Pricing Game Analysis of Forest Tourism PPP Project under Price Regulation

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Abstract

Based on the quasi-public good property of the forest tourism PPP project, the price-setting process is the result of a balanced game between the public sector and the private sector. In this paper, the pricing rules are established taking into consideration of the property of the quasi-public good, public willingness to pay, government price regulation, government compensation, etc. In addition, the game decision model of government departments and private departments is constructed based on different pricing rules and the relationship between the pricing level of the quasi-public goods and the decision-making parameter under different pricing rules is obtained, which provides basis for the pricing of the forest tourism PPP project, and some theoretical guidance for achieving the balance of interest of the government, the private sector and the public.

Key words

Quasi-public goods, PPP projects, Pricing rules, Game decision making.
1. Introduction

As a kind of quasi-public good, the forest tourism PPP project follows the pricing rules different from the general project. For the general project, the price is adjusted by the market mechanism. But for PPP project, due to its commonweal character, the price setting shall take the social benefits into consideration. Therefore, the governmental departments need to regulate the price of PPP projects to both ensure the compensatory cost and embody the property of forest tourism as public products. Moreover, this kind of price regulation will have an impact on the investment decision-making of the government sector and private sector. Therefore, it has become a problem to be solved urgently to choose a way to reasonably set the price and make the best decisions under the premise of taking into consideration of both the public interest and the private interest.

There are few studies on the pricing methods of PPP projects. In foreign countries, theories such as incentive theory, system dynamics and real options are applied in the study of PPP project concession pricing model, but systematic and comprehensive pricing methods are still relatively few [1-4]. In China, from the perspective of quasi-public good, Li Ning and Xu Feng proposed reasonable solutions to the ticket pricing for the tourist attractions [5-6], but they only presented institutional solutions. From the perspective of governmental regulation, Zhang Shuibo analyzed the connotation, objectives and system of the regulations of PPP project [7]. Yi Xin put forward the pricing mechanism of PPP rail transit project based on dynamic multi-objectives through the effective integration of government regulations and market mechanism, and established the pricing model [8], but he did not quantify the pricing process. In terms of the pricing models, many scholars have built PPP project pricing models based on different theories such as net present value, real option, capital asset pricing and game [9-12]. From the perspective of stakeholders, Sun Chunling introduced the “fairness concern” parameter considering the game between the interest of private enterprise and the public interest, and established the pricing model by using the Nash equilibrium game theory [13]. Song Bo et al. studied the pricing mechanism of the government and the private sector and its influence on the relevant parameters under the condition of different public needs and verified the relationship between the pricing of public goods and the elasticity of product demand [14]; but they did not consider the pricing game process of the public and private sectors in PPP projects. Therefore, based on the shortcomings of existing researches and from multiple perspectives, this paper takes into account the attribute of public good, public willingness to pay, government price regulation and government compensation in the PPP project pricing model. Taking forest tourism PPP project as
the specific research subject, this paper probes into the game decision-making model of the public and private sectors under different pricing rules and discusses the optimal investment strategy of the two parties.

2. Pricing of Forest Tourism PPP Project under Price Regulation

The forest tourism PPP project is relying on public resources, and is non-competitive, but when the project sets up the collecting fee system to control the number of tourists, this kind of forest tourism products become exclusive in consumption [15]. Therefore, the forest tourism PPP project in this paper has the attribute of quasi-public goods. As a kind of quasi-public good, the forest tourism products, unlike private products, do not take the profit maximization as the goal, but takes the social benefits into consideration. A too high price can only lead to the loss of its public welfare characteristics. Therefore, during the pricing process of the forest tourism PPP project, the government exerts impact on the decisions or behaviors of the private sectors utilizing the price regulations to ensure that the consumer prices are within a reasonable range under the premise of guaranteeing the minimum income and controlling the maximum income of the private sectors.

Assume that \( \varepsilon \) refers to the consumer’s willingness to pay, and \( \varepsilon \in [\varepsilon_{\text{min}}, \varepsilon_{\text{max}}] \); \( p \) is the consumer price at the project operation stage, \( R \) is the actual operating income of the project, \( R_0 \) means the minimum expected income of the private sector and \( F \) stands for the operating period compensation provided by the governmental sectors to the private sectors. The government departments stipulate the regulating range of the price, i.e. \( p \in [m, M] \). In this value section, when \( p \) is given the lower limit value of \( m \), it can be guaranteed that the actual income of the private sector is still larger than the minimum expected income. This paper sets out the pricing rules of forest tourism PPP project products by comparing consumers’ willingness to pay and governmental price regulation range.

(1). \( \varepsilon_{\text{min}}>M \)

On the public product supply market, when the minimum willingness to pay of the consumer is greater than the maximum chargeable price of the operated PPP project in government regulation, the actual price of the public product \( p \leq M \), and the private sector will definitely choose \( p=m \). In this case, the government departments think that the actual income of the private sector is relatively large. They will claim the excess income in certain form to guarantee the social benefits of public goods.

(2). \( m \leq \varepsilon \leq M \)
On the public product supply market, when the willingness to pay of the consumer is within the range of the minimum charging price $m$ and the maximum charging price $M$ regulated by the governmental departments, the public products are traded freely according to the market supply and demand. At this time, the product pricing rights are completely handed over to the private sectors. They will set the price according to the market and require that $p \in [m, M]$. Since the private sectors set price independently, and their starting point is to pursue the maximum profit, they will not consider the governmental compensation.

(3). $\varepsilon_{\text{max}} < m$

On the public product supply market, when the maximum willingness to pay of the consumer is smaller than the minimum charging price regulated by the governmental departments, $p < m$. At this time, the government thinks that the actual income of the private sector is fairly small and will provide the price subsidy-based compensation $F$ for the operational period to the private sectors, so as to ensure the lowest revenue and the normal operation of PPP project of the private sectors.

3. Game Decision-making Model of the Forest Tourism PPP Project

3.1 Model Assumption

Assume that the total amount of the initial investment in the forest tourism PPP project is $C$, $T_f$ refers to the concession term of the project construction and operation, the project construction period is $T_c$ and the project operation and maintenance cost is $v$. In general, the project operating and maintenance cost drops with the increase of the project investment amount and constantly increases with the passage of time. It can be expressed as:

$$v(C,t) = k C^{-\alpha} t^\beta$$  \hspace{1cm} (1)

Where $k$, $\alpha$ and $\beta$ are constants and are all larger than 0.

The yearly average number of tourists of the project is $Q$. Assume that $Q$ is only relevant to $p$, the demand function can be expressed as:

$$Q(p) = a - bp$$  \hspace{1cm} (2)
Where, $b$ is the slope of the demand function. $a/b$ is the price level when the quantity demand is zero. $a$ and $b$ are positive constants.

The actual income of the project in unit time can be expressed as:

$$R = pQ(p) = p(a - bp)$$ (3)

According to the principles of public economics, the social benefits of public goods are composed of producer surplus and consumer surplus. In this paper, the producer surplus refers to the profits earned by the private sector; the consumer surplus refers to the difference between the highest price that the tourism consumers are willing to pay and the price actually paid. Thus, the consumer surplus can be expressed as:

$$W = \frac{1}{2}(a - bp)\left(\frac{a}{b} - p\right)(T_f - T_c)$$ (4)

### 3.2 Model Building

In the forest tourism PPP project, the government department’s goal is to maximize the social benefits of forestry resources, while the objective of the investment of private sectors is to maximize their own profits, thus a kind of mutually gaming relationship was formed. The optimal investment strategies of the governmental department and private sector are obtained through constructing the following gaming model.

The gaming model of the private sector:

$$\max E[\pi_i] = \int_{T_c}^{T_f} R dt + F - C - \int_{T_c}^{T_f} v(C, t) dt$$ (5)

The gaming model of the government department:

$$\max E[U_i] = \pi_i + W - \mu F$$

$$s.t. \quad E[\pi_i] \geq R_0$$ (6)
In (5) and (6), the two sides of the game do not take actions simultaneously, but take actions step by step. Therefore, this is a kind of multistage game. Nash equilibrium solution of the multistage game can be determined by adopting the backward recursion method.

4. Game Analysis

1. When \( \varepsilon_{\min} > M \),
   Assume that the proportion of the excess earnings obtained by the government is \( \phi \), then the function of the profit of the private sector is:

   \[
   \pi_i = (1 - \phi)M \cdot (a - bM) \cdot (T_f - T_c) - C - kC^{-\alpha} \frac{T_f^{\beta+1} - T_c^{\beta+1}}{\beta + 1}
   \]  
   \[
   (7)
   \]

   The function of the social benefit of the government sector is:

   \[
   U_i = (1 - \phi)M \cdot (a - bM) \cdot (T_f - T_c) - C - kC^{-\alpha} \frac{T_f^{\beta+1} - T_c^{\beta+1}}{\beta + 1} + \frac{1}{2} (a - bp) \cdot \left( \frac{a}{b} - p \right) \cdot (T_f - T_c)
   \]  
   \[
   (8)
   \]

   (1) First, analyze and determine the profit of the private sector based on the concession term and charging price given by the government sector to determine the optimal initial investment amount. No matter what decisions the government will make, the private sector will always choose the initial investment amount which can maximize their profit.

   Let \( \frac{\partial E[\pi_i]}{\partial C} = 0 \), and we have

   \[
   C_i^* = \sqrt[\alpha+1]{\frac{\alpha k \cdot (T_f^{\beta+1} - T_c^{\beta+1})}{(\beta + 1)}}
   \]  
   \[
   (9)
   \]

   Nature 1: When \( \frac{\partial C_i^*}{\partial T_f} > 0 \), it suggests that with the increase of the concession term, the project’s initial investment amount also rises correspondingly, because the late operation and
maintenance costs of the project can be lowered only when the initial investment amount is actively increased and the project quality is enhanced.

(2) When it is predicted that the private sector makes decisions based on Formula (8), the optimal decision of the government becomes the following optimization problem:

$$\text{max } E(U_i) = (1 - \phi) M \cdot (a - bM) \cdot (T_f - T_c) - C_i^*$$
$$- k(C_i^*)^{-a} \frac{T_f^\beta - T_c^\beta}{\beta + 1} + \frac{1}{2} (a - bM) \left( \frac{a}{b} - M \right) (T_f - T_c)$$  \hspace{1cm} (10)

s.t.  \( E[\pi] \geq R_0 \)

The above optimization problem shows that when the government makes decisions, in addition to maximizing the social benefits of the project, it must satisfy the requirement of participation by the private sector, i.e. the investment profit of the private sector cannot be lower than the minimum expected return.

Seek the solution to this problem. Assume \( \lambda_i \) is the multiplier of Lagrange in Formula (9), then the Lagrange function of the above problem is: \( L_i = E(U_i) + \lambda_i [E[\pi] - R_0] \)

Let \( \frac{\partial L_i}{\partial M} = 0, \frac{\partial L_i}{\partial T_f} = 0, \frac{\partial L_i}{\partial \lambda_i} = 0 \), and we have:

\[
T_f^* = \beta \sqrt{\frac{(1 - \phi) a \left( \frac{a}{b} - M \right)}{2k(C_i^*)^{-a}}} \hspace{1cm} (11)
\]

\[
M^* = \frac{a}{b} \frac{2k(C_i^*)^{-a} T_f^\beta}{a(1 - \phi)} \hspace{1cm} (12)
\]

Eqs. (11) and (12) show the optimal decision of the government department when the minimum willingness to pay of the consumer is greater than the upper limit of the toll price. The following nature can be obtained based on the above formula.

Nature 2: Since \( ((\partial T_f^*)/(\partial C)) > 0 \), the larger the initial investment amount is, the longer the concession period is, i.e. it will take longer for the project to recoup the capital outlay and get the expected earnings. \( ((\partial T_f^*)/(\partial M)) < 0 \) and \( ((\partial T_f^*)/(\partial \phi)) < 0 \), suggesting that the higher the charging price and the proportion of the excess earnings of the government is, the shorter the concession period.
term will be, i.e. the actual income of the project will be larger. The government controls the excess earnings of the private sector through manipulating the length of the concession term.

Nature 3: Formula (12) suggests that the upper limit value $M$ stipulated by the government is positively correlated to the initial investment amount $C$ and the proportion $\phi$ of the excess earnings and is negatively correlated to the concession term $T_f$. It is positively correlated to the related parameter $a$ of the demand elasticity and negatively correlated to another related parameter $b$ of the demand elasticity. When $a$ remains unchanged, the larger $b$ is, the lower the upper limit of the stipulated charging price will be; when $b$ remains unchanged, the larger $a$ is, the upper limit value of the stipulated charging price will be.

2. When $m \leq \varepsilon \leq M$,

The private function of the private sector is:

$$\pi_2 = p(a-bp)(T_f - T_c) - C - kC^{-a}T_f^{\frac{\beta+1}{\beta+1}} - T_c^{\frac{\beta+1}{\beta+1}}$$

(13)

The function of the social benefit of the governmental department is:

$$U_2 = p(a-bp)(T_f - T_c) - C - kC^{-a}T_f^{\frac{\beta+1}{\beta+1}} - T_c^{\frac{\beta+1}{\beta+1}}$$

$$+ \frac{1}{2}(a-bp)\left(\frac{a}{b} - p\right)(T_f - T_c)$$

(14)

In this case, the private sector will set the price independently and choose the cost and charging price which will make its profit maximized.

Let $\frac{\partial E[\pi_2]}{\partial C} = 0$, and $\frac{\partial E[\pi_2]}{\partial p} = 0$, and we can get

$$C^*_2 = C^*_1, \quad p^*_2 = \frac{a}{2b}$$

(15)

Nature 4: The price set by the private sector independently is related to the relevant parameters $a$ and $b$ of the demand elasticity. When $a$ remains unchanged, the larger $b$ is, the lower
the equilibrium price on the supply market is; when \( b \) remains unchanged, the larger \( a \) is, the larger the market demand is and the equilibrium price will be.

When the private sector makes investment decisions as per the above formula, the function of the government’s social benefits is:

\[
U_2 = \frac{3}{8} \frac{a^2}{b} (T_f - T_c) - \left( \frac{\alpha k \left( T_f^{\beta+1} - T_c^{\beta+1} \right)}{\beta + 1} \right) \left( 1 + \frac{1}{a} \right)
\]  

(16)

2. When \( \varepsilon_{\text{max}} < m \)

The aim of the financial compensation of the governmental department is to secure the minimum income of the private sector, so the compensation function can be expressed as:

\[
F = (m - p)Q^*(T_f - T_c)
\]  

(17)

Then, the function of the private sector’s profit is:

\[
\pi_3 = p\cdot Q^*(T_f - T_c) + (m - p)\cdot Q^*(T_f - T_c) - C - kC^{-a} \frac{T_f^{\beta+1} - T_c^{\beta+1}}{\beta + 1}
\]  

(18)

Given that the governmental compensation comes from tax revenue and other public financial capital, deadweight loss will occur generally. Assume that per unit compensation will lead to \( \mu (\mu > 1) \) unit social cost, then the function of the government sector’s social benefits will be:

\[
U_3 = p\cdot Q^*(T_f - T_c) + (m - p)\cdot Q^*(T_f - T_c) - C - kC^{-a} \frac{T_f^{\beta+1} - T_c^{\beta+1}}{\beta + 1} \\
+ \frac{1}{2} \left( a - bp \right) \left( \frac{a}{b} - p \right) \cdot (T_f - T_c) - \mu (m - p)\cdot Q^*(T_f - T_c)
\]  

(19)

Using the abovementioned method we can get
\[ C_1^* = C_1^* = \alpha \frac{\sqrt{tp(T_f^{\beta_1} - T_c^{\beta_1})}}{\beta + 1} \]  

(20)

\[ T_f^{*} = \beta \left( a - bp \right) \frac{\mu p + \frac{1}{2} \left( \frac{a}{b} - p \right)}{\mu \cdot k(C_1^*)^{\alpha}} \]  

(21)

\[ p_3^* = \frac{a(\mu - 1)}{b(2\mu - 1)} \]  

(22)

\[ m^* = \frac{C_1^* + k(C_1^*)^{-\alpha} T_f^{\beta_1} - T_c^{\beta_1} + R_0}{\beta + 1} \]  

(23)

Formula (21), (22) and (23) indicate the government sector’s optimal decision under the circumstance that the consumer’s maximum willingness to pay is smaller than the lower limit value of the charging price. The following nature can be obtained based on the above formulas:

Nature 5: Take the derivative of \( p \) using formula (21). When

\[ p < \frac{a(\mu - 1)}{b(2\mu - 1)}, \frac{\partial T_f^{*}}{\partial p} > 0; \text{ when } p > \frac{a(\mu - 1)}{b(2\mu - 1)}, \frac{\partial T_f^{*}}{\partial p} < 0; \text{ when } p = \frac{a(\mu - 1)}{b(2\mu - 1)} , \text{ the concession term reaches its maximum value.} \]

Nature 6: When \( ((\partial T_f^{*})/\partial \mu) < 0 \), it suggests that the larger the governmental compensation cost is, the shorter the concession term will be, i.e. the government reduces its compensation to the private sector through shortening the concession term, thus reducing the compensation cost.

Nature 7: When \( ((\partial p)/(\partial \mu)) > 0 \), it suggests that the larger the governmental compensation cost is, the higher the charging price is, i.e. the charging price needs to be raised to reduce the gap between the price and the lower limit value of the charging price, thus reducing the governmental compensation cost.

Nature 8: Formula (23) suggests that the lower limit value \( m \) of the charging price stipulated by the government sector is related to the initial investment amount \( C \), concessionary time \( T_f \), the minimum expected income of the private sector \( R_0 \) and relevant parameters \( a \) and \( b \) of the demand elasticity.
Conclusion

Through the analysis of the optimal decisions of the public and private sectors under three different pricing mechanisms, we can get the following conclusions:

(1) By comparing the consumer’s willingness to pay with the upper and lower limit values of the price stipulated by the government sector, the pricing rules of the forest tourism PPP project under different circumstances are obtained. This kind of pricing rule comprehensively takes into account the pricing principle of quasi-public goods, government price regulation, consumer’s affordability, government compensation and income distribution to ensure that the interests of the three stakeholders of the PPP project are balanced.

(2) When $\varepsilon_{\text{min}} > M$, the price will be set based on the upper limit value of the price stipulated by the government; when $m \leq \varepsilon \leq M$, the private sector will set the price independently as per the market situation; when $\varepsilon_{\text{max}} < m$, the price will be set as per the consumer’s willingness to pay.

(3) Based on the influence of different pricing rules on the decisions of the public sector and private sector, the game decision-making model is established. The following points are obtained after the comparative analysis:

(a) When making decisions under different pricing rules, the optimal initial investment amount of the private sectors is equal, and is related to the concession period.

(b) When $\varepsilon_{\text{min}} > M$ and $\varepsilon_{\text{max}} < m$, the concession terms are related to the proportion $\phi$ of the excess earnings and the coefficient $\mu$ of the governmental compensation cost respectively; and they are both related to the related parameters $a$ and $b$ of the market demand elasticity.

(c) Under different pricing rules, the actual charging price is positively correlated with the parameter $a$ of the market demand elasticity and is negatively correlated to the parameter $b$ of the market demand elasticity.

(d) The optimal upper limit value and lower limit value of the charging price stipulated by the government sector are related to the initial investment amount, concession term, and the relevant parameters $a$ and $b$ of the market demand.

(4) With the application of PPP project in China’s infrastructure construction, more and more problems will be exposed, therefore, to further improve the theories relevant to PPP project is of great significance to promote public-private cooperation.

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