$$v^{(j)}(0) = v_j, \quad j = 0,1. \tag{19}$$

Observe that the above equation is a special case of Eq. (1) with

$$\beta = 1.75, \gamma = 0.25, m = 2, n = 1, a = 1,$$
$$g(\tau) = \ln(\tau^2),\ k(\tau,\eta) = \tau\eta,\ \tau,\eta \in [0,1],$$

$$(G\sigma)(\eta) = \frac{1}{9}\sigma(\eta),\ \eta \in [0,1], \sigma \in C[0,1].$$

The functions g and k are continuous and G is increasing linear transformation that satisfies in assumption (ii). To check assumption (iii), let's put

$$L(\tau,\eta) = \frac{1}{\Gamma(\frac{3}{2})}\int_\eta^\tau \mu\eta\sqrt{\tau-\mu d\mu}$$
$$= \frac{2\eta}{\sqrt{\pi}}\left[\frac{2\tau}{3}(\tau-\eta)^{\frac{3}{2}} - \frac{2}{5}(\tau-\eta)^{\frac{5}{2}}\right].$$

Since $\sup_{\tau\in[0,1]}\int_0^1 L^2(\tau,\eta)\, d\eta \le 1$, then applying Theorem 2, we deduce that Eq. (19) has a unique solution in χ.

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