

## **The Probability Lattice in the Random Decision Formal Context**

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### **Abstract**

Concept lattice theory, based on binary logic, establishes the binary relation between object and attribute in the form background, forming concept and constructing concept lattice. The data of concept lattice analysis were generally described in the formal context. As a powerful tool for conceptual data analysis and knowledge processing, the formal concept analysis was widely used in data mining, knowledge discovery of artificial intelligence, and many other areas. In different formal contexts, the concept lattice structure, rule extraction, object reduction and attribute reduction are the hot issues of research. In this paper, we proposed the concept of the probability lattice in the random decision formal context, the corresponding theorems and algorithm. Also, we proved its effectiveness and application with the examples, especially in solving the risk decision problem.

### **Key Words**

Concept lattice, random decision formal context, probability lattice in the random decision formal context, risk decision problem.

## 1. Introduction

Theory of concept lattice was German professor Wille who first proposed in 1982 as a mathematical theory. It based on the binary logic, established the binary relation between object and attributes in the formal context, and constructed the concept lattice. As a powerful tool for conceptual data analysis and knowledge processing, the formal concept analysis was widely used in data mining, knowledge discovery of artificial intelligence, and many other areas<sup>[5-7]</sup>.

With the development of technology, in most cases, the information that human dealing with was often uncertain, which were classified as fuzzy information and random information. For fuzzy information, the fuzzy concept<sup>[2]</sup> of fuzzy relation between object and property is presented, basing on fuzzy logic. And it had been widely used in the data analysis, knowledge processing and information retrieval. Since 2005, when the professor W. Zhang proposed the decision formal context, all kinds of extended decision formal context had been put forward, such as: fuzzy decision formal context, real value decision formal context and incomplete decision formal context<sup>[9-12]</sup>. In different contexts, the concept lattice structure, rule extraction, object reduction and attribute reduction are the hot issues of research. In 2013, professor B. Liu and Y. Li put forward the random concept<sup>[3]</sup>. So far, there are very few studies that use the concept lattice to describe and process random information, but it is a development trend.

In this paper, we construct the probability lattice in the random decision formal context. As a mathematical theory, it can easily and efficiently reflect and process random information.

### Definition 1<sup>[4]</sup>

If a binary relation  $\hat{E}$  is called a partial order relationship, when it meets:

- (1) Reflexive: " $a \hat{E} a$ ;
- (2) Anti-symmetric: " $a, b \hat{E} A, a \hat{E} b, b \hat{E} a \Rightarrow a = b$ ;
- (3) Transitive: " $a, b, c \hat{E} A, a \hat{E} b, b \hat{E} c \Rightarrow a \hat{E} c$ ."

Then,  $(A, \hat{E})$  is called poset.

### Theorem 2<sup>[4]</sup>

Let  $L$  is a non-empty set.  $\cup, \cap$  are the binary operations of  $L$ . If it's satisfy:

Idempotent law:  $a \cup a = a, a \cap a = a$ ;

Commutative law:  $a \dot{\cup} b = b \dot{\cup} a, a \dot{\cup} b = b \dot{\cup} a$ ;

Associative law:  $a \dot{\cup} (b \dot{\cup} c) = (a \dot{\cup} b) \dot{\cup} c, a \dot{\cup} (b \dot{\cup} c) = (a \dot{\cup} b) \dot{\cup} c$ ;

Absorption law:  $a \dot{\cup} (a \dot{\cup} b) = a, a \dot{\cup} (a \dot{\cup} b) = a$ .

And there is: " $a, b \in L, a \leq b \Leftrightarrow a \dot{\cup} b = b$ ,

Then,  $(L, \dot{\cup}, \dot{\cup})$  is a lattice.

### Definition 3<sup>[4]</sup>

Let  $(L, \dot{\cup}, \dot{\cup})$  is a lattice, if " $x, y, z \in L$ , there is  $x \dot{\cup} (y \dot{\cup} z) = (x \dot{\cup} y) \dot{\cup} (x \dot{\cup} z)$  (or  $x \dot{\cup} (y \dot{\cup} z) = (x \dot{\cup} y) \dot{\cup} (x \dot{\cup} z)$ ). Then  $(L, \dot{\cup}, \dot{\cup})$  is a distributive lattice.

### Definition 4<sup>[2]</sup>

Let  $(U, A, I)$  is a formal context, of which  $U = \{x_1, x_2, \dots, x_n\}$  is an object set and each  $x_i (i \in n)$  is called an object; For the condition attribute set  $A = \{a_1, a_2, \dots, a_m\}$ , each  $a_j (j \in m)$  is called a condition attribute; Let  $I$  is the binary relationship from  $U$  to  $A, I \subseteq U \otimes A$ . If  $(x, a) \in I$ , it means  $x$  has a condition attribute  $a$ ; If  $(x, a) \notin I$ , it means  $x$  has no condition attribute  $a$ . We define that 1 represents  $(x, a) \in I$ , and 0 represents  $(x, a) \notin I$ .

### Definition 5<sup>[2]</sup>

Let  $(U, A, I, C, J)$  and  $(U, C, J)$  are two formal contexts, with the same area of discussion, then  $(U, A, I, C, J)$  is called the decision formal context.

### Definition 6<sup>[3]</sup>

Let  $(U, A, I, B, J)$  is a random decision formal context, of which  $U = \{x_1, x_2, \dots, x_n\}$  is an object set and each  $x_i (i \in n)$  is called an object; For the condition attribute set  $A = \{a_1, a_2, \dots, a_m\}$ , each  $a_j (j \in m)$  is called an condition attribute; For the decision attribute set  $B = \{b_1, b_2, \dots, b_k\}$ , each  $b_q (q \in k)$  is called a decision attribute; Let  $I$  is the binary relationship from  $U$  to  $A, I \subseteq U \otimes A$ . If  $(x, a) \in I$ , it means  $x$  has a condition attribute  $a$ ; If  $(x, a) \notin I$ , it means  $x$  has no condition attribute  $a$ . Let  $J$  is the binary relationship from  $U$  to  $B, J \subseteq U \otimes B$ . If  $(x, b) \in J$ , it means  $x$  may make decision  $b$ ; If  $(x, b) \notin J$ , it means

$x$  not make decision  $b$ .

We define 1 represents  $(x,a) \hat{\in} I$ , and 0 represents  $(x,a) \notin I$ . When  $(x,b) \hat{\in} J$ , let  $p \hat{\in} (0,1]$  be the probability of  $x$  to make decision  $b$ ; and 0 represents  $(x,b) \notin J$ . Thereof, the random decision formal context transforms into a table in  $[0,1]$ . If all probability  $p$  is 0 or 1, the random decision formal context is degraded to a classical formal context.

**Example 1**<sup>[3]</sup>

Let  $U = \{1,2,3,4\}$ ,  $A = \{a_1, a_2, a_3\}$  and  $B = \{b_1, b_2, b_3\}$ . We get the table 1. (Noticing the decision attributes is not independent of each other, so the sum of the probabilities of each row is not necessarily equal to 1).

Table 1, the random decision formal context of example 1

$U$	Condition attributes $A$			Decision attributes $B$		
	$a_1$	$a_2$	$a_3$	$b_1$	$b_2$	$b_3$
1	1	1	0	0.7	0.2	0.8
2	0	1	1	0.1	0.3	0.9
3	1	0	1	0.2	0.9	0.7
4	0	1	0	0.1	0.3	0.6

**2. Main Results**

**Definition**

Let  $(U,A,I,B,J)$  is a random decision formal context.  $X^A = (A, \mathcal{C}, \tilde{E})$  is a condition attribute space.  $X^B = (B, \mathcal{C}, \tilde{E})$  is a decision attribute space. We call the set  $P_{X^A, X^B}$  is the probability set in the random decision formal context:

$$P_{X^A, X^B} = \{p_{s,t}(x) \mid " x \hat{\in} U, " s \hat{\in} X^A, " t \hat{\in} X^B, (x,s) \hat{\in} I, (x,t) \hat{\in} J\}.$$

In which,  $p_{s,t}(x)$  is the probability of  $x$ , that has the condition attribute  $s$ , to make decision  $t$ .

Obviously, there is  $P_{X^A, X^B} \subseteq (0, 1]$ .

**Definition 8**

Let  $P_{X^A, X^B}$  is the probability set in the random decision formal context. Let the “ $\leq$ ” is the less than or equal to in real number field  $R$ . Then,  $(P_{X^A, X^B}, \leq)$  is a poset.

**Theorem 9**

Let  $P_{X^A, X^B}$  is a probability set in the random decision formal context. Let the “ $\leq$ ” is the less than or equal to in real number field  $R$ . Let for “ $p_1, p_2 \in P_{X^A, X^B}$ ”, there are  $p_1 \dot{\cup} p_2 = \max\{p_1, p_2\}$ ,  $p_1 \dot{\cap} p_2 = \min\{p_1, p_2\}$ . Then,  $(P_{X^A, X^B}, \dot{\cup}, \dot{\cap})$  is a lattice, and it is a distributive lattice.

**Proof:**

(1) In  $(P_{X^A, X^B}, \leq)$ , for “ $a, b, c \in P_{X^A, X^B}$ ”, there is

Idempotent law:  $a \dot{\cup} a = \max\{a, a\} = a$ ,  $a \dot{\cap} a = \min\{a, a\} = a$ ;

Commutative law:  $a \dot{\cup} b = \max\{a, b\} = b \dot{\cup} a$ ,  $a \dot{\cap} b = \min\{a, b\} = b \dot{\cap} a$ ;

Associative law:

$$a \dot{\cup} (b \dot{\cup} c) = \max\{a, \max\{b, c\}\} = \max\{a, b, c\} = \max\{\max\{a, b\}, c\} = (a \dot{\cup} b) \dot{\cup} c$$

$$a \dot{\cap} (b \dot{\cap} c) = \min\{a, \min\{b, c\}\} = \min\{a, b, c\} = \min\{\min\{a, b\}, c\} = (a \dot{\cap} b) \dot{\cap} c.$$

Absorption law:

$$a \dot{\cap} (a \dot{\cup} b) = \min\{a, \max\{a, b\}\} = a,$$

$$a \dot{\cup} (a \dot{\cap} b) = \max\{a, \min\{a, b\}\} = a.$$

And meets the rules: “ $a, b \in P_{X^A, X^B}$ ,  $a \leq b \Leftrightarrow a \dot{\cup} b = \max\{a, b\} = b$ ”.

So by the theorem 2,  $(P_{X^A, X^B}, \dot{\cup}, \dot{\cap})$  is a lattice.

(2) In lattice  $(P_{X^A, X^B}, \dot{\cup}, \dot{\cap})$ , for “ $a, b, c \in P_{X^A, X^B}$ ”, there is

$$a \dot{\cup} (b \dot{\cap} c) = \max\{a, \min\{b, c\}\} = \max\{\min\{a, b\}, \min\{a, c\}\} = (a \dot{\cap} b) \dot{\cup} (a \dot{\cap} c)$$

So by the definition 3, the lattice  $(P_{X^A, X^B}, \dot{\cup}, \dot{\cap})$  is a distributive lattice.

### Definition 10

We call the lattice  $(P_{X^A, X^B}, \hat{U}, \hat{U})$  is the probability lattice in the random decision formal context.

### Algorithm 11

The algorithm of probability lattice in the random decision formal context:

Let  $(U, A, I, B, J)$  is a random decision formal context, of which  $U = \{x_1, x_2, \dots, x_n\}$  is an object set and each  $x_i (i \in n)$  is called an object; For the condition attribute set  $A = \{a_1, a_2, \dots, a_m\}$ , each  $a_j (j \in m)$  is called an condition attribute; For the decision attribute set  $B = \{b_1, b_2, \dots, b_k\}$ , each  $b_q (q \in k)$  is called a decision attribute; Let  $I$  is the binary relationship from  $U$  to  $A, I \subseteq U \otimes A$ . If  $(x, a) \hat{I}$ , it means  $x$  has a condition attribute  $a$ ; If  $(x, a) \notin I$ , it means  $x$  has no condition attribute  $a$ . Let  $J$  is the binary relationship from  $U$  to  $B, J \subseteq U \otimes B$ . If  $(x, b) \hat{J}$ , it means the  $x$  may make decision  $b$ ; If  $(x, b) \notin J$ , it means the  $x$  not make decision  $b$ .

**Step 1.** Find out the condition attribute space  $X^A = (A, \zeta, \hat{E})$  and the decision attribute space  $X^B = (B, \zeta, \hat{E})$ .

**Step 2.** Find out the probability set in the random decision formal context:

$$P_{X^A, X^B} = \{p_{s,t}(x) \mid x \hat{I} U, s \hat{I} X^A, t \hat{I} X^B, (x,s) \hat{I} I, (x,t) \hat{I} J\}$$

**Step 3.** Obtain the probability lattice in the random decision formal context  $(P_{X^A, X^B}, \hat{U}, \hat{U})$  and come to a solution.

## 3. The application case

### Example 2.

The relationship between the occurrence of the disease and the clinical performance is not a certainty. When a doctor visits a patient, it is often based on his knowledge or experience, (for example, his previous clinical record), and the clinical manifestation of the patient to make a

preliminary decision about whether or not the patient had a particular disease. Of course, patients with the same clinical manifestations should be diagnosed with the same disease. This is a subjective probability, like a doctor who points out that a patient has an operation under certain conditions, "The success rate is 50%."

Suppose set  $U$  is for a group of patients,  $U = \{x_1, x_2, x_3, x_4, x_5\}$ . Set  $A$  is for a collection of disease symptoms provided by the patient (conditional properties),  $A = \{\text{have a headache, a stuffy nose, a lot of snot}\} = \{a_1, a_2, a_3\}$ . Set  $B$  is for a collection of initial judgment results (decision properties),  $B = \{\text{rhinitis}\} = \{b_1\}$ . The  $p_{a_i, b_j}(x_k)$  is the probability of  $x_k$  to make the correct diagnosis  $b_j$ , when disease symptoms  $a_i$  occurs.

Table 2. the random decision formal context of example 2

	Condition attributes $A$			Decision attributes $B$
$U$	Has a headache $a_1$	A stuffy nose $a_2$	A lot of snot $a_3$	Rhinitis $b_1$
$x_1$	1	1	0	0.6
$x_2$	0	1	1	0.1
$x_3$	1	0	1	0.3
$x_4$	0	1	0	0.1
$x_5$	0	0	1	0.1

**The solution is:**

By the algorithm 11 of probability lattice in the random decision formal context, we get the following:

**Step 1.** The condition attribute space:

$$\begin{aligned}
 X^A &= (A, \zeta, \tilde{E}) \\
 &= \{a_1, a_2, a_3, a_1 \zeta a_2, a_1 \zeta a_3, a_2 \zeta a_3, a_1 \zeta a_2 \zeta a_3, a_1 \tilde{E} a_2, a_1 \tilde{E} a_3, a_2 \tilde{E} a_3, a_1 \tilde{E} a_2 \tilde{E} a_3\}
 \end{aligned}$$

The decision attribute space:

$$X^B = (B, \zeta, \tilde{E}) = \{b_1\}.$$

**Step 2.** The probability set in the random decision formal context:

$$\begin{aligned} P_{X^A, X^B} &= \{p_{s,t}(x) \mid "x \hat{I} U, "s \hat{I} X^A, "t \hat{I} X^B, (x,s) \hat{I} I, (x,t) \hat{I} J\} \\ &= \{p_{a_2, b_1}(x_4), p_{a_3, b_1}(x_5), p_{a_1 \zeta a_2, b_1}(x_1), p_{a_1 \zeta a_3, b_1}(x_3), p_{a_2 \zeta a_3, b_1}(x_2)\} \\ &= \{0.1, 0.1, 0.6, 0.3, 0.1\} \end{aligned}$$

**Step 3.** Let the “ $\leq$ ” is the less than or equal to in real number field  $\mathbb{R}$ . Let for “ $p_1, p_2 \hat{I} P_{X^A, X^B}$ ”, there are  $p_1 \dot{\cup} p_2 = \max\{p_1, p_2\}$ ,  $p_1 \dot{\cap} p_2 = \min\{p_1, p_2\}$ . We get the probability lattice in the random decision formal context:  $(P_{X^A, X^B}, \dot{\cup}, \dot{\cap})$ .

Therefore, we come to the solutions:

(1) Because of  $p_{a_1 \zeta a_2, b_1}(x_1) \dot{\cup} p_{a_1 \zeta a_3, b_1}(x_3) = \min\{0.6, 0.3\} = 0.3$ , according to disease symptoms  $a_1$ , the probability of making correct diagnosis is only 0.3;

Because of  $p_{a_2, b_1}(x_4) \dot{\cup} p_{a_1 \zeta a_2, b_1}(x_1) \dot{\cup} p_{a_2 \zeta a_3, b_1}(x_2) = \min\{0.1, 0.6, 0.1\} = 0.1$ , according to disease symptoms  $a_2$ , the probability of making correct diagnosis is only 0.1;

Because of  $p_{a_3, b_1}(x_5) \dot{\cup} p_{a_1 \zeta a_3, b_1}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_1}(x_2) = \min\{0.1, 0.3, 0.1\} = 0.1$ , according to disease symptoms  $a_3$ , the probability of making correct diagnosis is only 0.1;

Because of  $p_{a_1 \zeta a_2, b_1}(x_1) = 0.6$ , according to disease symptoms  $a_1 a_2$ , the probability of making correct diagnosis is only 0.6;

Because of  $p_{a_1 \zeta a_3, b_1}(x_3) = 0.3$ , according to disease symptoms  $a_1 a_3$ , the probability of making correct diagnosis is only 0.3;

Because of  $p_{a_2 \zeta a_3, b_1}(x_2) = 0.1$ , according to disease symptoms  $a_2 a_3$ , the probability of making correct diagnosis is only 0.1;

(2) Having both disease symptoms  $a_1 a_2$  is an important basis for diagnosing patients with rhinitis;

(3) When you have disease symptoms  $a_1$ , you need to be alert for rhinitis.



**Example 3.**

In recent years, with the rapid economic growth, China's ports have been developing rapidly. In 2016, seven of the world's top 10 ports came from China. There are many studies on the effect of shipping safety and port operation. The factors affecting the port are mainly climatic factors (such as storm surge, fog, ice and snow, high temperature, etc.), environmental factors, and other factors (such as policy). Among them, the bad weather factors will affect the normal operation of the port, thus making it uncertain to complete the transportation task on time. According to past experience, in a certain season, it is different probability for different ports to be able to complete the transport task on time, in different bad weather conditions. How to choose the proper port in order to transport the task as timely as possible, under the influence of various adverse weather conditions?

Suppose set  $U$  is for a group of cargos,  $U = \{x_1, x_2, x_3, x_4, x_5\}$ . Set  $A$  is for a collection of the weather factors (conditional properties),  $A = \{\text{typhoon, high temperature, others factors of weather (e.g. policy)}\} = \{a_1, a_2, a_3\}$ . Set  $B$  is for a collection of ports (decision properties),  $B = \{\text{port } b_1, \text{port } b_2, \text{port } b_3\}$ . The  $p_{a_i, b_j}(x_k)$  is the probability of  $x_k$  to complete the transportation task on time for choosing port  $b_j$ , when bad weather factors  $a_i$  occurs.

Table 3, the random decision formal context of example 3

$U$	Condition attributes $A$			Decision attributes $B$		
	Typhoon $a_1$	High temperature $a_2$	Other factors of weather (e.g. policy) $a_3$	Port $b_1$	Port $b_2$	Port $b_3$
$x_1$	1	0	0	0.9	0.95	0.8
$x_2$	1	1	0	0.82	0.83	0.78
$x_3$	0	1	1	0.76	0.8	0.98
$x_4$	0	1	0	0.93	0.97	0.96
$x_5$	0	0	1	0.78	0.80	0.99

**The solution is:**

By the algorithm 11 of probability lattice in the random decision formal context, we get the following:

**Step 1.** The condition attribute space:

$$\begin{aligned} X^A &= (A, \zeta, \tilde{E}) \\ &= \{a_1, a_2, a_3, a_1 \zeta a_2, a_1 \zeta a_3, a_2 \zeta a_3, a_1 \zeta a_2 \zeta a_3, a_1 \tilde{E} a_2, a_1 \tilde{E} a_3, a_2 \tilde{E} a_3, a_1 \tilde{E} a_2 \tilde{E} a_3\}. \end{aligned}$$

The decision attribute space:

$$X^B = (B, \zeta, \tilde{E}) = \{b_1, b_2, b_3, b_1 \zeta b_2, b_1 \zeta b_3, b_2 \zeta b_3, b_1 \zeta b_2 \zeta b_3, b_1 \tilde{E} b_2, b_1 \tilde{E} b_3, b_2 \tilde{E} b_3, b_1 \tilde{E} b_2 \tilde{E} b_3\}.$$

**Step 2.** The probability set in the random decision formal context:

$$\begin{aligned} P_{X^A, X^B} &= \{p_{s,t}(x) \mid x \hat{=} U, s \hat{=} X^A, t \hat{=} X^B, (x,s) \hat{=} I, (x,t) \hat{=} J\} \\ &= \{p_{a_1, b_1}(x_1), p_{a_1, b_2}(x_1), p_{a_1, b_3}(x_1), p_{a_1 \zeta a_2, b_1}(x_2), p_{a_1 \zeta a_2, b_2}(x_2), p_{a_1 \zeta a_2, b_3}(x_2), p_{a_2 \zeta a_3, b_1}(x_3), p_{a_2 \zeta a_3, b_2}(x_3), \\ &\quad p_{a_2 \zeta a_3, b_3}(x_3), p_{a_2, b_1}(x_4), p_{a_2, b_2}(x_4), p_{a_2, b_3}(x_4), p_{a_3, b_1}(x_5), p_{a_3, b_2}(x_5), p_{a_3, b_3}(x_5)\} \\ &= \{0.9, 0.95, 0.8, 0.82, 0.83, 0.78, 0.76, 0.8, 0.98, 0.93, 0.97, 0.96, 0.78, 0.80, 0.99\}. \end{aligned}$$

**Step 3.** Let the “ $\hat{=}$ ” is the less than or equal to in real number field  $\mathbb{R}$ . Let for “ $p_1, p_2 \hat{=} P_{X^A, X^B}$ ”, there are  $p_1 \hat{\cup} p_2 = \max\{p_1, p_2\}$ ,  $p_1 \hat{\cap} p_2 = \min\{p_1, p_2\}$ . We get the probability lattice in the random decision formal context:  $(P_{X^A, X^B}, \hat{\cup}, \hat{\cap})$ .

Therefore, we come to the solutions:

(1) Because of

$$\begin{aligned} &p_{a_1, b_1}(x_1) \hat{\cap} p_{a_1, b_2}(x_1) \hat{\cap} p_{a_1, b_3}(x_1) \hat{\cap} p_{a_1 \zeta a_2, b_1}(x_2) \hat{\cap} p_{a_1 \zeta a_2, b_2}(x_2) \hat{\cap} p_{a_1 \zeta a_2, b_3}(x_2) \\ &= \min\{0.9, 0.95, 0.8, 0.82, 0.83, 0.78\} = 0.78 \end{aligned}$$

So, when a bad weather typhoon  $a_1$  occurs, it should be warned that the worst option is port  $b_3$ , because the probability that we can complete the transportation task on time is just 0.78;

Because of

$$p_{a_1, b_1}(x_1) \hat{\cup} p_{a_1, b_2}(x_1) \hat{\cup} p_{a_1, b_3}(x_1) \hat{\cup} p_{a_1 \zeta a_2, b_1}(x_2) \hat{\cup} p_{a_1 \zeta a_2, b_2}(x_2) \hat{\cup} p_{a_1 \zeta a_2, b_3}(x_2)$$

$$= \max\{0.9, 0.95, 0.8, 0.82, 0.83, 0.78\} = 0.95$$

So, when a bad weather typhoon  $a_1$  occurs, the most optimistic option is port  $b_2$ , because the probability that we can complete the transportation task on time is 0.95.

(2) Because of

$$\begin{aligned} & p_{a_1 \zeta a_2, b_1}(x_2) \dot{\cup} p_{a_1 \zeta a_2, b_2}(x_2) \dot{\cup} p_{a_1 \zeta a_2, b_3}(x_2) \dot{\cup} p_{a_2 \zeta a_3, b_1}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_2}(x_3) \\ & \dot{\cup} p_{a_2 \zeta a_3, b_3}(x_3) \dot{\cup} p_{a_2, b_1}(x_4) \dot{\cup} p_{a_2, b_2}(x_4) \dot{\cup} p_{a_2, b_3}(x_4) \} \\ & = \min\{0.82, 0.83, 0.78, 0.76, 0.8, 0.98, 0.93, 0.97, 0.96\} = 0.76 \end{aligned}$$

So, when a bad weather high temperature  $a_2$  occurs, it should be warned that the worst option is port  $b_1$ , because the probability that we can complete the transportation task on time is just 0.76;

Because of

$$\begin{aligned} & p_{a_1 \zeta a_2, b_1}(x_2) \dot{\cup} p_{a_1 \zeta a_2, b_2}(x_2) \dot{\cup} p_{a_1 \zeta a_2, b_3}(x_2) \dot{\cup} p_{a_2 \zeta a_3, b_1}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_2}(x_3) \\ & \dot{\cup} p_{a_2 \zeta a_3, b_3}(x_3) \dot{\cup} p_{a_2, b_1}(x_4) \dot{\cup} p_{a_2, b_2}(x_4) \dot{\cup} p_{a_2, b_3}(x_4) \} \\ & = \max\{0.82, 0.83, 0.78, 0.76, 0.8, 0.98, 0.93, 0.97, 0.96\} = 0.98 \end{aligned}$$

So, when a bad weather high temperature  $a_2$  occurs, the most optimistic option is port  $b_3$ , because the probability that we can complete the transportation task on time is 0.98.

(3) Because of

$$\begin{aligned} & p_{a_2 \zeta a_3, b_1}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_2}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_3}(x_3) \dot{\cup} p_{a_3, b_1}(x_5) \dot{\cup} p_{a_3, b_2}(x_5) \dot{\cup} p_{a_3, b_3}(x_5) \\ & = \min\{0.76, 0.8, 0.98, 0.78, 0.80, 0.99\} = 0.76 \end{aligned}$$

So, when other factors of weather (e.g. policy)  $a_3$  occurs, it should be warned that the worst option is port  $b_1$ , because the probability that we can complete the transportation task on time is just 0.76;

Because of

$$\begin{aligned} & p_{a_2 \zeta a_3, b_1}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_2}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_3}(x_3) \dot{\cup} p_{a_3, b_1}(x_5) \dot{\cup} p_{a_3, b_2}(x_5) \dot{\cup} p_{a_3, b_3}(x_5) \\ & = \max\{0.76, 0.8, 0.98, 0.78, 0.80, 0.99\} = 0.99 \end{aligned}$$

So, when other factors of weather (e.g. policy)  $a_3$  occur, the most optimistic option is port  $b_3$ , because the probability that we can complete the transportation task on time is 0.99.

(4) Because of

$$\begin{aligned} & p_{a_1 \zeta a_2, b_1}(x_2) \dot{\cup} p_{a_1 \zeta a_2, b_2}(x_2) \dot{\cup} p_{a_1 \zeta a_2, b_3}(x_2) \\ & = \min\{0.82, 0.83, 0.78\} = 0.78 \end{aligned}$$

So, when both typhoon and high temperature  $a_1 a_2$  occurs together, it should be warned that the worst option is port  $b_3$ , because the probability that we can complete the transportation task on time is just 0.78;

Because of

$$\begin{aligned} & p_{a_1 \zeta a_2, b_1}(x_2) \dot{\cup} p_{a_1 \zeta a_2, b_2}(x_2) \dot{\cup} p_{a_1 \zeta a_2, b_3}(x_2) \\ & = \max\{0.82, 0.83, 0.78\} = 0.83 \end{aligned}$$

So, when both typhoon and high temperature  $a_1 a_2$  occur together, the most optimistic option is port  $b_2$ , because the probability that we can complete the transportation task on time is 0.83.

(5) Because of

$$\begin{aligned} & p_{a_2 \zeta a_3, b_1}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_2}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_3}(x_3) \\ & = \min\{0.76, 0.8, 0.98\} = 0.76 \end{aligned}$$

So, when both high temperature and other factors of weather (e.g. policy)  $a_2 a_3$  occur together, it should be warned that the worst option is port  $b_1$ , because the probability that we can complete the transportation task on time is just 0.76;

Because of

$$\begin{aligned} & p_{a_2 \zeta a_3, b_1}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_2}(x_3) \dot{\cup} p_{a_2 \zeta a_3, b_3}(x_3) \\ & = \max\{0.76, 0.8, 0.98\} = 0.98 \end{aligned}$$

So, when both high temperature and other factors of weather (e.g. policy)  $a_2 a_3$  occur together, the most optimistic option is port  $b_3$ , because the probability that we can complete the

transportation task on time is 0.98.

Of course, in the end, we should still combine other factors, such as transportation costs and shipping periods, to consider which route and port to choose.

## Conclusion

The results of this paper are theorem 9 and algorithm 11. In this paper, we constructed the probability lattice model in the random decision formal context  $(P_{X^A, X^B}, \hat{U}, \hat{V})$ , and solve the risk decision problem and the problems of stochastic concept of random phenomena in nature. Also, the research on the probability lattice has yet to be continued. How to perfect and better use this model to solve practical problems and a better probability lattice algorithm is the next step.

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