AMSE JOURNALS –2015-Series: Advances A; Vol. 52; N° 1; pp 17-30
Submitted Oct. 2014; Revised Dec. 26, 2015; Accepted March 15, 2015

A Four-Step Hybrid Block Method for First Order Initial Value Problems in Ordinary Differential Equations

N. S. Yakusak and R. B. Adeniyi

Department of Mathematics, University of Ilorin, P. M.B. 1515, Ilorin, Nigeria
(yakusaksamuel@yahoo.com, raphade@unilorin.edu.ng)

Abstract

In this paper, we derived a Four-Step hybrid block method for the solution of general first order Initial Value Problem (IVP) in Ordinary Differential Equations (ODEs) by collocation and interpolation techniques and with Chebyshev polynomial of the first kind as basis function. The properties and feature of the method are analysed and numerical examples are also presented to illustrate the accuracy and effectiveness of the method.

Key words Collocation, Interpolation, Linear Multistep Method, Chebyshev Polynomial, Hybrid, Differential Equation, Ordinary Differential Equation.

1. Introduction

Mathematical models are developed to help in understanding physical phenomena. These models often yield equations that contain some derivative of an unknown function of one or several variables. Such equation are called Differential Equation (DE). The *LMMs* have the advantage of been self-stating and permitting easy change of step length. The Chebyshev polynomial of the first kind over the interval $[-1,1]$ is highly desirable in approximation of function as the error involve is evenly distributed in the entire range of consideration.

We consider an approximation for the solution of general first order Initial Value Problems (IVP) of the form;

$$y'(x) = f(x, y(x)), y(x_0) = y_0 \tag{1}$$

Where f is a continuous function over an interval of integration.

The solution of (1) is extensively discussed in literatures [1 - 9]. [10] develped a

Four-Step block hybrid method with one offstep point for numerical integration of (1). [11] Proposed a three-step first derivative numerical itegration scheme for IVPs in ODEs.

In this direction, we are then motivated in this paper to developed a Four-Step hybrid block method by introducing four off step points selected to guarantee zero stability to generate the method for solving *ODEs* .

In section 2, we shall present the construction of our proposed numerical scheme for problem (1) . Section 3 provides an analysis for derived scheme while section 4 illustrates the method using some selected test problems . Finally, the paper is ended in section 5 with some concluding remarks .

2. Derivation of Hybrid Method

In this section, we intend to construct the proposed four-step *LMMs* which will be used to generate the method. We consider an approximation of the form:

$$y(x) = \sum_{j=0}^{n=p+q-1} a_j T_j(x) \tag{1}$$

and

$$y'(x) = \sum_{j=0}^{n} a_j T_j'(x) \tag{2}$$

where a_j is unknown co-efficients and $T_j(x)$ are polynomial basis functions of degree $n = p+q-1$, where the number of interpolation points is p and the number of distinct collocation points q are, respectively, chosen to satisfy $1 \le p \le k$ and $q > 0$. The integer $k \ge 1$ denotes the step number of the method.

To derive this, four offstep points are introduced. These offstep points are carefully selected to guarantee zero stability condition. For the method, the offstep points are $v_i = (\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2})$ Using (2.1) and (2.2) with $p = 1, q = 9$, we have a polynomial of degree $p+q-1$ as follows:

$$y(x) = \sum_{j=0}^{9} a_j T_j \tag{3}$$

with first derivative,

$$y'(x) = \sum_{j=0}^{9} a_j T_j'(x) \tag{4}$$

interpolating (2.3) at x_n, while collocating (2.4) at $x_n = 0(\frac{1}{2})4$ we obtain.

$$
\begin{pmatrix}
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
0 & \frac{1}{2} & -2 & \frac{9}{2} & -8 & \frac{25}{2} & -18 & \frac{49}{2} & -32 & \frac{81}{2} \\
0 & \frac{1}{2} & \frac{-3}{2} & \frac{15}{8} & \frac{-3}{4} & -\frac{55}{32} & \frac{135}{32} & \frac{-135}{128} & \frac{93}{32} & \frac{765}{512} \\
0 & \frac{1}{2} & -1 & 0 & \frac{-3}{4} & 2 & -\frac{5}{2} & 0 & \frac{7}{2} & -4 \\
0 & \frac{1}{2} & \frac{-1}{2} & -\frac{9}{8} & \frac{7}{4} & \frac{25}{32} & \frac{-99}{32} & \frac{91}{128} & \frac{119}{32} & \frac{-1539}{512} \\
0 & \frac{1}{2} & 0 & \frac{-3}{2} & 0 & \frac{5}{2} & 0 & \frac{-7}{2} & 0 & \frac{9}{2} \\
0 & \frac{1}{2} & \frac{1}{2} & \frac{-9}{8} & \frac{-7}{4} & \frac{25}{32} & \frac{99}{32} & \frac{91}{128} & \frac{-119}{32} & \frac{-1539}{512} \\
0 & \frac{1}{2} & 1 & 0 & -2 & \frac{-5}{2} & 0 & \frac{7}{2} & 4 & 0 \\
0 & \frac{1}{2} & \frac{3}{2} & \frac{15}{8} & \frac{3}{4} & \frac{-55}{32} & -\frac{135}{32} & \frac{-637}{128} & \frac{-93}{32} & \frac{765}{512} \\
0 & \frac{1}{2} & 2 & \frac{9}{3} & 8 & \frac{25}{2} & 18 & \frac{49}{2} & 32 & \frac{81}{2}
\end{pmatrix}
\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \\ a_8 \\ a_9 \end{pmatrix}
=
\begin{pmatrix} y_n \\ hf_n \\ hf_{n+\frac{1}{2}} \\ hf_{n+1} \\ hf_{n+\frac{3}{2}} \\ hf_{n+2} \\ hf_{n+\frac{5}{2}} \\ hf_{n+3} \\ hf_{n+\frac{7}{2}} \\ hf_{n+4} \end{pmatrix}
$$

Solving this by Gaussian Elimination method, with the aid of Maple yields

$$
\left.\begin{aligned}
a_0 &= y_n + h(-\frac{1087}{56700} f_{n+3} - \frac{908}{2835} f_{n+2} - \frac{633}{56700} f_{n+1} + \frac{8501}{14175} f_{n+\frac{5}{2}} + \frac{1177}{1134600} f_{n+4} + \frac{14647}{113400} f_n \\
&\qquad + \frac{12491}{14175} f_{n+\frac{3}{2}} + \frac{9263}{14175} f_{n+\frac{1}{2}} + \frac{359}{2025} f_{n+\frac{7}{2}}) \\
a_1 &= h(\frac{88}{315} f_{n+\frac{7}{2}} + \frac{8}{15} f_{n+\frac{5}{2}} + \frac{8}{15} f_{n+\frac{3}{2}} + \frac{88}{315} f_{n+\frac{1}{2}} + \frac{11}{420} f_{n+4} + \frac{2}{15} f_{n+3} + \frac{1}{18} f_{n+2} \\
&\qquad + \frac{2}{15} f_{n+1} + \frac{11}{420} f_n) \\
a_2 &= h(\frac{22}{105} f_{n+\frac{7}{2}} - \frac{2}{15} f_{n+\frac{5}{2}} - \frac{2}{15} f_{n+\frac{3}{2}} - \frac{22}{105} f_{n+\frac{1}{2}} + \frac{11}{420} f_{n+4} + \frac{1}{15} f_{n+3} - \frac{1}{15} f_{n+1} - \frac{11}{420} f_n) \\
a_3 &= h(\frac{16}{105} f_{n+\frac{7}{2}} + \frac{16}{135} f_{n+\frac{5}{2}} + \frac{16}{135} f_{n+\frac{3}{2}} + \frac{16}{105} f_{n+\frac{1}{2}} + \frac{41}{1890} f_{n+4} - \frac{17}{135} f_{n+3} - \frac{1}{3} f_{n+2} \\
&\qquad - \frac{17}{135} f_{n+1} + \frac{41}{1890} f_n) \\
a_4 &= h(\frac{1}{15} f_{n+\frac{7}{2}} - \frac{1}{45} f_{n+\frac{5}{2}} + \frac{1}{45} f_{n+\frac{3}{2}} - \frac{1}{15} f_{n+\frac{1}{2}} + \frac{7}{360} f_{n+4} - \frac{23}{180} f_{n+3} + \frac{23}{180} f_{n+1} - \frac{7}{360} f_n) \\
a_5 &= h(\frac{32}{1575} f_{n+\frac{7}{2}} + \frac{32}{225} f_{n+\frac{5}{2}} + \frac{32}{225} f_{n+\frac{3}{2}} + \frac{16}{105} f_{n+\frac{1}{2}} + \frac{89}{6300} f_{n+4} - \frac{31}{225} f_{n+3} - \frac{7}{90} f_{n+2} \\
&\qquad - \frac{31}{225} f_{n+1} + \frac{89}{6300} f_n) \\
a_6 &= h(-\frac{2}{105} f_{n+\frac{7}{2}} + \frac{2}{27} f_{n+\frac{5}{2}} - \frac{1}{45} f_{n+\frac{2}{27}} + \frac{2}{105} f_{n+\frac{1}{2}} + \frac{2}{9} f_{n+4} - \frac{4}{135} f_{n+3} + \frac{4}{135} f_{n+1} - \frac{2}{189} f_n) \\
a_7 &= h(-\frac{8}{135} f_{n+\frac{7}{2}} + \frac{8}{135} f_{n+\frac{5}{2}} + \frac{8}{135} f_{n+\frac{3}{2}} - \frac{8}{135} f_{n+\frac{1}{2}} + \frac{4}{135} f_{n+4} - \frac{4}{135} f_{n+3} - \frac{4}{63} f_{n+2} \\
&\qquad + \frac{8}{315} f_{n+1} + \frac{2}{315} f_n) \\
a_8 &== h(-\frac{2}{105} f_{n+\frac{7}{2}} - \frac{2}{45} f_{n+\frac{5}{2}} + \frac{2}{45} f_{n+\frac{3}{2}} - \frac{8}{315} f_{n+\frac{1}{2}} + \frac{1}{315} f_{n+4} + \frac{2}{45} f_{n+3} - \frac{2}{45} f_{n+1} - \frac{1}{315} f_n) \\
a_9 &= h(-\frac{32}{2835} f_{n+\frac{7}{2}} - \frac{32}{405} f_{n+\frac{5}{2}} - \frac{32}{405} f_{n+\frac{3}{2}} - \frac{32}{2835} f_{n+\frac{1}{2}} + \frac{4}{2835} f_{n+4} + \frac{16}{405} f_{n+3} + \frac{8}{81} f_{n+2} \\
&\qquad + \frac{16}{405} f_{n+1} + \frac{4}{2835} f_n)
\end{aligned}\right\} \qquad (2.5)
$$

Substituting (2.5) into (2.3) gives

$$
\left. y(t) = \alpha_0(t) y_n + h(\sum_{j=0}^{4} \beta_j(t) f_{n+j} + \beta_{\frac{1}{2}}(t) f_{n+\frac{1}{2}} + \beta_{\frac{3}{2}}(t) f_{n+\frac{3}{2}} + \beta_{\frac{5}{2}}(t) f_{n+\frac{5}{2}} + \beta_{\frac{7}{2}}(t) f_{n+\frac{7}{2}}) \right\} \qquad (2.6)
$$

where $\alpha_j(t)$ and $\beta_j(t)$ are continuous coefficients obtained as

$$
\left.\begin{aligned}
\alpha_0(t) &= 1\\
\beta_0(t) &= -\frac{1}{70}t^8 + t + \frac{2}{2835}t^9 - \frac{761}{280}t^2 + \frac{1069}{600}t^5 - \frac{3}{15}t^6 + \frac{2632}{225}t^5 + \frac{29531}{7560}t^3 - \frac{267}{80}t^4\\
\beta_{\frac{1}{2}}(t) &= \frac{1}{9}t^8 - \frac{16}{2835}t^9 + 8t^2 - \frac{2632}{225}t^5 + \frac{115}{27}t^6 + \frac{1924}{105}t^3 + \frac{349}{18}t^4 - \frac{292}{315}t^7\\
\beta_1(t) &= -\frac{17}{45}t^8 + \frac{8}{405}t^9 - 14t^2 + \frac{15289}{450}t^5 - \frac{358}{27}t^6 + \frac{207}{5}t^3 - \frac{18353}{360}t^4 + \frac{756}{315}t^7\\
\beta_{\frac{3}{2}}(t) &= \frac{11}{15}t^8 - \frac{16}{405}t^9 + \frac{56}{3}t^2 - \frac{4288}{75}t^5 + \frac{71}{3}t^6 - \frac{8012}{135}t^3 + \frac{797}{10}t^4 - \frac{596}{105}t^7\\
\beta_2(t) &= -\frac{8}{9}t^8 + \frac{4}{81}t^9 - \frac{35}{2}t^2 + \frac{10993}{180}t^5 - \frac{716}{27}t^6 + \frac{691}{12}t^3 - \frac{1457}{18}t^4 + \frac{418}{63}t^7\\
\beta_{\frac{5}{2}}(t) &= \frac{31}{45}t^8 - \frac{16}{405}t^9 + \frac{56}{5}t^2 - \frac{9544}{225}t^5 + \frac{2581}{135}t^6 - \frac{188}{5}t^3 + \frac{4891}{90}t^4 - \frac{1564}{315}t^7\\
\beta_3(t) &= -\frac{1}{3}t^8 + \frac{8}{405}t^9 - \frac{14}{3}t^2 + \frac{2803}{150}t^5 - \frac{26}{3}t^6 + \frac{2143}{135}t^3 - \frac{187}{8}t^4 + \frac{244}{105}t^7\\
\beta_{\frac{7}{2}}(t) &= \frac{29}{315}t^8 - \frac{16}{2835}t^9 + \frac{8}{7}t^2 - \frac{1072}{225}t^5 + \frac{61}{27}t^6 - \frac{412}{105}t^3 + \frac{527}{90}t^4 - \frac{28}{35}t^7\\
\beta_4(t) &= -\frac{1}{90}t^8 + \frac{2}{2835}t^9 - \frac{1}{8}t^2 + \frac{967}{1800}t^5 - \frac{7}{27}t^6 + \frac{121}{280}t^3 - \frac{469}{720}t^4 + \frac{23}{315}t^7
\end{aligned}\right\} \qquad (2.7)
$$

Evaluating (2.7) at $x_{n+\frac{1}{2}}$, x_{n+1}, $x_{n+\frac{3}{2}}$, x_{n+2}, $x_{n+\frac{5}{2}}$, x_{n+3}, $x_{n+\frac{7}{2}}$, x_{n+4} the following discrete schemes are obtained

$$
y_{n+\frac{1}{2}} = y_n + h(\frac{1070017}{7257600}f_n + \frac{2233547}{3628800}f_{n+\frac{1}{2}} - \frac{2302297}{3628800}f_{n+1} + \frac{2797679}{3628800}f_{n+\frac{3}{2}}
- \frac{31457}{45360}f_{n+2} + \frac{1573169}{3628800}f_{n+\frac{5}{2}} - \frac{645607}{3628800}f_{n+3} - \frac{156437}{3628800}f_{n+\frac{7}{2}} - \frac{33953}{7257600}f_{n+4})
$$

$$
y_{n+1} = y_n + h(\frac{32377}{226800}f_n + \frac{22823}{28350}f_{n+\frac{1}{2}} - \frac{212447}{113400}f_{n+1} + \frac{15011}{28350}f_{n+\frac{3}{2}} - \frac{2903}{5670}f_{n+2}
+ \frac{9341}{28350}f_{n+\frac{5}{2}} - \frac{15577}{113400}f_{n+3} + \frac{953}{28350}f_{n+\frac{7}{2}} - \frac{119}{32400}f_{n+4})
$$

$$
y_{n+\frac{3}{2}} = y_n + h(\frac{12881}{89600}f_n + \frac{35451}{44800}f_{n+\frac{1}{2}} + \frac{1719}{44800}f_{n+1} + \frac{39967}{44800}f_{n+\frac{3}{2}} - \frac{351}{560}f_{n+2}
+ \frac{17217}{44800}f_{n+\frac{5}{2}} - \frac{7031}{44800}f_{n+3} + \frac{243}{6400}f_{n+\frac{7}{2}} - \frac{369}{89600}f_{n+4})
$$

$$y_{n+2} = y_n + h(\frac{4063}{28350} f_n + \frac{11288}{14175} f_{n+\frac{1}{2}} + \frac{122}{14175} f_{n+1} + \frac{16376}{14175} f_{n+\frac{3}{2}} - \frac{4616}{14175} f_{n+2}$$
$$+ \frac{1978}{14175} f_{n+\frac{5}{2}} - \frac{1978}{14175} f_{n+3} + \frac{488}{14175} f_{n+\frac{7}{2}} - \frac{107}{28350} f_{n+4})$$

$$y_{n+\frac{5}{2}} = y_n + h(\frac{41705}{290304} f_n + \frac{115075}{145152} f_{n+\frac{1}{2}} + \frac{3775}{145152} f_{n+1} + \frac{159175}{145152} f_{n+\frac{3}{2}} - \frac{125}{9072} f_{n+2}$$
$$+ \frac{85465}{145152} f_{n+\frac{5}{2}} - \frac{24575}{145152} f_{n+3} + \frac{5725}{145152} f_{n+\frac{7}{2}} - \frac{175}{41472} f_{n+4})$$

$$y_{n+3} = y_n + h(\frac{401}{2800} f_n + \frac{279}{350} f_{n+\frac{1}{2}} + \frac{9}{1400} f_{n+1} + \frac{403}{350} f_{n+\frac{3}{2}} - \frac{9}{70} f_{n+2}$$
$$+ \frac{333}{350} f_{n+\frac{5}{2}} - \frac{79}{1400} f_{n+3} + \frac{9}{350} f_{n+\frac{7}{2}} - \frac{9}{2800} f_{n+4})$$

$$y_{n+\frac{7}{2}} = y_n + h(\frac{149527}{1036800} f_n + \frac{408317}{518400} f_{n+\frac{1}{2}} + \frac{24353}{518400} f_{n+1} + \frac{542969}{518400} f_{n+\frac{3}{2}} + \frac{343}{6480} f_{n+2}$$
$$+ \frac{368039}{518400} f_{n+\frac{5}{2}} + \frac{261023}{518400} f_{n+3} + \frac{111587}{518400} f_{n+\frac{7}{2}} - \frac{8183}{1036800} f_{n+4})$$

$$y_{n+4} = y_n + h(\frac{1978}{14175} f_n + \frac{11776}{14175} f_{n+\frac{1}{2}} - \frac{1856}{14175} f_{n+1} + \frac{20992}{14175} f_{n+\frac{3}{2}} - \frac{1816}{2835} f_{n+2} \quad (2.8)$$
$$+ \frac{20992}{14175} f_{n+\frac{5}{2}} - \frac{1856}{14175} f_{n+3} + \frac{11776}{14175} f_{n+\frac{7}{2}} + \frac{1978}{14175} f_{n+4})$$

The equations y_{n+1}, $y_{n+\frac{1}{2}}$, y_{n+2}, $y_{n+\frac{3}{2}}$, $y_{n+\frac{5}{2}}$, y_{n+3}, $y_{n+\frac{7}{2}}$, y_{n+4} together form the Block Method as (2.8)

3. Analysis of the Method

3.1 Order of the Methods

We define a Linear operator L defined by:

$$L[y(x):h] = \sum_{j=0}^{k}[\alpha_j y(x_n + jh) - h\beta_j y'(x_n + jh)] \quad (3.1)$$

where y(x) is an arbitrary test function that is continuously differentiable in the interval [a,b]. Expanding $y(x_n + jh)$ and $y'(x_n + jh)$ in Taylor series about x_n and collecting like terms in h and y gives:

$$L[y(x):h] = C_0 y(x) + C_1^{(1)} h y'(x) + C_2^{(1)} y(x) + \ldots + C_p h^p y^p(x) \qquad (3.2)$$

Definition 3.1

The differential operator (3.1) and the associated LMM_s are said to be of order p if

$$C_0 = C_1 = C_2 = \ldots C_p = 0, C_{p+1} \neq 0 \qquad (3.3)$$

Definition 3.2

The term C_{p+1} is called error constant and it implies that the local truncation error is given by

$$E_{n+k} = C_{p+1} h^{p+1} y^{p+1}(x_n) + 0(h^{p+2}) \qquad (3.4)$$

Following Definition 3.1 and 3.2, the above Method obtain is of order

$C_0 = C_1 = C_3 = C_4 = C_5 = C_6 = C_7 = C_8 = C_9 = C_{10} = 0$

and error constant

$(8183/1061683200, 9/1433600, 25/3670016, 47/7257600, 25/3670016, 9/1433600, 8183/1061683200, -37/14968800)$

3.2 Consistency and Zero Stability

Definition 3.3

The linear Multistep Method is said to be consistent if it has order $p \geq 1$.

Definition 3.4

The Hybrid Block Method is said to be zero stable if the roots R of the characteristic polynomial $\bar{p}(R)$, defined by:

$$\bar{p}(R) = det[RA^0 - A']$$

satisfies $|R| \leq 1$ and every root with $|R_0| = 1$ has multiplicity not exceeding two in the limit as $h \to 0$

3.4 Convergence.

The main aim of a numerical method is to produce solution that behave similar to the theoretical solution at all times. The convergence of the continuous Hybrid Four-Step Method is considered in the light of the basic properties discussed earlier in conjunction with the fundamental theorem of Dahlquist [11] for linear multistep method. We state Dahlquist theorem without proof.

Theorem 3.1

The necessary and sufficient condition for a multistep method to be convergent is for it to be consistent and zero stable.

Putiing (2.8) in matrix form as a block we obtain

$$\begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix} \begin{bmatrix} f_{n+\frac{1}{2}}\\ f_{n+1}\\ f_{n+\frac{3}{2}}\\ f_{n+2}\\ f_{n+\frac{5}{2}}\\ f_{n+3}\\ f_{n+\frac{7}{2}}\\ f_{n+4} \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{bmatrix} \begin{bmatrix} y_{n-7}\\ y_{n-6}\\ y_{n-5}\\ y_{n-4}\\ y_{n-3}\\ y_{n-2}\\ y_{n-1}\\ y_n \end{bmatrix}$$

$$+h\begin{bmatrix} \frac{2233547}{3628800} & -\frac{2302297}{3628800} & \frac{2797679}{3628800} & -\frac{31457}{45360} & \frac{1573169}{3628800} & -\frac{5607}{3628800} & \frac{156437}{3628800} & -\frac{33953}{7257600}\\ \frac{22823}{28350} & -\frac{21247}{113400} & \frac{15011}{28350} & -\frac{2903}{5670} & \frac{9341}{28350} & -\frac{15577}{113400} & \frac{953}{28350} & -\frac{119}{32400}\\ \frac{35451}{44800} & \frac{1719}{44800} & \frac{39967}{44800} & -\frac{351}{560} & \frac{17217}{44800} & -\frac{7031}{44800} & \frac{243}{6400} & -\frac{369}{89600}\\ \frac{11288}{14175} & \frac{122}{14175} & \frac{16376}{14175} & -\frac{908}{2835} & \frac{4616}{14175} & -\frac{1978}{14175} & \frac{488}{14175} & -\frac{107}{28350}\\ \frac{115075}{145152} & \frac{3775}{145152} & \frac{159175}{145152} & -\frac{125}{9072} & \frac{85465}{145152} & -\frac{24575}{145152} & \frac{5725}{145152} & -\frac{175}{41472}\\ \frac{279}{350} & \frac{9}{1400} & \frac{403}{350} & -\frac{9}{70} & \frac{333}{350} & \frac{79}{1400} & \frac{9}{350} & -\frac{9}{2800}\\ \frac{408317}{518400} & \frac{24353}{518400} & \frac{542969}{518400} & \frac{343}{6480} & \frac{368039}{518400} & \frac{261023}{518400} & \frac{111587}{518400} & -\frac{8183}{1036800}\\ \frac{11776}{14175} & -\frac{1856}{14175} & \frac{20992}{14175} & -\frac{1816}{2835} & \frac{20992}{14175} & -\frac{1856}{14175} & \frac{11776}{14175} & \frac{1978}{14175} \end{bmatrix} \begin{bmatrix} f_{n+\frac{1}{2}}\\ f_{n+1}\\ f_{n+\frac{3}{2}}\\ f_{n+2}\\ f_{n+\frac{5}{2}}\\ f_{n+3}\\ f_{n+\frac{7}{2}}\\ f_{n+4} \end{bmatrix}$$

$$+h\begin{bmatrix} 0&0&0&0&0&0&0&\frac{1070017}{7257600}\\ 0&0&0&0&0&0&0&\frac{32377}{226800}\\ 0&0&0&0&0&0&0&\frac{12881}{89600}\\ 0&0&0&0&0&0&0&\frac{4063}{28350}\\ 0&0&0&0&0&0&0&\frac{41705}{290304}\\ 0&0&0&0&0&0&0&\frac{401}{2800}\\ 0&0&0&0&0&0&0&\frac{149527}{1036800}\\ 0&0&0&0&0&0&0&\frac{1978}{14175} \end{bmatrix}\begin{bmatrix} f_{n-7}\\ f_{n-6}\\ f_{n-5}\\ f_{n-4}\\ f_{n-3}\\ f_{n-2}\\ f_{n-1}\\ f_n \end{bmatrix}$$

The first characteristic polynomial of the above matrix is given by $\rho(R) = det[RA^\circ - A']$

$$A^\circ = \begin{bmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{bmatrix},\ A' = \begin{bmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{bmatrix}$$

By definition 3.4

$$\rho(R) = det\left[R\begin{pmatrix} 1&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&1 \end{pmatrix} - \begin{pmatrix} 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&1 \end{pmatrix}\right]$$

$$\rho(R) = det\begin{bmatrix} R & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & R & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & R & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & R & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & R & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & R & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & R & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R-1 \\ \end{bmatrix}$$

$$\rho(R) = [R^7(R-1)]$$

Therefore $R = 0$ and $R = 1$. The hybrid method is zero stable.

The block method (2.8) is convergent in the light of Theorem 3.1.

4. **Numerical Example**

We now illustrate the self starting scheme (2.8) with numerical examples below. All calculations and programs were carried out with the aid of Maple.

The following notations are adopted in the tables below.

x : Value of the independent variable where numerical value is taken.

$y(x_n)$: Exact result at x_n

y_n : Our numerical result at x.

Example 1

$$y'(x) + y(x) = 0, y(0) = 1, h = 0.1, \quad 0 \le x \le 1$$

Exact Solution $y(x) = e^{-x}$ [9]

Example 2

$$y'(x) - 5y(x) = 0, y(0) = 1, h = 0.01, \quad 0 \le x \le 1$$

Exact Solution $y(x) = e^{5x}$ [8]

Table 1a :Numerical Solution for Example 1

X	y(x_n)	y_n	[9]
0.0	1.0000000000	1.0000000000	1.0000000000

0.1	0.9048374180	0.9048374180	0.9048374180
0.2	0.818730753	0.818730753	0.818730753
0.3	0.7408182207	0.7408182207	0.7408182205
0.4	0.6703200460	0.6703200460	0.6703200461
0.5	0.6065306597	0.6065306597	0.6065306603
0.6	0.5488116361	0.5488116360	0.5488116368
0.7	0.4965853038	0.4965853038	0.4965853042
0.8	0.4493289641	0.4493289641	0.4493289649
0.9	0.4065696597	0.4065696597	0.4065696606
1.0	0.3678794412	0.3678794411	0.3678794420

Table 1b Comparison of Error for Example 1

x	Proposed Scheme	[9]
0.1	0.0	0.0
0.2	$1.0 \text{ x } 10^{-10}$	0.0
0.3	0.0	$2.0 \text{ x } 10^{-10}$
0.4	0.0	$1.0 \text{ x } 10^{-10}$
0.5	0.0	$6.0 \text{ x } 10^{-10}$
0.6	$1.0 \text{ x } 10^{-10}$	$7.0 \text{ x } 10^{-10}$
0.7	0.0	$4.0 \text{ x } 10^{-10}$
0.8	0.0	$8.0 \text{ x } 10^{-10}$
0.9	0.0	$9.0 \text{ x } 10^{-10}$
1.0	$1.0 \text{ x } 10^{-10}$	$8.0 \text{ x } 10^{-10}$

Table 2a :Numerical Solution for Example 2

X	$y(x_n)$	y_n	[8]
0.0	1.000000000	1.000000000	1.000000000
0.01	1.051271096	1.051271096	1.051271097
0.02	1.105170918	1.105170918	1.105170917
0.03	1.161834243	1.161834243	1.161834244
0.04	1.221402758	1.221402758	1.221402758
0.05	1.284025417	1.284025417	1.284025418
0.06	1.349858808	1.349858807	1.349858808

0.07	1.419067549	1.419067548	1.419067551
0.08	1.491824698	1.491824698	1.4918247
0.09	1.568312185	1.568312185	1.568312188

Table 2.b Comparison of Error for Example 2

X	Proposed Scheme	[8]
0.01	0.0	6.2×10^{-10}
0.02	0.0	1.1×10^{-09}
0.03	0.0	1.3×10^{-09}
0.04	0.0	1.6×10^{-10}
0.05	0.0	1.3×10^{-09}
0.06	1.0×10^{-09}	4.2×10^{-10}
0.07	1.0×10^{-09}	2.4×10^{-10}
0.08	1.0×10^{-09}	2.4×10^{-09}
0.09	0.0	2.5×10^{-09}

4 Discussion

Computer program is written for the implementation of the Hybrid Block Method. The method developed where tested respectively on two numerical examples for general first order ordinary differential equation in the last section. The derived method converge

more faster to the exact solution, when campared to [9] and [8] as we can see in TABLE 1b and TABLE 2b. Therefore our method is comparable with the existen methods.

5. Conclusion

The approach adopted for the derivation of the block method involve interpolation and collocation at appropriate selected points. The proposed order ten hybrid block method for general first order ODEs was found to be zero-stable, consistent and convergent. Numerical evidences shows that the method proposed here perform favourable when compared with existing scheme as it yielded better accuracy.

References

[1] Okunuga S. A. And Onamanyi P. (1985): An Accurate Collocation Method for me Solving *ODE*. AMSE journals, France, Series Advances A, 4(4): 45-48.

[2] Adeniyi, R. B. , Adeyefa, E. O. and Alabi, M.O. (2006) . A continuous formulation of some classical initial value solvers by non-perturbed multistep collocation approach using chebyshev polynomial as basis function . Journal of the Nigerian Association of Mathematical Physics ,10:261-274 .

[3] Adeniyi, R.B. and Alabi, M.O. (2007). Continuous Formulation Of a class Accurate Implicit Linear Multistep Methods with Chebyshev Basis Function in a collocaion technique. Journal of Mathematical Association of Nigeria (ABACUS), 34(2A):58-77.

[4] Awoyemi, D.O. (1992). On some continuous linear multistep methods for initial value problem Unpublished Phd thesis, University of Ilorin, Ilorin Nigeria.

[5] Areo, E.A., Ademiluyi, R.A. and Babatola, P.O. (2008). Accurate collocation multistep method for integration of first order ordinary differential equation, Int.J.Comp.maths, 2(1):15-27.

[6] Ehigie, J.O., Okunuga, S.A. and Sofoluwa, A.B. (2011). A class of 2-step continuous hybrid implicit linear multistep method for IVP's, Journal of Nigerian Mathematical Society, 30 145-161.

[7] Odejide, S.A and Adeniran, A.O. (2012). A Hybrid linear collocation multistep scheme for solving first order initial value problems. Journal of Nigerian mathematical society. 31:229-241.

[8] Yahaya, Y. A. (2004). Some Theory and Application of Continuous Linear Multi-Step Methods for Ordinary Differential Equation. Ph. D Thesis (Unpublished). University of Jos, Nigeria.

[9] Abdullahi, A., Chollom, J. P and Yahaya, Y. A. (2014). A family of Two-Step block generalized Adams Methods for the solution of Non-Stiff IVP in ODEs. Journal of Mathematical Association Of Nigeria (ABACUS), 41(2A):67-77.

[10] Mohammed, U., Ma'ali, A.I. and Badeggi, A.Y. (2014). Derivation of Block

Hybrid Method for the solution of first order initial value problems in ODEs. Pacific Journal of Science and Technology. 15(1):93-98.
[11] Bolaji, B. (2014). A Three-Step First derivative numerical integration scheme for IVP in ODEs. Journal of Mathematical Association Of Nigeria (ABACUS), 41(2A):89-98.
[12] Henrici, P. (1962). Discrete Variable Methods for ODEs, John Wiley and Sons, New York, USA.