OPTIMAL EXPERIMENT DURATION AND MODEL CHOICE FOR THERMAL DIFFUSIVITY ESTIMATION USING THE FLASH APPARATUS

Foued Mzali* and Fethi Albouchi

Laboratoire d'Etudes des Systèmes Thermiques et Energétiques (LESTE-ENIM),
Ecole Nationale d'Ingénieurs de Monastir, Avenue Ibn El Jazzar, Monastir 5019, Tunisia;
* E-Mail: Foued.mzali@enim.rnu.tn; Tel. (216)-73.500.514

ABSTRACT

In this study, optimal experiment design is performed in order to choice the thermal model and the optimal duration of the flash experiment. The thermal diffusivity estimation is performed using the Levenberg–Marquardt algorithm by minimizing the ordinary least squares function, describing the gap between measured and calculated temperature response. Different thermal models based on analytical or numerical resolutions of the heat equation are studied. The developed models take into account heat losses on front, rear, or/and lateral surfaces. Experimental thermal diffusivity measurements are then completed on 2017 A Aluminum alloy.

KEYWORDS Flash method, parameter estimation, inverse problem, finite volume method, optimal experiment design.

1 INTRODUCTION

The flash method is a commonly used technique for thermal diffusivity measurement of solids. It consists to apply a brief heat pulse on the front face of a cylindrical sample. The resulting temperature rise on the opposite face is recorded versus time. Then, the thermal diffusivity is computed using the measured temperature evolution and an identification method based on a simple analytical model [1].
The Parker and Degiovanni identification methods permit to calculate the thermal diffusivity directly using some particular times from the experimental data [1-3]. The Parker method considers only adiabatic systems. Whereas, Degiovanni considers low heat losses for limited geometrical conditions.
In this study, the thermal diffusivity is computed by minimizing the ordinary least squares function, describing the gap between measured temperature and calculated response [4]. This response is given by different thermal models, based on analytical or numerical resolutions of the heat equation. The developed models take into account heat losses on front, rear, or/and lateral surfaces. The parameter estimation is performed using the Levenberg–Marquardt [5]. Optimal experiment design is performed in order to choice the thermal model and the optimal duration of the flash experiment. Afterwards experimental measurements are performed on 2017 A samples.

2. DIRECT PROBLEM

2.1 System description

The system under investigation is a cylindrical sample, supposed to be homogeneous, opaque and having constant thermal properties and density (figure 1). This sample is submitted to an irradiation flux on the upper face. The sample exchanges heat throw lateral and bases areas (h_r, h_e, h_0).

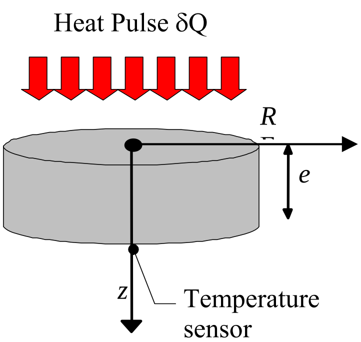

Figure 1 Flash system

Taking into account the symmetry about the principal axis z, the temperature evolution at position (z, r) of the sample can be obtained by solving the two-dimensional heat conduction equation in cylindrical coordinates [6].
Considering the following variables:

$$r = r^+/e \; ; \; z = z^+/e \tag{1}$$

The heat equation is then written, in its dimensionless form, as:

$$\frac{1}{(\alpha/e^2)}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} \tag{2.a}$$

Subject to the boundary conditions:

$$\frac{\partial T}{\partial z} = -\frac{T_m \phi(t)}{(\alpha/e^2)} + Bi_o T \quad at\ z=0 \tag{2.b}$$

$$\frac{\partial T}{\partial r} = 0 \quad at\ \ r=0 \tag{2.c}$$

$$\frac{\partial T}{\partial z} = -Bi_e T \quad at\ z=1 \tag{2.d}$$

$$\frac{\partial T}{\partial r} = -Bi_r T \quad at\ r=R/e \tag{2.e}$$

Where $T_m = Q / \rho \cdot c \cdot e$ is the heat flux term and $\phi(t)$ is the heat flux shape.

The initial condition is taken as :

$$T = 0 \ \ for\ \tau = 0 \tag{2.f}$$

Where F_{0z} is the axial Fourier number defined as :

$$F_{oz} = \frac{\alpha_z \cdot t_0}{e^2}; \tag{3}$$

The Biot numbers are defined as:

$$Bi_i = \frac{h_i \cdot e}{\lambda} \tag{4}$$

Where h_r , h_0 , h_e are respectively the heat transfer coefficients at the lateral area ($r=R/e$) and the bases areas (at $z=0$ and $z=1$).

2.2 Model Resolutions

Different thermal models are developed considering heat transfer mode (1D or 2D), heat losses and type of excitation (Dirac or Crenel).

In the first model (adiabatic 1D), all the sample surfaces are isolated and the heat transfer occurs along the axial direction (z axis). The front face of the sample is subjected to a brief heat pulse (Dirac). Considering these approximations, the temperature evolution is analytically represented by Fourier series [1] as following:

$$T(z,t) = T_m \cdot \left(1 + 2 \cdot \sum_{n=1}^{\infty} \cos\frac{n\pi z}{e} \cdot \exp\left(\frac{-n^2\pi^2}{e^2} \cdot a\,t\right)\right) \tag{5}$$

Where T_m is the maximal temperature (reached at t=∞).

In the second model (1D with heat losses) numerically resolved by the finite volume method [7], the heat flux has a finite duration (crenel), the lateral surface is isolated ($h_r = 0$) and the heat transfer occurs along the axial direction (z axis). The heat transfer coefficients at the front and the rear faces, are taken equal ($h_0 = h_e$).

In the third model (2D with heat losses), also solved numerically by the finite volume method [7], the heat transfer occurs along the radial and axial directions (r and z axis) with supposing all the heat transfer coefficients equal ($h_r = h_0 = h_e$).

3. INVERSE PROBLEM

3.1 OLS formulation

Following the ordinary least squares (OLS) formulation given by Beck and Arnold [4], the sum of the squared temperature residuals to be minimised is:

$$J = (Y - T\ (\beta))^T (Y - T\ (\beta)) = \sum_{i=1}^{n} (Y^i - T\ (t_i, \beta))^2 \tag{6}$$

Where β is the vector of unknown parameters. Y and T are respectively the measured and the calculated temperature vectors.

3.2 Optimal Experiment Design

The sensitivity analysis is classically used for the optimal experimental design. It consists of analyzing the evolution of the different reduced sensitivity coefficients versus time.

$$\overline{X}_{i,k} = \beta_k X_{i,k} \tag{7}$$

Where $X_{i,k}$ are the sensitivity coefficients defined as:

$$X_{i,k} = \frac{\partial T(t_i, \beta)}{\partial \beta_k} \tag{8}$$

The optimality criteria permit a better analysis of the estimation possibility and give an evaluation of the parameter accuracy. These criteria are based on the information matrix X^TX which summarizes the information content of an experiment. The D-optimality is used to minimize the volume of the confidence region (ellipsoid) of the estimated parameters by maximizing the determinant of X^TX. However, this ellipsoid can be skinny and long if one parameter has a very large variance compared to the others. So minimizing its volume may be misleading. Another approach is to study the condition number of X^TX (modified E-optimality) that can be taken as the ratio of its largest eigenvalue to its smallest one. The more this value is near one, the more the problem is well posed. In other words, the modified E-optimality criterion optimizes the shape of the hyper ellipsoid by equalizing all its axes and by minimizing the condition number of X^TX [8].

3.3 Estimation Algorithm

Owing to the nonlinearity of the inverse problem, the minimisation process is constructed with successive approximations with the use of the Gauss-Newton method. In vectoriel form, the solution vector β_{n+1} at the iteration $n+1$ can be written as:

$$\beta_{n+1} = \beta_n + [X(\beta_n)^T X(\beta_n)]^{-1} X(\beta_n)^T (Y - T(\beta_n)) \tag{9}$$

In this iterative procedure, when the problem is ill posed, the matrix X^TX is ill conditioned and the computation of its inverse is difficult.

To overcome these problems, a new diagonal matrix term $\mu_n \cdot \Omega^n$ is added to X^TX in order to damp oscillations and instabilities due to the ill-conditioned character of the problem. We obtain then the Levenberg-Marquardt method:

$$\beta_{n+1} = \beta_n + [X(\beta_n)^T X(\beta_n) + \mu_n \cdot \Omega^n]^{-1} X(\beta_n)^T (Y - T(\beta_n)) \tag{10}$$

This method has the advantage of varying smoothly between the steepest descent method, when the damping parameter μ_n is high, and the Gauss-Newton method by decreasing the value of μ_n [5,9]. This iterative process stops when a minimum of J is reached.

4. RESULTS AND DISCUSSIONS

4.1 Choice of an optimal duration for the experiment

In this study, the optimal experiment design is performed using three models, presented in Section 2.2. In order to determine the optimal experiment duration, the condition number of X^TX (E- modified) is analyzed.

In all three models, the parameters to be taken into account for the identification are (α/e², Tm and Bi). If the temperature is reduced by its maximum, the term Tm will be eliminated from the identification since the temperature is not more sensible to it.

On figure 2, the condition number of X^TX (CN) is calculated for the case of a poorly conductive solid (α/e²=0.03 s^{-1}). We remark that, the adiabatic 1D model always gives a CN equal to one, which indicated that the matrix X^TX is well conditioned. All other models have minima at certain measurement duration. Both 1D and 2D models with losses give CN having a low minimum (between 2 and 3) but the optimal experiment duration of the 1D model (150s) is higher then the 2D one (80s). If three parameters (α/e², Tm and Bi) are taken into account in the estimation using the 2D model, the miminum of the condition number becomes higher (about 40) but the experiment duration becomes lower (50s).

On figure 3, the modified E-criterion (condition number) is presented for a conductive solid (α/e²=0.7 s^{-1}). We note that like for poorly conductive materials, the 1D model with losses gives the lowest CN (10) and the 2D model with 3 parameters gives the highest CN (10^3). However the highest optimal experiment duration (4s) is given for 1D model.

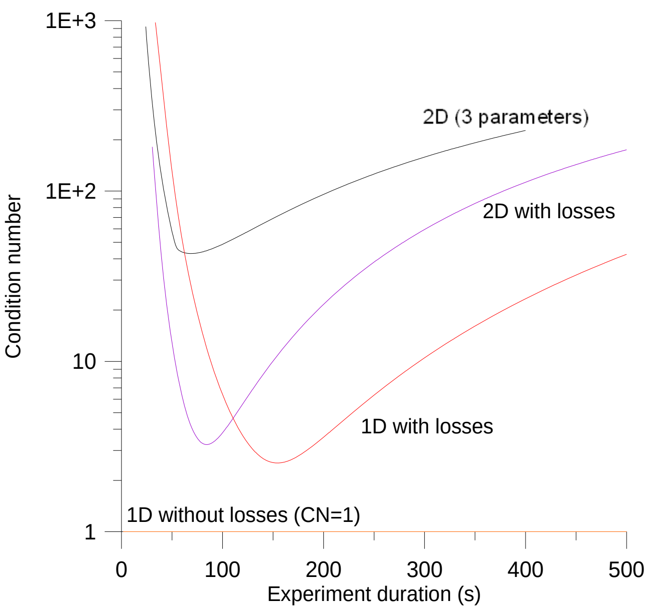

Figure 2: Condition number of X^TX as function of experiment duration in the case of poorly conductive solid (Δt=0.5s and Bi = 0.01 and α/e²=0.03 s^{-1} and Tm=0.2K)

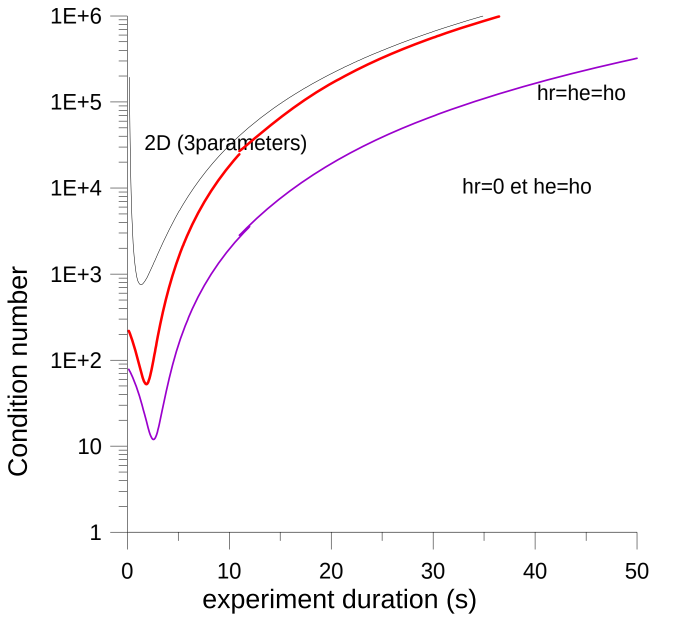

Figure 3: Condition number of X^TX criteria as function of experiment duration in the case of conductive solid (Δt=0.05s and Bi = 0.001 and α/e²=0.7 s^{-1} and Tm=0.2K)

4.2 Thermal diffusivity estimation

The experimental device presented on figure 2 includes a flash source, the sample and an open junction thermocouple (Bismuth Telluride Bi_2Te_3). The heat pulse is generated by five xenon lamps, which provides a uniform heat pulse of a variable power and with 6 ms pulse duration. The sample is supported in horizontal position by the thermocouple.
For this study, the heat rise at the rear sample face is little, so it was necessary to relate the output of the thermocouple to a conditioning-amplifier (2210A). Then the response is recorded on a numerical oscilloscope HM-407.

Figure 4: Flash Apparatus

Experimental measurements were made on 2017 A samples. The initial time (t=0) is marked on the curve by a peak due to a magnetic phenomena generated by the flash.
The Parameter estimation is performed by minimizing the gap between the measured temperatures and the calculated temperatures, using the Levenberg-Marquardt algorithm. Different experiment durations are used for the estimation. The results are presented respectively on figure 5.
We note a difference of the estimated thermal diffusivity between the 2 and the 3 parameters in the 2D model. In fact,

increasing the number of parameters increase the correlation between them. This induces errors on the estimation. Comparing the results issued from the different models with heat losses (1D and 2D), we find a neglecting difference. However, the adiabatic 1D model gives lower thermal diffusivity values.

We see in figure 6 that the confidence region of the estimated thermal diffusivity decreases by increasing the estimation durations and become invariant up to an experiment duration of 5s which is in concordance with the optimal design.

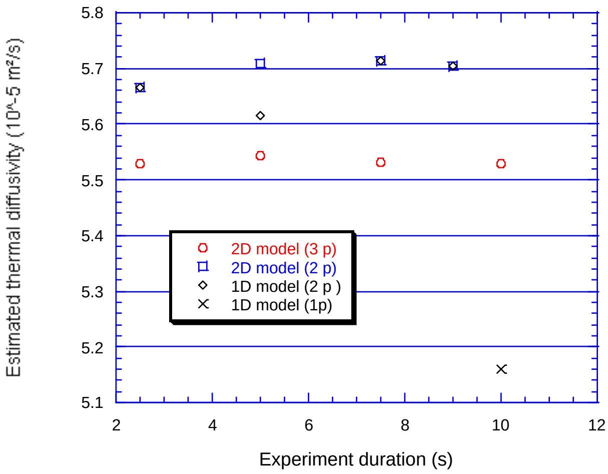

Figure 5 Estimated thermal diffusivity in the case of 2017 A sample

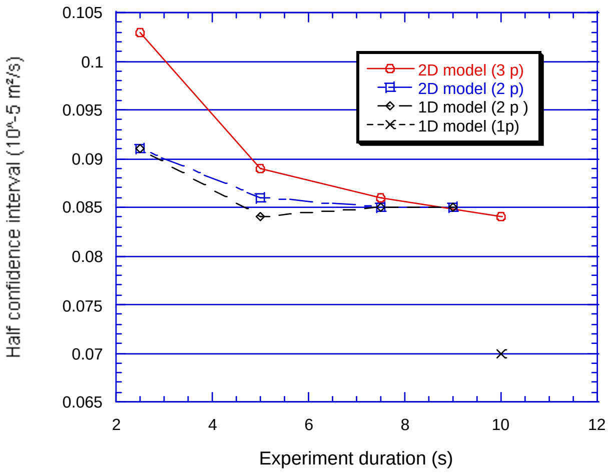

Figure 6 Half confidence interval (2.57×σ) of the estimated thermal diffusivity of 2017 A sample

5. CONCLUSION

Optimal experiment duration and choice of thermal model for thermal diffusivity estimation using the flash experiment is studied in this paper. The 1D model with heat losses is as well precise as the 2D-model and is sufficient for the estimation of the thermal diffusivity. Errors in diffusivity measurement of conductive solids are avoided by taking in to account the heat flux duration. Two parameter (α/e^2 and Bi) models give better results as three parameter (α/e^2 , Tm and Bi) models. Whereas, the error in the evaluation of T_{max} can induce small errors in the estimation. This has been avoided by calculating a mean value of different T_{max} suspect points.

6. REFERENCES

1. W J Parker, R J Jenkins, C P Butler and G L Abbott, *J. Appl. Phys.* **32:** 1679 (1961).
2. A. Degiovanni, *Rev. Gén. Therm.* **185**: 417 (1977).
3. L. Vozar, *Flash Method for thermal diffusivity measurement: Theory and Praxis* (Constantine the Philosopher University in Nitra, Oto Kanas Nitra, 2001).
4. J V Beck, K Arnold, *Parameter estimation in engineering and science* (John Wiley and Sons, NewYork, 1977).
5. M N Ozisik, H R B Orlande, *Inverse heat transfer* (Taylor & Francis, NewYork, 2000)
6. M. N. Ozisik, *Heat transfer* (Wiley -interscience publication, USA, 1985).
7. S.V. Patankar, *Numerical Heat transfer and fluid flow* (Hemisphere Mc. Graaw-Hill, NewYork, 1980).
8. F Mzali, L Sassi, A Jemni, S Ben Nasrallah, D Petit, *Inverse Problems in Science Engineering*, **12: 2**: 193 (2004).
9. W.H. Press, B.P. Flannery, S.A. Teukolsky,W.T. Vetterling, *Numerical recipes in PASCAL* (Cambrigde University Press, 1990).