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EFFECTS OF PRESSURE WORK ON MHD NATURAL CONVECTION FLOW ALONG A VERTICAL WAVY SURFACE

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ABSTRACT

An analysis is presented to investigate the influences of pressure work on MHD natural convection flow along a uniformly heated vertical wavy surface. The governing equations are transformed into dimensionless non-similar equations by using set of suitable transformations and solved numerically by the implicit finite difference method, known as Keller-box scheme. Numerical results for the velocity profiles, temperature profiles, skin friction coefficient, the rate of heat transfers, the streamlines and the isotherms are shown graphically and skin friction coefficient and rate of heat transfer have been shown in tabular form for different values of the selective set of parameters.

Keywords: Natural convection, uniform surface temperature, wavy surface, magnetic parameter, Prandtl number, pressure work.

1. INTRODUCTION

The pressure work effect plays an important role in natural convection in various devices which are subjected to large deceleration or which operate at high rotational speeds and also in strong gravitational field processes on large scales (on large planets) and in geological processes. Joshi and Gebhart [1] first investigated the effect of pressure stress work and viscous dissipation in some natural convection flows. The natural convection along a vertical wavy surface was first studied by Yao [2] and using an extended Prantdl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces, such as sinusoidal surface. Moulic and Yao [3] also investigated mixed convection heat transfer along a vertical wavy surface. Alam et al. [4] have also studied the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field. Combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface have been investigated by Hossain and Rees [5]. Hossain et al. [6] have studied the problem of natural convection of fluid with temperature dependent viscosity along a heated vertical wavy surface. Natural and mixed convection heat and mass transfer along a vertical wavy surface have been investigated by Jang. [7]. Molla et al. [8] have studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Tashtoush and Al-Odat [9] investigated magnetic field effect on heat and fluid flow over a wavy surface with a variable heat flux. Recently Parveen and Alim [10-11] investigated Joule heating effect on Magnetohydrodynamic natural convection flow along a vertical wavy surface with viscosity dependent on temperature and studied effect of temperature dependent thermal conductivity on magnetohydrodynamic natural convection flow along a vertical wavy surface. They found that the temperature distribution within the boundary layer rises considerably for the higher values of Joule heating. Significant effects Joule heating have also been found on the local skin friction rate of heat transfer whereas these to physical quantities rise due to the values of thermal conductivity variation parameter enhancement. Alim et al. [12] considerd the effects of Temperature Dependent Thermal Conductivity on Natural Convection Flow along a Vertical Wavy Surface with Heat Generation. Miraj et al.[13] investigated effects of Pressure Work and Radiation on Natural Convection Flow around a Sphere with Heat Generation. The thermal conductivity of the fluid had been assumed to be constant in all the above studies. However, it is known that this physical property may be change significantly with temperature.

The present study is to incorporate the idea of the effects of pressure work on MHD natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface. Numerical results of the velocity profiles, temperature profiles, local skin friction coefficient, rate of heat transfer, the streamlines and the isotherms are shown graphically. Some selected results of skin friction coefficient and rate of heat transfer for different values of pressure work parameter Ge have been shown tabular form and then discussed.

2. FORMULATION OF THE PROBLEM

Steady two dimensional laminar free convection boundary layer flow of a viscous incompressible and electrically conducting fluid along a vertical wavy surface in presence of uniform transverse magnetic field of strength B_0 with physical properties is considered. It is assumed that the wavy surface is electrically insulated and is maintained at a uniform temperature T_w., The fluid is stationary above the wavy plate and is kept at a temperature T_∞. The surface temperature T_w is greater than the ambient temperature T_∞ that is, $T_w > T_\infty$. The flow configuration of the wavy surface and the two-dimensional Cartesian coordinate system are shown in figure1

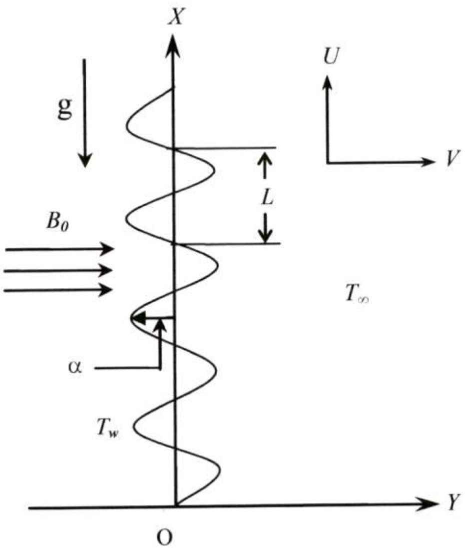


Figure 1: Physical model and coordinate system

The boundary layer analysis outlined below allows $\bar{\sigma}(X)$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be defined by

$$Y_w = \bar{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right) \qquad (1)$$

Where, α is the amplitude and L is the wave length associated with the wavy surface.
The governing equations of such flow of magnetic field along a vertical wavy surface under the usual Boussinesq approximations can be written in a dimensional form as:
Continuity Equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (2)$$

X-Momentum Equation

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \frac{1}{\rho}\nabla.\ \mu\nabla U + g\beta(T - T_\infty) - \frac{\sigma_0 B_0^2}{\rho}U \qquad (3)$$

Y-Momentum Equation

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \frac{1}{\rho}\nabla.\ \mu\nabla V \qquad (4)$$

Energy Equation

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{k}{\rho C_P}\nabla^2 T + \frac{T\beta}{\rho C_P}U\frac{\partial p}{\partial X} \qquad (5)$$

where (X, Y) are the dimensional coordinates along and normal to the tangent of the surface and (U, V) are the velocity components parallel to (X, Y), g is the acceleration due to earth gravity, P is the dimensional pressure of the fluid, T is the temperature of the fluid in the boundary layer, C_P is the specific heat at constant pressure, μ is the dynamic viscosity of the fluid in the boundary layer region depending on the fluid temperature, ρ is the density, ν is the kinematic viscosity, where $\nu = \mu / \rho$, k is the thermal conductivity of the fluid, β is the volumetric coefficient of thermal expansion, B_0 is the strength of magnetic field, σ_0 is the electrical conductivity of the fluid and ∇^2 is the Laplacian operator, where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

The boundary conditions for the present problem are

$$U = 0, V = 0, T = T_w \text{ at } Y = Y_w = \bar{\sigma}(X);$$
$$U = 0, T = T_\infty, P = p_\infty \text{ as } Y \to \infty \qquad (6)$$

Where, P_∞ is the pressure of fluid outside the boundary layer.
Using Prandtl's transposition theorem to transform the irregular wavy surface into a flat surface as extended by Yao [2] and boundary layer approximation, the following dimensionless variables are introduced for non-dimensional governing equations

$$x = \frac{X}{L},\ y = \frac{Y - \bar{\sigma}}{L}Gr^{\frac{1}{4}},\ p = \frac{L^2}{\rho\nu^2}Gr^{-1}P$$
$$u = \frac{\rho L}{\mu}Gr^{1/2}U,\ v = \frac{\rho L}{\mu}Gr^{1/4}\ V - \sigma_x U\ , \qquad (7)$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \sigma_x = \frac{d\bar{\sigma}}{dX} = \frac{d\sigma}{dx},\ Gr = \frac{g\beta(T_w - T_\infty)}{\nu^2}L^3$$

Where, θ is the dimensionless temperature function and (u, v) are the dimensionless velocity components parallel to (x, y). Here p is the dimensionless pressure of the fluid, L is the wave length associated with the wavy surface and Gr is the Grashof number. Introducing the above dimensionless dependent and independent variables into equations (2)–(5), the following dimensionless form of the governing equations are obtained after ignoring terms of smaller orders of magnitude in Gr.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (8)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + Gr^{1/4}\sigma_x\frac{\partial p}{\partial y} + 1 + \sigma_x^2\ \frac{\partial^2 u}{\partial y^2} - Mu + \theta \qquad (9)$$

$$\sigma_x\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -Gr^{1/4}\frac{\partial p}{\partial y} + \sigma_x\ 1 + \sigma_x^2\ \frac{\partial^2 u}{\partial y^2} - \sigma_{xx}u^2 \qquad (10)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}\ 1 + \sigma_x^2\ \frac{\partial^2 \theta}{\partial y^2} - Ge\left(\frac{T_\infty}{T_w - T_\infty} + \theta\right)u \qquad (11)$$

It is worth noting that the σ_x and σ_{xx} indicate the first and second derivatives of σ with respect to x, therefore, σ_x = $d\bar{\sigma}/dX = d\sigma/dx$ and $\sigma_{xx} = d\sigma_x/dx$.

In the above equations *Pr*, *M* and *Ge* are respectively known as the Prandtl number, the magnetic parameter and pressure work parameter which are defined as

$$\Pr = \frac{C_p \mu}{k},\ M = \frac{\sigma_0 B_0^2 L^2}{\mu Gr^{1/2}},\ Ge = \frac{g\beta L}{c_p} \quad (12)$$

For the present problem this pressure gradient ($\partial p/\partial x = 0$) is zero. Thus, the elimination of $\partial p/\partial y$ from equations (9) and (10) leads to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 1+\sigma_x^2\ \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_x \sigma_{xx}}{1+\sigma_x^2}u^2 - \frac{M}{1+\sigma_x^2}u + \frac{1}{1+\sigma_x^2}\theta \quad (13)$$

The corresponding boundary conditions for the present problem then turn into

$$\left.\begin{aligned} &u = v = 0,\ \theta = 1 \text{ at } y = 0 \\ &u = 0,\ \theta = 0 \quad \text{as } y \to \infty \end{aligned}\right\} \quad (14)$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi = x^{3/4} f(x,\eta), \quad \eta = yx^{-1/4}, \quad \theta = \theta(x,\eta) \quad (15)$$

Where, $f(x, \eta)$ is the dimensionless stream function, η is the dimensionless similarity variable and ψ is the stream function that satisfies the continuity equation (8) and is related to the velocity components in the usual way as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (16)$$

Introducing the transformations given in equation (15) and using (16) into equations (13) and (11) are transformed into the new co-ordinate system. Thus the resulting equations are

$$(1+\sigma_x^2)f''' + \frac{3}{4}ff'' - \left(\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2 + \frac{1}{1+\sigma_x^2}\theta - \frac{Mx^{1/2}}{1+\sigma_x^2}f' = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \quad (17)$$

$$\frac{1}{\Pr}(1+\sigma_x^2)\theta'' + \frac{3}{4}f\theta' - Ge\left(\frac{T_\infty}{T_w - T_\infty} + \theta\right)xf' = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \quad (18)$$

The boundary conditions (14) now take the following form:

$$\left.\begin{aligned} &f(x,0) = f'(x,0) = 0,\ \theta(x,0) = 1 \\ &f'(x,\infty) = 0,\ \theta(x,\infty) = 0 \end{aligned}\right\} \quad (19)$$

Here prime denote the differentiation with respect to η.

However, once we know the values of the functions f and θ and their derivatives, it is important to calculate the values of the shearing stress τ_w in terms of the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of local Nusselt number Nu_x from the following relations:

$$C_{fx} = \frac{2\tau_w}{\rho U^2};\ Nu_x = \frac{q_w X}{k(T_w - T_\infty)} \quad (20)$$

where, $q_w = -k(\bar{n}.\nabla T)_{y=0}$,

$$\tau_w = (\mu\ \ \bar{n}.\nabla\bar{u})_{y=0}; U = \mu_\infty Gr^{1/2}/\rho L \quad (21)$$

Here $\bar{n} = \dfrac{\bar{i}f_x + \bar{j}f_y}{\sqrt{f_x^2 + f_y^2}}$ is the unit normal to the surface.

Using the transformation (15) and (21) into equation (20) the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x take the following forms:

$$\frac{1}{2}(Gr/x)^{1/4}C_{fx} = \sqrt{1+\sigma_x^2}\ \ f''(x,0) \quad (22)$$

$$Gr^{-1/4}x^{-3/4}Nu_x = -\sqrt{1+\sigma_x^2}\,\theta'(x,0) \quad (23)$$

For the computational purpose the period of oscillations in the waviness of this surface has been considered to be π.

3. METHOD OF SOLUTION

The governing partial differential equations are reduced to dimensionless local non-similar equations by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using Keller box method described by Keller [14], Cebeci and Bradshaw [15] and used by many other authors.

4. RESULTS AND DISCUSSION

The effect of pressure work on MHD natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface has been investigated. Although there are four parameters of interest in the present problem, the effects of pressure work parameter Ge, the magnetic parameter M, Prandtl number Pr and the amplitude of the wavy surface α on the surface shear stress, the rate of heat transfer, the velocity and temperature, the streamlines and the isotherms are focused. Numerical values of local shearing stress and the rate of heat transfer are calculated from equations (22) and (23) in terms of the skin-friction coefficients Cfx and Nusselt number Nux respectively for a wide range of the axial distance variable x starting from the leading edge for different values of the parameters Pr, M, Ge and α.

The velocity and temperature of the flow field is found to change more or less with the variation of the flow parameters. The effect of the flow parameters on the velocity and temperature fields, the skin friction coefficients, the rate of heat transfer are analyzed with the help of graphs.

The effects of the different values of Prandtl number *Pr* on velocity and temperature have been shown in figures 2(a) and 2(b). For the higher values of Prandtl number *Pr* both the velocity and the temperature decreases such that there exists a local maximum of the velocity within the boundary layer. Figure 2(a) shows that the velocity fall down slowly.The maximum values of velocities are recorded as 0.46694, 0.42603, 0.37733, 0.34447 and 0.25209 for Prandtl number Pr = 0.72, 1.0, 1.5, 2.0 and 5.0 at the position of η = 1.73814, 1.65930, 1.58311, 1.50946 and 1.36929 respectively and the maximum velocity decreases by 46.01 %. The values of temperature are recorded as 0.67087, 0.63111, 0.57801, 0.53793 and 0.40023 for Prandtl number Pr = 0.72, 1.0, 1.5, 2.0 and 5.0 at the position of η = 1.23788 and the temperature decreases by 40.34 %. Figure 2(b) displays the results that the change of temperature profiles in the η-direction reveals the typical temperature profiles for natural convection boundary layer flow, i.e., the temperature is zero at the boundary wall. It is observed that the velocity as well as the boundary layer thickness decreases and the temperature as well as the thermal boundary layer thickness decreases for the increasing values of Prandtl number.

The effects for different values of magnetic parameter M on the velocity and temperature profiles have been presented graphically in figures 3(a) and 3(b). It is seen from the figure 3(a) that for the values of magnetic parameter M the velocity decreasing up to the position of $\eta = 5.5$ from the wall. At the position of $\eta = 5.5$ velocity becomes constant that is velocity profiles meet at a point and then cross the side and increasing with magnetic parameter M. This is cause of the velocity profiles having lower peak values for higher values of magnetic parameter M tend to decreases comparatively slower along η-direction. The maximum values of velocities are recorded as 0.47070, 0.45227, 0.41811, 0.38739 and 0.34726 for magnetic parameter $M = 0.0, 0.5, 1.5, 2.5$ and 4.0 respectively which occur at the same position $\eta = 1.73814$. Here, it is observed that at $\eta = 1.73814$, the maximum velocity decreases by 26.23 % as the magnetic parameter M change from 0.0 to 4.0. The values of temperature are recorded as 0.66954, 0.67611, 0.68875, 0.70068 and 0.71717 for magnetic parameter $M = 0.0, 0.5, 1.5, 2.5$ and 4.0 at the same position of $\eta = 1.23788$ and the temperature increases by 7.11 %. The change of temperature profiles in the η-direction also shows the typical temperature profiles for natural convection boundary layer flow that is the value of temperature profiles is 1.0 (one) at the boundary wall then the temperature profiles decreases gradually along η-direction to the asymptotic value.

Figures 4(a) and 4(b) demonstrates the velocity and temperature distribution for different values of the pressure work parameter Ge. It has been seen from figure 4(a) that as the pressure work parameter Ge increases, the velocities rising up to the position of $\eta = 1.8822$ for the pressure work parameter Ge and from that position of η velocities fall down slowly and finally approaches to zero. It is also observed from figure 4(b) that the temperature profiles increases with the pressure work parameter Ge. The maximum values of velocities are recorded as 0.48723, 0.48570, 0.48418, 0.48265 and 0.48113 for the pressure work parameter $Ge = 0.0, 0.02, 0.04, 0.06, 0.08$ respectively which occur at the same position $\eta = 1.73814$ and the maximum velocity decreases by 1.25 %. Temperatures are recorded as 0.71063, 0.70769, 0.70476, 0.70184 and 0.69893 for the pressure work parameter $Ge = 0.0, 0.02, 0.04, 0.06, 0.08$ respectively at the same position of $\eta = 1.23788$ and the temperature profiles decreases by 1.65 %. Both the velocity and temperature profiles accumulate nearly in the following points where $\eta = 2.27434$ for the pressure work parameter $Ge = 0.0$ to 0.08.

In figures 5(a) and 5 (b) the skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of Prandtl number Pr have been displayed. It is observed from the figure 5(a) that for higher values of Prandtl number the skin friction decreasing upto the axial position of x = 1.4 and then skin friction becomes constant for all values of Prandtl number Pr that is, skin friction coefficient meet together at the position of x = 1.4 and cross the sides that means after the axial position of x = 1.4 skin friction is increasing with Prandtl number but frictional force at the wall always rising towards downstream. It is seen from the figure 5(b) that for higher values of Prandtl number the rate of heat transfer decreases that is heat transfer slows down for higher Prandtl number.

In figures 6(a) and 6(b) effects of magnetic parameter M on skin friction and the rate of heat transfer have been presented. From figure 6(a) it is found that skin friction decreases significantly for greater magnetic field strength. This is physically realizable as the magnetic field retards the velocity field and consequently reduces the frictional force at the wall. However rate of heat transfer opposite pattern due to the higher values of magnetic parameter M which are presented in figure 6(b).

The effect of different values of the pressure work parameter Ge on the skin friction coefficients and the rate of heat transfer are shown graphically in figures 7(a) and 7 (b) respectively. In this case the values of local skin friction coefficient C_{fx} are recorded to be 1.13690, 1.08073, 1.02523, 0.97061 and 0.91713 for $Ge = 0.0, 0.02, 0.04, 0.06, 0.08$ which occur at same point $x = 1.0$. From the figure 7(a), it is observed that at $x = 1.0$, the skin friction coefficient decreases by 19.33 % due to the higher value of the pressure work parameter Ge. However, the values of rate of heat transfer are found to be -2.49159, -2.15027, -1.82783, -1.52539 and -1.24406 for $Ge = 0.0, 0.02, 0.04, 0.06, 0.08$ which occur at same point $x = 1.0$. The rate of heat transfer coefficient increases by 50.06 % due to the increased value of the pressure work parameter Ge. It is seen from the figure 7(b) that for higher values of the pressure work parameter Ge the rate of heat transfer increases that is heat transfer rising up for the higher pressure work parameter Ge.

Figure 8(a) and 8(b) show that streamlines and isotherms for selected values of the pressure work parameter $Ge = 0.0$ and 0.09 with amplitude of waviness of the surface $\alpha = 0.2$, Prandtl number $Pr = 0.72$ and magnetic parameter $M = 0.1$ respectively. In Figure 8 (a) have been shown the value of stream function ψ is 0.0 near the wall and then ψ increases gradually in the downstream within the boundary layer and away from the wall. In this case the maximum values of stream function ψ_{max} are found as 4.8 and 3.3 for the values of the pressure work parameter Ge equal to 0.0 and 0.09 respectively. The isolines of temperature (isotherms) distribution show that temperature decreases significantly as the values of the pressure work parameter Ge increases which have been presented in figure 8(b). The value of isotherm is 1.0 at the wall and isotherms decreases slowly along the y-direction and finally approach to zero. The maximum values of isotherms are recorded as 3.2 and 1.7 for the values of the pressure work parameter Ge equal to 0.0 and 0.09 respectively.

Some numerical values of skin friction coefficient C_{fx} and rate of heat transfer Nu_x are calculated from equations (22) and (23) for the wavy surface from lower stagnation point at x =0.0 to x =2.0 presented in tabular form in the Table 1.

Table 1: Skin friction coefficient and rate of heat transfer against x for different values of the pressure work parameter Ge with other controlling parameters $Pr = 0.72$, $\alpha = 0.2$, M = 0.1 and $Ec = 10.0$.

x	$Ge = 0.0$		$Ge = 0.05$	
	C_{fx}	Nu_x	C_{fx}	Nu_x
0.00000	0.74379	0.32836	0.74379	0.32836
0.10500	0.76926	0.21752	0.76002	0.23920
0.20500	0.83203	0.06873	0.81239	0.11983
0.30500	0.92525	-0.17575	0.89223	-0.07829
0.40500	1.02919	-0.54708	0.97926	-0.37571
0.50500	1.10382	-0.96659	1.03533	-0.69553
0.60500	1.11319	-1.24284	1.02890	-0.87541
0.70500	1.06473	-1.29673	0.97017	-0.86429
0.80500	0.99911	-1.24022	0.89811	-0.76772
0.90500	0.95159	-1.20868	0.84436	-0.69606
1.00500	0.94034	-1.28619	0.82379	-0.70861
1.10500	0.97383	-1.54252	0.84207	-0.84783
1.20500	1.05693	-2.07866	0.90115	-1.17371
1.30500	1.18834	-3.02902	0.99715	-1.76117
1.40500	1.34659	-4.42222	1.10937	-2.60161
1.50500	1.47550	-5.85268	1.19151	-3.40197
1.60500	1.51353	-6.56203	1.19853	-3.69180
1.70500	1.46037	-6.35288	1.13631	-3.40983
1.80500	1.37292	-5.76369	1.05205	-2.93078
1.90500	1.30541	-5.32574	0.98665	-2.57213
2.00000	1.28639	-5.29208	0.96020	-2.46405

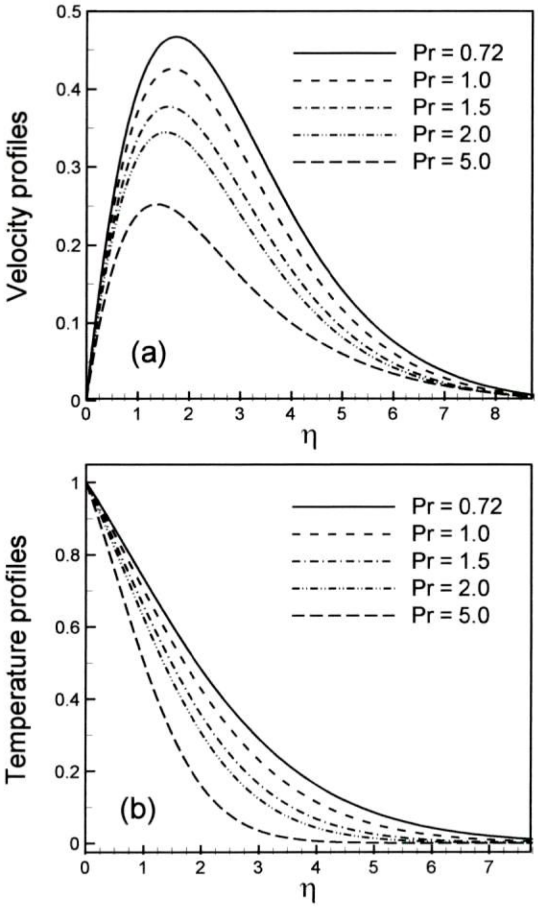


Figure 2: (a) Velocity and (b) Temperature profiles against η or different values of Pr with α = 0.2, M = 0.1 and Ge = 0.01.

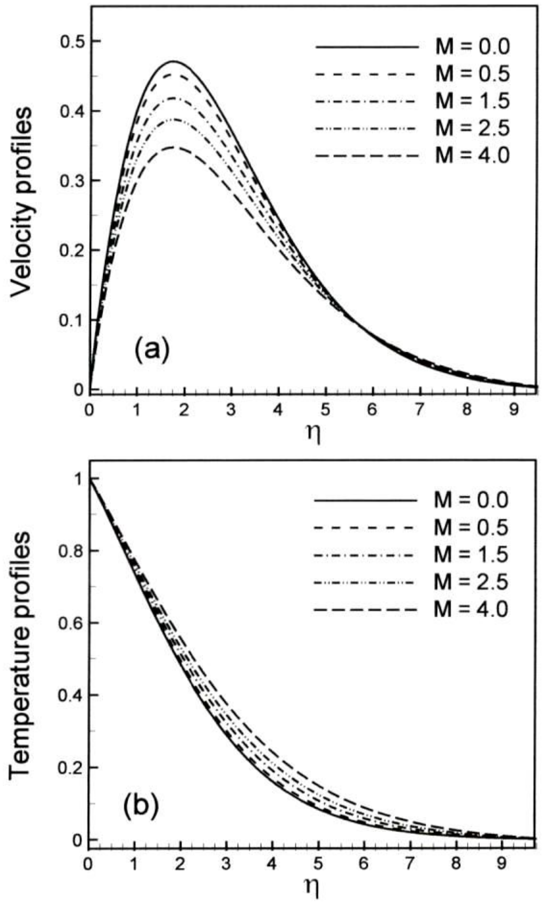
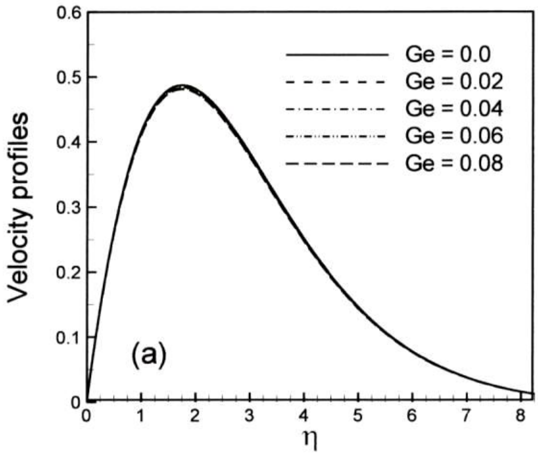


Figure 3: (a) Velocity and (b) Temperature profiles against η for different values of M with α = 0.2, Pr = 0.72 and Ge = 0.01.



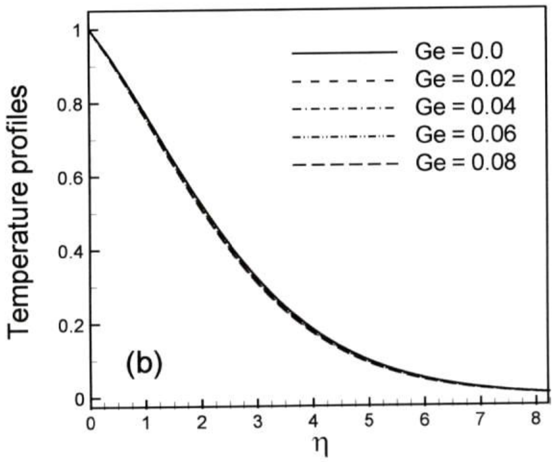
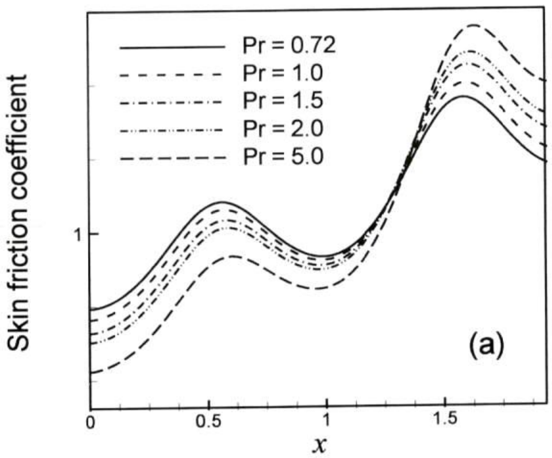


Figure.4: (a) Velocity and (b) Temperature profiles against η for different values of Ge with $\alpha = 0.2$, $Pr = 0.72$ and M = 0.1.



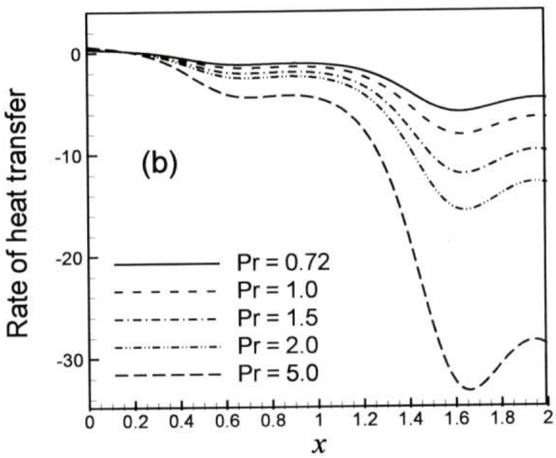
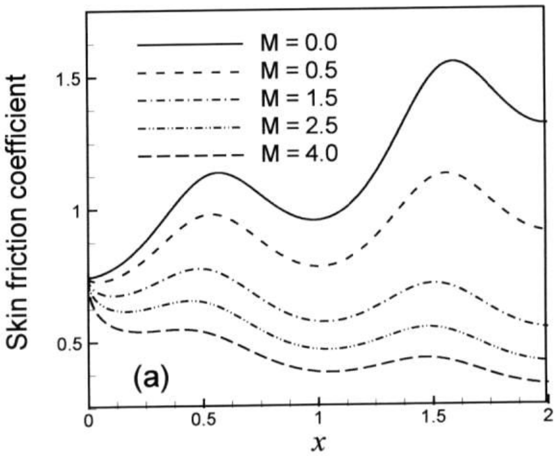


Figure 5: (a) Skin friction coefficient and (b) Rate of heat transfer against x for different values of Pr with $\alpha = 0.2$, $M = 0.1$ and $Ge = 0.01$.



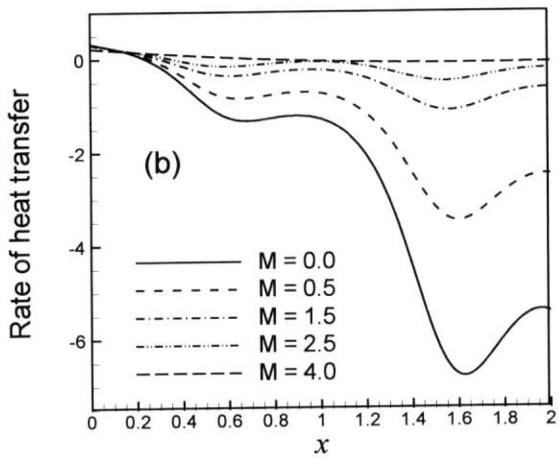
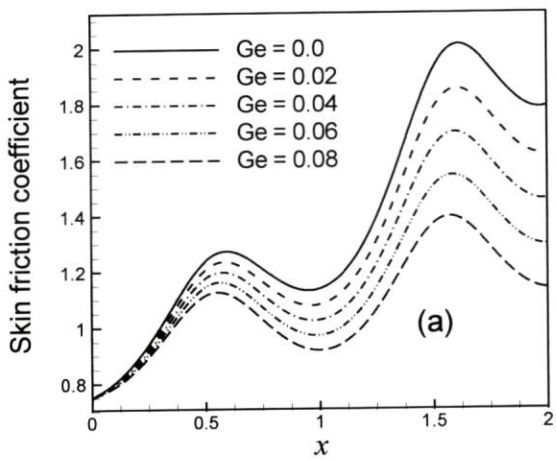


Figure 6: (a) Skin friction coefficient and (b) Rate of heat transfer against x for different values of M with $\alpha = 0.2$, $Pr = 0.72$ and $Ge = 0.01$.



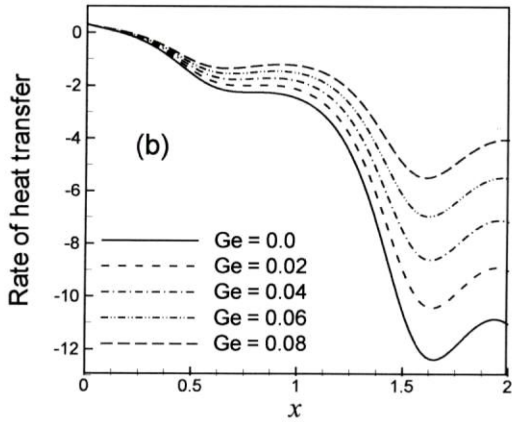


Figure 7: (a) Skin friction coefficient and (b) Rate of heat transfer against x for different values of Ge with $\alpha = 0.2$, $Pr = 0.72$ and $M = 0.1$.

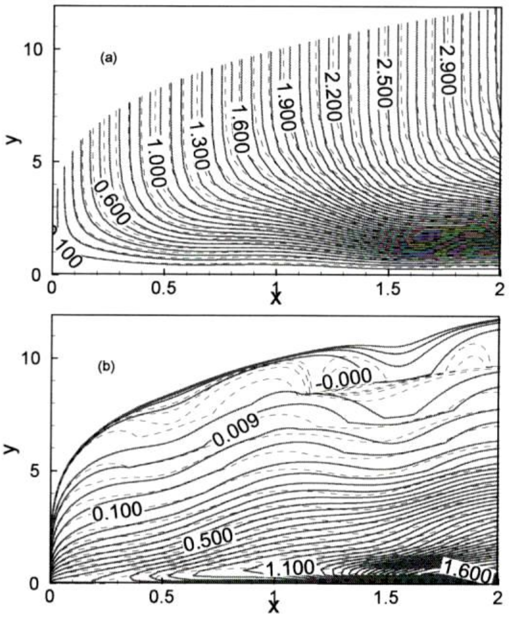


Figure 8: (a) Streamlines and (b) Isotherms for Ge = 0.0 (Red solid lines), Ge = 0.09 (Black dashed lines), with $\alpha = 0.2$, $Pr = 0.72$ and $M = 0.1$.

5. COMPARISON WITH PREVIOUS WORK

A comparison of the present numerical results of the skin friction coefficient C_{fx}, and the heat transfer coefficient Nu_x with Parveen and Alim [11] have been shown in Table 2. Here, the magnetic parameter M and the pressure work parameter Ge are ignored to make the numerical data comparable with Parveen and Alim [11] for different values of Prandtl number Pr. It is obvious from the comparison table that the present results agreed well with the results of Parveen and Alim [11].

Table 2: Comparison of the values of the skin friction coefficient C_{fx} and the heat transfer coefficient Nu_x with Parveen and Alim [11] and present work for the variation of Prandtl number Pr while M = 0.0 and Ge = 0.0 with $\alpha = 0.2$.

Pr	C_{fx}		Nu_x	
	Parveen and Alim [11]	Present work	Parveen and Alim [11]	Present work
0.73	0.96181	0.96176	0.35267	0.35272
1.73	0.83837	0.83842	0.47511	0.47509
4.24	0.71227	0.71232	0.63141	0.63139
7.00	0.64567	0.64572	0.73367	0.73373

6. CONCLUSION

The effects of the Prandtl number Pr, the magnetic parameter M, the pressure work parameter Ge and the amplitude of wavy surface α on MHD natural convection flow of viscous incompressible fluid along a uniformly heated vertical wavy surface have been studied. From the present investigations the following conclusions may be drawn:

Improved value of the pressure work parameter Ge the velocity profiles, the temperature profiles increases slowly and the rate of heat transfer coefficient increases with pressure work parameter. The local skin friction coefficient decreases due to the increased value of the pressure work parameter.

For the higher values of Prandtl number Pr the velocity, temperature decreases slowly and finally approach to zero, the rate of heat transfer decreases while the skin friction initialy decreases, becomes constant near x = 1.4 after that position skin friction increase with Prandtl number but frictional force at the wall always rising towards downstream.

Magnetic field strength enhancement causes the temperature and the rate of heat transfer rise and the velocity and skin friction coefficient reduction within the boundary layer. At the position of $\eta = 5.5$ the velocity becomes constant and then cross the side and increasing with magnetic parameter.

For increasing values of pressure work parameter Ge the stream function ψ leads to increase. The isolines of temperature (isotherms) show that temperature is 1.0 at the wall and decreases slowly away from the wall and finally approach to zero..

7. NOMENCLATURE

B_0	Applied magnetic field strength
C_{fx}	Local skin friction coefficient
Cp	Specific heat at constant pressure [J.kg-1.K-1]
f	Dimensionless stream function
g	Acceleration due to gravity [ms^{-2}]
Gr	Grashof number
k	Thermal conductivity [$Wm^{-1}K^{-1}$]
k_∞	Thermal conductivity of the ambient fluid [$Wm^{-1}K^{-1}$]
L	Characteristic length associated with the wavy surface [m]
$\overline{n}$	Unit normal to the surface
Nu_x	Local Nusselt number
P	Pressure of the fluid [Nm^{-2}]
Pr	Prandtl number
q_w	Heat flux at the surface [Wm^{-2}]
T	Temperature of the fluid in the boundary layer [K]
T_w	Temperature at the surface [K]
T_∞	Temperature of the ambient fluid [K]

u, v Dimensionless velocity components along the (x, y) axes [ms^{-1}]

x, y Axis in the direction along and normal to the tangent of the surface

Greek symbols

α Amplitude of the surface waves

β Volumetric coefficient of thermal expansion [K^{-1}]

η Dimensionless similarity variable

θ Dimensionless temperature function

ψ Stream function [m^2s^{-1}]

μ Viscosity of the fluid [$kgm^{-1}s^{-1}$]

μ_∞ Viscosity of the ambient fluid

v Kinematic viscosity [m^2s^{-1}]

ρ Density of the fluid [kgm^{-3}]

σ_0 Electrical conductivity

τ_w Shearing stress.

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