152	0.3425

Table.2 comparison of transient temperature T/T_W at time $\xi=0.05$ for β=1.0, $T_E=0.0$, $T_W=1.0$, $\omega=0.5$, $N=0.1$ and two sets of wall reflectivities

		x/L = 0.25	**x/L = 0.5**	**x/L = 0.75**
$\varepsilon_W=1.0$	[23]	0.4888	0.1778	0.0591
	[24]	0.4889	0.1773	0.0588
$\varepsilon_E=1.0$	LBM	0.4893	0.1787	0.05724
$\varepsilon_W=1.0$	[23]	0.5030	0.2005	0.0833
	[24]	0.5031	0.2001	0.0830
$\varepsilon_E=0.0$	LBM	0.5037	0.1993	0.0841

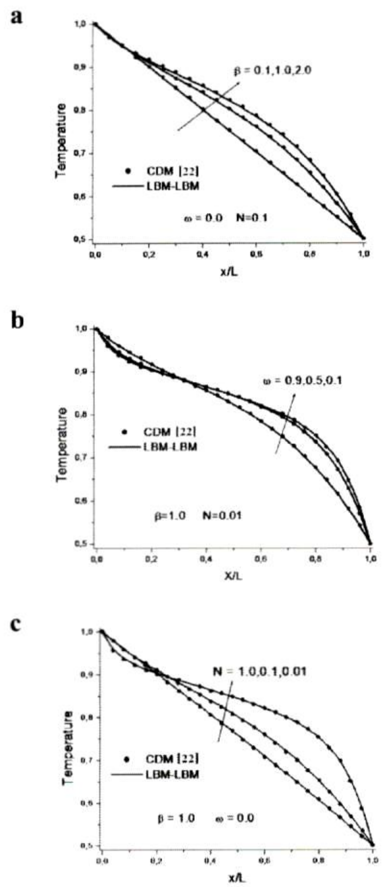


Fig.1 Comparison of steady-state results (T/T_w) for several (a) extinction coefficient, (b) scattering albedo and (c) conduction–radiation parameter

Fig.1b shows the effect of the scattering albedo by comparing the LBM results (T/T_w) and those published for β=1.0, N=0.01 and $\varepsilon_w = \varepsilon_E$ = 1.0. **Fig.1c** shows the effect of the conduction–radiation parameter by comparing the LBM results (T/T_w) and those published for β=1.0, ω=0.0 and $\varepsilon_w = \varepsilon_E$ = 1.0. Excellent agreement is also found.

Fig.2a and **b** show the effect of the emissivity by comparing the LBM results (T/T_w) and those published for β=1.0, ω=0.0 and respectively N=0.1 and 0.01. **Fig.2c** shows the effect of the emissivity for N=0.01, β=1.0, ω=0.0 and ε_E = 1.0. It is shown for all the cases studied that the LBM results are in good agreements with the published results.

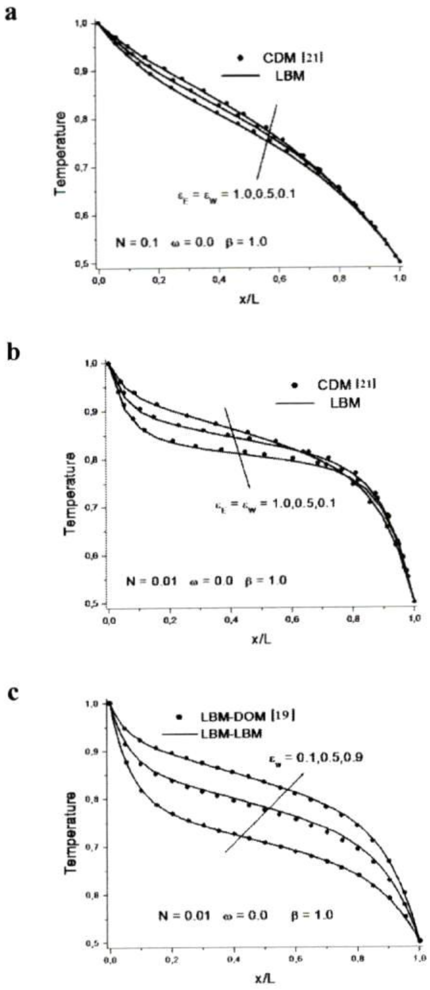


Fig.2 Comparison of steady-state results (T/T_w) for β=1.0, ω=0.0 (a) N=0.1 (b) N=0.01 (c) ε_E=1.0 and N=0.01

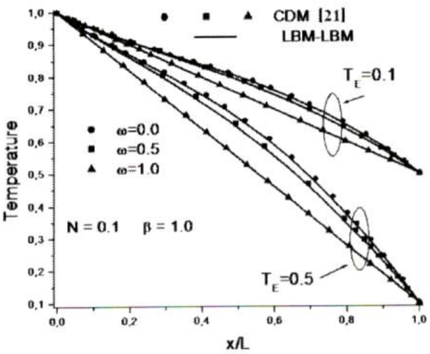


Fig.3 Comparison of steady-state results (T/T_w) for β=1.0, N=0.1 $\varepsilon_E = \varepsilon_w$ =1.0

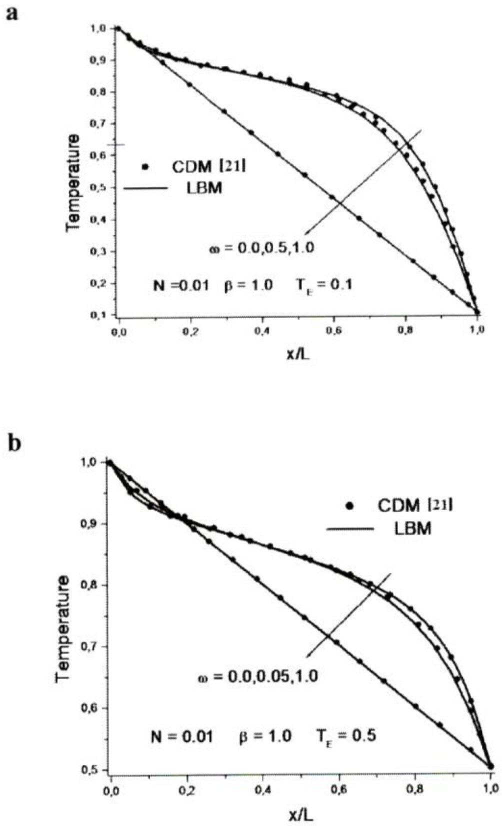
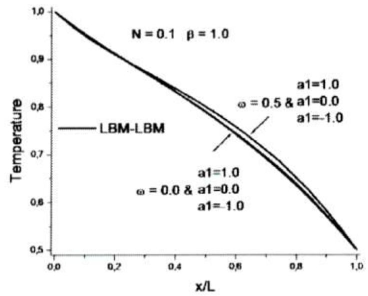


Fig.4 Comparison of steady-state results (T/T_w) for β=1.0, N=0.01 $\varepsilon_E = \varepsilon_w$ =1.0 and (a) T_E=0.1 (b) T_E=0.5

Fig. 3 and **4** show the effect of scattering albedo on temperature distribution. Both boundaries are assumed black. Three different values of scattering albedo (ω=0.0, 0.5 and 1.0) are considered. The LBM results (T/T_w) are given for two sets of boundary temperatures (T_E=0.1 and 0.5). **Fig.3** shows the non-dimensional temperature (T/T_w) for N=0.1 and β=1.0. **Fig.4a** and **b** show the non-dimensional temperature for N=0.01, β=1.0 and respectively for T_E=0.1 and 0.5. The found results are in good agreement with those published.

Figs. 5a and **b** show the effect of anisotropy factor *a* on non dimensional temperature distribution, for N=0.1, for two values of the scattering albedo ω=0.0 and 0.5 and for two values of the extinction coefficient β=0.1 and 1.0. Results are presented for *a* = -1.0, 0.0 and 1.0. It is shown that the anisotropy factor *a* has not an appreciable effect on the temperature distribution in the medium.

a



b

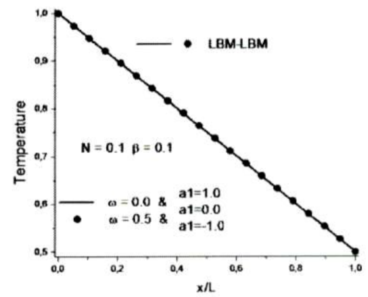


Fig.5 Effect of the anisotropy factor *a* for (a) β=1.0 (b) β=0.1

4. Conclusions

Combined conduction–radiation problem in one-dimensional gray planar absorbing, emitting and anisotropically scattering medium has been investigated by the LBM. In order to examine the accuracy and the computational efficiency of the proposed method, the non-dimensional temperature (T/T_w) is compared with the published results for various values of the extinction coefficient, conduction–radiation parameter, boundary emissivity, scattering albedo, anisotropy factor and east boundary temperature. For all cases studied, a good agreement is obtained.

Nomenclature

a	Anisotropy factor
c	speed of light (m/s)
c_p	Specific heat at constant pressure ($JKg^{-1}K^{-1}$)
e_i	Lattice speed (m/s)
f	Internal energy distribution functions (K)
f^{eq}	Equilibrium internal energy distribution (K)
G	incident radiation (W/m^2)
I	intensity of radiation (W/m^2)
I^{eq}	Equilibrium intensity (W/m^2)
k	thermal conductivity ($Wm^{-1}K^{-1}$)
L	length of the planar geometry (m)
M	total number of discrete directions
N	conduction-radiation parameter($=k\beta/4\sigma T_w^3$)
N_x	total number of node
p	scattering phase function
q_R	heat flux (W/m^2)
S	source term (W/m^2)
t	time (s)
T	Temperature (K)
w	weight in the LBM

Greek symbols

Δx	Lattice spacing (m)
Δt	Time increment (s)
α	Thermal diffusivity (m^2/s)
β	extinction coefficient (1/m)
ε	emissivity
γ	polar angle (rad)
σ	Stefan–Boltzmann constant,5.67 10^{-8} ($W/m^2 K^4$)
ρ	density (Kg/m^3)
ξ	non dimensional time ($=\alpha\beta^2 t$)
τ	Relaxation time for temperature (s)
τ_R	Relaxation time for radiation (s)
ω	scattering albedo

Subscript

E	East
i	index for lattice direction
w	West

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