Hybrid Algorithm for Rough Multi-level Multi-objective Decision Making Problems

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ABSTRACT

The purpose of this paper is to generate compromise solutions for the multi-level multi-objective decision making (MLMODM) problems with rough parameters in the objective functions (RMLMODM) based on TOPSIS method and "Lower & Upper" approximations method. We introduce a computational hybrid algorithm for solving RMLMODM problems. Also, we solved illustrative numerical example and compared the solution of the proposed algorithm with the solution of Global Criterion (GC) method. The engineers and the scientists can apply the introduced hybrid algorithm to various practical RMLMODM problems to obtain numerical solutions.

1. INTRODUCTION

Rough set theory is an important mathematical tool for dealing with the description of vague objects. Rough set methodology has been introduced by Pawlak (1991), [1].

Linear optimization problem which is considered where some or all of its coefficients in the objective function and/or constraints are rough intervals is introduced by Hamzehee et al. [2].

Various hybrid algorithms for solving several kinds of multi-objective optimization problems based on TOPSIS approach are presented in [3].

S. F. El-Feky and T.H.M. Abou-El-Enien, [4], develop a methodology to find compromise solutions for rough multi-objective optimization problems.

In the following section, the formulation of RMLMODM problems is given. A new hybrid algorithm based on the TOPSIS method [5-7], and "Lower & Upper" approximations method [1-2], for solving RMLMODM problems is proposed in section (3). For the sake of illustration, we present an example and compared the solution of the example by the proposed algorithm with the ideal solutions and the solution of global criterion (GC) method.

2. PROBLEM DEFINITION

Consider the following linear multi-level multi-objective decision making (LMLMODM) problem with rough parameters in the objective functions:

$[DM_{L_1}]$

$$\underset{X_1}{Maximize}\left(f_{11}(X_1, \ldots, X_h), \ldots, f_{1k_1}(X_1, \ldots, X_h)\right)$$

where, X_2 solves the 2nd level

$[DM_{L_2}]$

$$\underset{X_2}{Maximize}\left(f_{21}(X_1, \ldots, X_h), \ldots, f_{2k_2}(X_1, \ldots, X_h)\right)$$

where, X_3 solves the 3rd level

$[DM_{L_3}]$

$$\underset{X_3}{Maximize}\left(f_{31}(X_1, \ldots, X_h), \ldots, f_{3k_3}(X_1, \ldots, X_h)\right)$$

where, X_4 solves the 4th level

where, X_{I_h} solves the hth level

$[DM_{L_h}]$

$$\underset{X_h}{Maximize}\left(f_{h1}(X_1, \ldots, X_h), \ldots, f_{hk_h}(X_1, \ldots, X_h)\right)$$

subject to   (1)

$$X \in \mathbb{M} = \{X \in \mathbb{R}^n : DX \leq b\}$$

where

$$f_{iq} = \sum_{j=1}^{n} c_{iq,j}\left([Q_{iq,j}^{LL}, Q_{iq,j}^{HL}], [Q_{iq,j}^{LH}, Q_{iq,j}^{HH}]\right)x_j, i = 1, 2, \ldots, h, q = 1, 2, \ldots, k_i,$$ (2)

m: The number of constraints,
n: The number of variables,
h: The number of levels,

k: The number of objective functions,

DM_{L_i}: j^{th} level decision maker, i=1,2,…,h,

k_i: The number of objective functions of the DM_{L_i}, i=1,2,….,h,

$$K = K_1 \cup K_2 \cup ….\cup K_j = \{1,2, ….., k\},$$

n_i: The number of variables of the DM_{L_i}, i=1,2,….,h,

$c_{iq,j}$: Real constants coefficients of the objective functions , $(i = 1,2, …. , h, q = 1,2, …. , k_i, j = 1, …. , n)$.

b: An m-dimensional column vector of right-hand sides of constraints

D: An $(m \times n)$coefficient matrix,

$\mathbb{R}$: The set of all real numbers,

X: An n-dimensional column vector of variables,

X_i: An n_i -dimensional column vector of variables for the ith level, i=1,2,…,h,

$$\mathbb{N}=\{1,2,…..,n\},$$

$$\mathbb{R}^n = \{X = (x_1, x_2, …. , x_n)^T : x_i \in \mathbb{R}, i \in \mathbb{N}\},$$

$\left[Q_{iq,j}^{LL}, Q_{iq,j}^{HL}\right], \left[Q_{iq,j}^{LH}, Q_{iq,j}^{HH}\right]$ are rough interval coefficients of the objective functions , $(i = 1,2, …. , h, q = 1,2, …. , k_i, j = 1, …. , n)$.

Using the upper and lower approximation method, [2], the multi-level multi-objective decision making problems with rough parameters in the objective functions (RMLMODM) can be transformed to the following four deterministic linear multi-objective decision making (LMODM) problems :

$\underline{\boldsymbol{P^{LL}}}$: (Lower interval coefficients-Lower interval)

$[DM_{L_1}]$

$$\underset{X_1}{Maximize} \left(F_{11}^{LL}(X_1, …, X_h), …, F_{1k_1}^{LL}(X_1, …, X_h)\right)$$

where, X_2 solves the 2nd level

$[DM_{L_2}]$

$$\underset{X_2}{Maximize} \left(F_{21}^{LL}(X_1, …, X_h), …, F_{2k_2}^{LL}(X_1, …, X_h)\right)$$

where, X_3 solves the 3rd, level

$[DM_{L_3}]$

$$\underset{X_3}{Maximize} \left(F_{31}^{LL}(X_1, …, X_h), …, F_{3k_3}^{LL}(X_1, …, X_h)\right)$$

where, X_4 solves the 4th level

where, X_{l_h} solvesthe h^{th} level

$[DM_{L_h}]$

$$\underset{X_h}{Maximize} \left(F_{h1}^{LL}(X_1, …, X_h), …, F_{hk_h}^{LL}(X_1, …, X_h)\right)$$

subject to   (3)

$X \in \mathbb{M}$

$\underline{\boldsymbol{P^{HL}}}$:(Upper interval coefficients-Lower interval)

$[DM_{L_1}]$

$$\underset{X_1}{Maximize} \left(F_{11}^{HL}(X_1, …, X_h), …, F_{1k_1}^{HL}(X_1, …, X_h)\right)$$

where, X_2 solves the 2^{nd}level

$[DM_{L_2}]$

$$\underset{X_2}{Maximize} \left(F_{21}^{HL}(X_1, …, X_h), …, F_{2k_2}^{HL}(X_1, …, X_h)\right)$$

where, X_3 solves the 3rd level

$[DM_{L_3}]$

$$\underset{X_3}{Maximize} \left(F_{31}^{HL}(X_1, …, X_h), …, F_{3k_3}^{HL}(X_1, …, X_h)\right)$$

where, X_4 solves the 4th level

where, X_{l_h} solves the hth level

$[DM_{L_h}]$

$$\underset{X_h}{Maximize} \left(F_{h1}^{HL}(X_1, …, X_h), …, F_{hk_h}^{HL}(X_1, …, X_h)\right)$$

subject to   (4)

$X \in \mathbb{M}$

$\underline{\boldsymbol{P^{LH}}}$: (Lower interval coefficients -Upper interval)

$[DM_{L_1}]$

$$\underset{X_1}{Maximize} \left(F_{11}^{LH}(X_1, …, X_h), …, F_{1k_1}^{LH}(X_1, …, X_h)\right)$$

where, X_2 solves the 2nd level

$[DM_{L_2}]$

$$\underset{X_2}{Maximize} \left(F_{21}^{LH}(X_1, …, X_h), …, F_{2k_2}^{LH}(X_1, …, X_h)\right)$$

where, X_3 solves the 3rd level

$[DM_{L_3}]$

$$\underset{X_3}{Maximize} \left(F_{31}^{LH}(X_1, …, X_h), …, F_{3k_3}^{LH}(X_1, …, X_h)\right)$$

where, X_4 solves the 4th level

where, X_{l_h} solves the hth level

$[DM_{L_h}]$

$$\underset{X_h}{Maximize} \left(F_{h1}^{LH}(X_1, …, X_h), …, F_{hk_h}^{LH}(X_1, …, X_h)\right)$$

subject to (5)

$X \in \mathbb{M}$

$\underline{P^{HH}}$:(Upper interval coefficients -Upper interval)

$[DM_{L_1}]$

$\underset{X_1}{\text{Maximize}} \left(F_{11}^{HH}(X_1, \ldots, X_h), \ldots, F_{1k_1}^{HH}(X_1, \ldots, X_h) \right)$

where, X_2 solves the 2nd level

$[DM_{L_2}]$

$\underset{X_2}{\text{Maximize}} \left(F_{21}^{HH}(X_1, \ldots, X_h), \ldots, F_{2k_2}^{HH}(X_1, \ldots, X_h) \right)$

where, X_3 solves the 3rd level

$[DM_{L_3}]$

$\underset{X_3}{\text{Maximize}} \left(F_{31}^{HH}(X_1, \ldots, X_h), \ldots, F_{3k_3}^{HH}(X_1, \ldots, X_h) \right)$

where, X_4 solves the 4th level

where, X_{I_h} solves the hth level

$[DM_{L_h}]$

$\underset{X_h}{\text{Maximize}} \left(F_{h1}^{HH}(X_1, \ldots, X_h), \ldots, F_{hk_h}^{HH}(X_1, \ldots, X_h) \right)$

subject to (6)

$X \in \mathbb{M}$

where

$F_{iq}^{LL} = \sum_{j=1}^{n} c_{iq,j} Q_{iq,j}^{LL} x_j, i = 1,2,\ldots,h, q = 1,2,\ldots,k_i,$ (7)

$F_{iq}^{LH} = \sum_{j=1}^{n} c_{iq,j} Q_{iq,j}^{LH} x_j, i = 1,2,\ldots,h, q = 1,2,\ldots,k_i,$ (8)

$F_{iq}^{HL} = \sum_{j=1}^{n} c_{iq,j} Q_{iq,j}^{HL} x_j, i = 1,2,\ldots,h, q = 1,2,\ldots,k_i,$ (9)

$F_{iq}^{HH} = \sum_{j=1}^{n} c_{iq,j} Q_{iq,j}^{HH} x_j, i = 1,2,\ldots,h, q = 1,2,\ldots,k_i,$ (10)

3. HYBRID ALGORITHM FOR RMLMODEM

A modified version of TOPSIS method, [3, 5], is introduced to find compromise solutions, [8-10], for the RMLMODM problems. Modified equations for the distance function equation from the positive ideal solution (PIS) and the distance function equation from the negative ideal solution (NIS) are introduced. Thus, we present the following hybrid algorithm based on a modified version of TOPSIS method and the upper and lower approximation method to generate compromise solutions, [4], for RMLMODM problems.

Algorithm:
Phase (1):
Step 1:

Let h = the number of the levels of the RMLMODM problem (1). Set i=1, "The 1st level".
Step 2:
Use the "Lower & Upper" approximations method to transform the RMLMODM Problem (1) into the four deterministic LMLMODM problems (3)-(6).
Step 3:
Construct the positive ideal solution (PIS) payoff tables, [4], of the following problems, for i=1:

$[DM_{L_i}]$

$\underset{X_i}{\text{Maximize}} \left(F_{i1}^{LL}(X_1, \ldots, X_h), \ldots, F_{ik_i}^{LL}(X_1, \ldots, X_h) \right)$

subject to (11)

$X \in \mathbb{M},$

$\underset{X_i}{\text{Maximize}} \left(F_{i1}^{HL}(X_1, \ldots, X_h), \ldots, F_{ik_i}^{HL}(X_1, \ldots, X_h) \right)$

subject to (12)

$X \in \mathbb{M},$

$\underset{X_i}{\text{Maximize}} \left(F_{i1}^{LH}(X_1, \ldots, X_h), \ldots, F_{ik_i}^{LH}(X_1, \ldots, X_h) \right)$

subject to (13)

$X \in \mathbb{M},$

$\underset{X_i}{\text{Maximize}} \left(F_{i1}^{HH}(X_1, \ldots, X_h), \ldots, F_{ik_i}^{HH}(X_1, \ldots, X_h) \right)$

subject to (14)

$X \in \mathbb{M},$

and obtain the PIS:

$(F_{1q}^{*LL-DM_{L_1}})$, $(F_{1q}^{*HL-DM_{L_1}})$, $(F_{1q}^{*LH-DM_{L_1}})$ and $(F_{1q}^{*HH-DM_{L_1}})$, $q = 1,2,\ldots,k_1.$

Step 4:

Construct the negative ideal solution (NIS) payoff tables, [4], of the problems (11-14) for i=1 and obtain the NIS: $\left(F_{1q}^{-LL-DM_{L_1}} \right), \left(F_{1q}^{-HL-DM_{L_1}} \right), \left(F_{1q}^{-LH-DM_{L_1}} \right)$ and $\left(F_{1q}^{-HH-DM_{L_1}} \right)$, $q = 1,2,\ldots,k_1.$

Step 5:

Let $w_{1q} = w_{1q}^*$ where $\sum_{q=1}^{k_1} w_{1q} = 1$ and $p = p^*$, $p^* \in \{1,2,\ldots,\infty\}.$
Step 6:

Construct distance functions $d_p^{PIS^{LL-DM_{L_1}}}$ and $d_p^{NIS^{LL-DM_{L_1}}}$, [4], by the above steps (3, 4 & 5):

$d_p^{PIS^{LL-DM_{L_1}}} =$

$$\left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{*LL-DM_{L_1}} - F_{1q}^{LL-DM_{L_1}}(X)}{F_{1q}^{*LL-DM_{L_1}} - F_{1q}^{-LL-DM_{L_1}}}\right)^p\right)^{1/p} \tag{15}$$

And

$$d_p^{NIS^{LL-DM_{L_1}}} =$$

$$\left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{LL-DM_{L_1}}(X) - F_{1q}^{-LL-DM_{L_1}}}{F_{1q}^{*LL-DM_{L_1}} - F_{1q}^{-LL-DM_{L_1}}}\right)^p\right)^{1/p} \tag{16}$$

Construct distance functions $d_p^{PIS^{HL-DM_{L_1}}}$ and $d_p^{NIS^{HL-DM_{L_1}}}$ by the above steps (3, 4 & 5):

$$d_p^{PIS^{HL-DM_{L_1}}} =$$

$$\left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{*HL-DM_{L_1}} - F_{1q}^{HL-DM_{L_1}}(X)}{F_{1q}^{*HL-DM_{L_1}} - F_{1q}^{-HL-DM_{L_1}}}\right)^p\right)^{1/p} \tag{17}$$

and

$$d_p^{NIS^{HL-DM_{L_1}}} =$$

$$\left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{HL-DM_{L_1}}(X) - F_{1q}^{-HL-DM_{L_1}}}{F_{1q}^{*HL-DM_{L_1}} - F_{1q}^{-HL-DM_{L_1}}}\right)^p\right)^{1/p} \tag{18}$$

Construct distance functions $d_p^{PIS^{LH-DM_{L_1}}}$ and $d_p^{NIS^{LH-DM_{L_1}}}$ by the above steps (3, 4 & 5):

$$d_p^{PIS^{LH-DM_{L_1}}} =$$

$$\left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{*LH-DM_{L_1}} - F_{1q}^{LH-DM_{L_1}}(X)}{F_{1q}^{*LH-DM_{L_1}} - F_{1q}^{-LH-DM_{L_1}}}\right)^p\right)^{1/p} \tag{19}$$

and

$$d_p^{NIS^{LH-DM_{L_1}}} =$$

$$\left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{LH-DM_{L_1}}(X) - F_{1q}^{-LH-DM_{L_1}}}{F_{1q}^{*LH-DM_{L_1}} - F_{1q}^{-LH-DM_{L_1}}}\right)^p\right)^{1/p} \tag{20}$$

Construct distance functions $d_p^{PIS^{HH-DM_{L_1}}}$ and $d_p^{NIS^{HH-DM_{L_1}}}$ by the above steps (3, 4 & 5):

$$d_p^{PIS^{HH-DM_{L_1}}} =$$

$$\left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{*HH-DM_{L_1}} - F_{1q}^{HH-DM_{L_1}}(X)}{F_{1q}^{*HH-DM_{L_1}} - F_{1q}^{-HH-DM_{L_1}}}\right)^p\right)^{1/p} \tag{21}$$

and

$$d_p^{NIS^{HH-DM_{L_1}}} =$$

$$\left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{HH-DM_{L_1}}(X) - F_{1q}^{-HH-DM_{L_1}}}{F_{1q}^{*HH-DM_{L_1}} - F_{1q}^{-HH-DM_{L_1}}}\right)^p\right)^{1/p} \tag{22}$$

<u>Step 7:</u>

Construct the following bi-objective problem with two commensurable (but conflicting) objectives, [4], using the distance functions $d_p^{PIS^{LL-DM_{L_1}}}$ and $d_p^{NIS^{LL-DM_{L_1}}}$:

$Minimize\ d_p^{PIS^{LL-DM_{L_1}}}(X)$

$Maximize\ d_p^{NIS^{LL-DM_{L_1}}}(X)$

subject to $\qquad\qquad\qquad\qquad$ (23)

$X \in \mathbb{M}$

where

$p = 1,2,\dots\dots,\infty.$

Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{HL-DM_{L_1}}}$ and $d_p^{NIS^{HL-DM_{L_1}}}$:

$Minimize\ d_p^{PIS^{HL-DM_{L_1}}}(X)$

$Maximize\ d_p^{NIS^{HL-DM_{L_1}}}(X)$

subject to $\qquad\qquad\qquad\qquad$ (24)

$X \in \mathbb{M}$

where

$p=1,2,\dots..,\infty.$

Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{LH-DM_{L_1}}}$ and $d_p^{NIS^{LH-DM_{L_1}}}$:

$Minimize\ d_p^{PIS^{LH-DM_{L_1}}}(X)$

$Maximize\ d_p^{NIS^{LH-DM_{L_1}}}(X)$

subject to $\qquad\qquad\qquad\qquad$ (25)

$X \in \mathbb{M}$

where

$p=1,2,\dots..,\infty.$

Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{HH-DM_{L_1}}}$ and $d_p^{NIS^{HH-DM_{L_1}}}$:

$Minimized_p^{PIS^{HH-DM_{L_1}}}(X)$

Maximize $d_p^{NIS^{HH-DM_{L_1}}}(X)$

subject to (26)

$X \in \mathbb{M}$

where

$p = 1, 2, \ldots, \infty$.

<u>Step 8:</u>

Construct PIS Payoff table for problem (23) and obtain:

$d_p^{-^{LL-DM_{L_1}}} = \left(\left(d_p^{PIS^{LL-DM_{L_1}}} \right)^-, \left(d_p^{NIS^{LL-DM_{L_1}}} \right)^- \right)$,

$d_p^{*^{LL-DM_{L_1}}} = \left(\left(d_p^{PIS^{LL-DM_{L_1}}} \right)^*, \left(d_p^{NIS^{LL-DM_{L_1}}} \right)^* \right)$.

where

$\left(d_p^{PIS^{LL-DM_{L_1}}} \right)^*$
$= \underset{X \in \mathbb{M}}{Minimize}\, d_p^{PIS^{LL-DM_{L_1}}}(X)$ and the solution is $X^{PIS^{LL-DM_{L_1}}}$,

$\left(d_p^{NIS^{LL-DM_{L_1}}} \right)^*$
$= \underset{X \in \mathbb{M}}{Maximize}\, d_p^{NIS^{LL-DM_{L_1}}}(X)$ and the solution is $X^{NIS^{LL-DM_{L_1}}}$,

$\left(d_p^{PIS^{LL-DM_{L_1}}} \right)^- = d_p^{PIS^{LL-DM_{L_1}}} \left(X^{NIS^{LL-DM_{L_1}}} \right)$ and

$\left(d_p^{NIS^{LL-DM_{L_1}}} \right)^- = d_p^{NIS^{LL-DM_{L_1}}} \left(X^{PIS^{LL-DM_{L_1}}} \right)$.

Construct PIS Payoff table for problem (24) and obtain:

$d_p^{-^{HL-DM_{L_1}}} = \left(\left(d_p^{PIS^{HL-DM_{L_1}}} \right)^-, \left(d_p^{NIS^{HL-DM_{L_1}}} \right)^- \right)$,

$d_p^{*^{HL-DM_{L_1}}} = \left(\left(d_p^{PIS^{HL-DM_{L_1}}} \right)^*, \left(d_p^{NIS^{HL-DM_{L_1}}} \right)^* \right)$.

Construct PIS Payoff table for problem (21) and obtain:

$d_p^{-^{LH-DM_{L_1}}} = \left(\left(d_p^{PIS^{LH-DM_{L_1}}} \right)^-, \left(d_p^{NIS^{LH-DM_{L_1}}} \right)^- \right)$,

$d_p^{*^{LH-DM_{L_1}}} = \left(\left(d_p^{PIS^{LH-DM_{L_1}}} \right)^*, \left(d_p^{NIS^{LH-DM_{L_1}}} \right)^* \right)$.

Construct PIS Payoff table for problem (22) and obtain:

$d_p^{-^{HH-DM_{L_1}}} = \left(\left(d_p^{PIS^{HH-DM_{L_1}}} \right)^-, \left(d_p^{NIS^{HH-DM_{L_1}}} \right)^- \right)$,

$d_p^{*^{HH-DM_{L_1}}} = \left(\left(d_p^{PIS^{HH-DM_{L_1}}} \right)^*, \left(d_p^{NIS^{HH-DM_{L_1}}} \right)^* \right)$

<u>Step 9:</u>

Construct the following satisfactory level model, [11-13], (for finite value of p) for problem (23):

Maximize $\delta^{LL-DM_{L_1}}$

subject to (27)

$\left(\dfrac{d_p^{PIS^{LL-DM_{L_1}}}(X) - (d_p^{PIS^{LL-DM_{L_1}}})^*}{\left(d_p^{PIS^{LL-DM_{L_1}}} \right)^- - \left(d_p^{PIS^{LL-DM_{L_1}}} \right)^*} \right) \geq \delta^{LL-DM_{L_1}}$,

$\left(\dfrac{\left(d_p^{NIS^{LL-DM_{L_1}}} \right)^* - d_p^{NIS^{LL-DM_{L_1}}}(X)}{\left(d_p^{NIS^{LL-DM_{L_1}}} \right)^* - \left(d_p^{NIS^{LL-DM_{L_1}}} \right)^-} \right) \geq \delta^{LL-DM_{L_1}}$,

$X \in \mathbb{M}, \delta^{LL-DM_{L_1}} \in [0,1]$

Construct the following satisfactory level model (for finite value of p) for problem (24):

Maximize $\delta^{HL-DM_{L_1}}$

subject to (28)

$\left(\dfrac{d_p^{PIS^{HL-DM_{L_1}}}(X) - (d_p^{PIS^{HL-DM_{L_1}}})^*}{\left(d_p^{PIS^{HL-DM_{L_1}}} \right)^- - \left(d_p^{PIS^{HL-DM_{L_1}}} \right)^*} \right) \geq \delta^{HL-DM_{L_1}}$,

$\left(\dfrac{\left(d_p^{NIS^{HL-DM_{L_1}}} \right)^* - d_p^{NIS^{HL-DM_{L_1}}}(X)}{\left(d_p^{NIS^{HL-DM_{L_1}}} \right)^* - \left(d_p^{NIS^{HL-DM_{L_1}}} \right)^-} \right) \geq \delta^{HL-DM_{L_1}}$,

$X \in \mathbb{M}, \delta^{HL-DM_{L_1}} \in [0,1]$

Construct the following satisfactory level model (for finite value of p) for problem (25):

Maximize $\delta^{LH-DM_{L_1}}$

subject to (29)

$\left(\dfrac{d_p^{PIS^{LH-DM_{L_1}}}(X) - (d_p^{PIS^{LH-DM_{L_1}}})^*}{\left(d_p^{PIS^{LH-DM_{L_1}}} \right)^- - \left(d_p^{PIS^{LH-DM_{L_1}}} \right)^*} \right) \geq \delta^{LH-DM_{L_1}}$,

$\left(\dfrac{\left(d_p^{NIS^{LH-DM_{L_1}}} \right)^* - d_p^{NIS^{LH-DM_{L_1}}}(X)}{\left(d_p^{NIS^{LH-DM_{L_1}}} \right)^* - \left(d_p^{NIS^{LH-DM_{L_1}}} \right)^-} \right) \geq \delta^{LH-DM_{L_1}}$,

$X \in \mathbb{M}, \delta^{LH-DM_{L_1}} \in [0,1]$

Construct the following satisfactory level model (for finite value of p) for problem (26):

Maximize $\delta^{HH-DM_{L_1}}$

subject to (30)

$\left(\dfrac{d_p^{PIS^{HH-DM_{L_1}}}(X) - (d_p^{PIS^{HH-DM_{L_1}}})^*}{\left(d_p^{PIS^{HH-DM_{L_1}}} \right)^- - \left(d_p^{PIS^{HH-DM_{L_1}}} \right)^*} \right) \geq \delta^{HH-DM_{L_1}}$,

$$\left(\frac{\left(d_p^{NIS^{HH-DM_{L_1}}}\right)^* - d_p^{NIS^{HH-DM_{L_1}}}(X)}{\left(d_p^{NIS^{HH-DM_{L_1}}}\right)^* - \left(d_p^{NIS^{HH-DM_{L_1}}}\right)^-}\right) \geq \delta^{HH-DM_{L_1}},$$

$$X \in \mathbb{M}, \delta^{HH-DM_{L_1}} \in [0,1]$$

<u>Step 10:</u>

Solve problems (27-30) to obtain the satisfactory levels $\delta^{*^{LL-DM_{L_1}}}$, $\delta^{*^{HL-DM_{L_1}}}$, $\delta^{*^{LH-DM_{L_1}}}$ and $\delta^{*^{HH-DM_{L_1}}}$ for the compromise solutions $X^{*^{LL-DM_{L_1}}}$, $X^{*^{HL-DM_{L_1}}}$, $X^{*^{LH-DM_{L_1}}}$ and $X^{*^{HH-DM_{L_1}}}$. If the DM_{L_1} is satisfied with the solutions, then go to step (11). Otherwise, go to step (5).

<u>Step 11:</u>

Ask the DM_{L_1} to select the maximum acceptable negative and positive tolerance (relaxation) values, [14, 15]: $\tau_i^{LL-DM_{L_1}-L}, \tau_i^{LL-DM_{L_1}-R}, \tau_i^{HL-DM_{L_1}-L}, \tau_i^{HL-DM_{L_1}-R}, \tau_i^{LH-DM_{L_1}-L}, \tau_i^{LH-DM_{L_1}-R}, \tau_i^{HH-DM_{L_1}-L}$ and $\tau_i^{HH-DM_{L_1}-R}$,
$i = 1,2,\ldots,n_1$ on the decision vectors:

$$X_1^{*^{LL-DM_{L_1}}} = \left(x_{11}^{*^{LL-DM_{L_1}}}, x_{12}^{*^{LL-DM_{L_1}}}, \ldots, x_{1n_1}^{*^{LL-DM_{L_1}}}\right), X_1^{*^{HL-DM_{L_1}}} = \left(x_{11}^{*^{HL-DM_{L_1}}}, x_{12}^{*^{HL-DM_{L_1}}}, \ldots, x_{1n_1}^{*^{HL-DM_{L_1}}}\right),$$

$$X_1^{*^{LH-DM_{L_1}}} = \left(x_{11}^{*^{LH-DM_{L_1}}}, x_{12}^{*^{LH-DM_{L_1}}}, \ldots, x_{1n_1}^{*^{LH-DM_{L_1}}}\right),$$
and

$$X_1^{*^{HH-DM_{L_1}}} = \left(x_{11}^{*^{HH-DM_{L_1}}}, x_{12}^{*^{HH-DM_{L_1}}}, \ldots, x_{1n_1}^{*^{HH-DM_{L_1}}}\right).$$

Set i = i+1, go to the next phase.

<u>Phase (2):</u>

<u>Step 12:</u>

Set i=2, "The 2nd level".
Construct the PIS payoff table of problems (11-14) for i=2, and obtain the PIS: $(F_{2q}^{*^{LL-DM_{L_2}}})$, $(F_{2q}^{*^{HL-DM_{L_2}}})$, $(F_{2q}^{*^{LH-DM_{L_2}}})$ and $(F_{2q}^{*^{HH-DM_{L_2}}})$, $q = 1,2,\ldots,k_2$.

<u>Step 13:</u>

Construct the negative ideal solution (NIS) payoff tables of the problems (11-14) for i=2 and obtain the NIS: $\left(F_{2q}^{-^{LL-DM_{L_2}}}\right), \left(F_{2q}^{-^{HL-DM_{L_2}}}\right), \left(F_{2q}^{-^{LH-DM_{L_2}}}\right)$ and $\left(F_{2q}^{-^{HH-DM_{L_2}}}\right)$, $q = 1,2,\ldots,k_2$.

<u>Step 14:</u>
Let $w_{2q} = w_{2q}^*$ where $\sum_{q=1}^{k_1} w_{2q} = 1$ and $p = p^*$, $p^* \in \{1,2,\ldots,\infty\}$.

<u>Step 15:</u>

Construct distance functions $d_p^{PIS^{LL-DM_{L_2}}}$ and $d_p^{NIS^{LL-DM_{L_2}}}$:

$$d_p^{PIS^{LL-DM_{L_2}}} = \left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{*^{LL-DM_{L_1}}} - F_{1q}^{LL-DM_{L_1}}(X)}{F_{1q}^{*^{LL-DM_{L_1}}} - F_{1q}^{-^{LL-DM_{L_1}}}}\right)^p + \sum_{q=1}^{k_2} w_{2q} \left(\frac{F_{2q}^{*^{LL-DM_{L_2}}} - F_{2q}^{LL-DM_{L_2}}(X)}{F_{2q}^{*^{LL-DM_{L_1}}} - F_{2q}^{-^{LL-DM_{L_2}}}}\right)^p\right)^{1/p} \quad (31)$$

And

$$d_p^{NIS^{LL-DM_{L_2}}} = \left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{LL-DM_{L_1}}(X) - F_{1q}^{-^{LL-DM_{L_1}}}}{F_{1q}^{*^{LL-DM_{L_1}}} - F_{1q}^{-^{LL-DM_{L_1}}}}\right)^p + \sum_{q=2}^{k_2} w_{2q} \left(\frac{F_{2q}^{LL-DM_{L_2}}(X) - F_{2q}^{-^{LL-DM_{L_2}}}}{F_{2q}^{*^{LL-DM_{L_2}}} - F_{2q}^{-^{LL-DM_{L_2}}}}\right)^p\right)^{1/p} \quad (32)$$

Construct distance functions $d_p^{PIS^{HL-DM_{L_2}}}$ and $d_p^{NIS^{HL-DM_{L_2}}}$:

$$d_p^{PIS^{HL-DM_{L_2}}} = \left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{*^{HL-DM_{L_1}}} - F_{1q}^{HL-DM_{L_1}}(X)}{F_{1q}^{*^{HL-DM_{L_1}}} - F_{1q}^{-^{HL-DM_{L_1}}}}\right)^p + \sum_{q=1}^{k_2} w_{2q} \left(\frac{F_{2q}^{*^{HL-DM_{L_2}}} - F_{2q}^{HL-DM_{L_2}}(X)}{F_{2q}^{*^{HL-DM_{L_2}}} - F_{2q}^{-^{HL-DM_{L_2}}}}\right)^p\right)^{1/p} \quad (33)$$

and

$$d_p^{NIS^{HL-DM_{L_2}}} = \left(\begin{array}{l}\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{HL-DM_{L_1}}(X) - F_{1q}^{-^{HL-DM_{L_1}}}}{F_{1q}^{*^{HL-DM_{L_1}}} - F_{1q}^{-^{HL-DM_{L_1}}}}\right)^p + \\ \sum_{q=1}^{k_2} w_{2q} \left(\frac{F_{2q}^{HL-DM_{L_2}}(X) - F_{2q}^{-^{HL-DM_{L_2}}}}{F_{2q}^{*^{HL-DM_{L_2}}} - F_{2q}^{-^{HL-DM_{L_2}}}}\right)^p\end{array}\right)^{1/p} \quad (34)$$

Construct distance functions $d_p^{PIS^{LH-DM_{L_2}}}$ and $d_p^{NIS^{LH-DM_{L_2}}}$:

$$d_p^{PIS^{LH-DM_{L_2}}} = \left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{*^{LH-DM_{L_1}}} - F_{1q}^{LH-DM_{L_1}}(X)}{F_{1q}^{*^{LH-DM_{L_1}}} - F_{1q}^{-^{LH-DM_{L_1}}}}\right)^p + \sum_{q=1}^{k_2} w_{2q} \left(\frac{F_{2q}^{*^{LH-DM_{L_2}}} - F_{2q}^{LH-DM_{L_2}}(X)}{F_{2q}^{*^{LH-DM_{L_2}}} - F_{2q}^{-^{LH-DM_{L_2}}}}\right)^p\right)^{1/p} \quad (35)$$

and

$$d_p^{NIS^{LH-DM_{L_2}}} = \left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{LH-DM_{L_1}}(X) - F_{1q}^{-^{LH-DM_{L_1}}}}{F_{1q}^{*^{LH-DM_{L_1}}} - F_{1q}^{-^{LH-DM_{L_1}}}}\right)^p + \sum_{q=1}^{k_2} w_{2q} \left(\frac{F_{2q}^{LH-DM_{L_2}}(X) - F_{2q}^{-^{LH-DM_{L_2}}}}{F_{2q}^{*^{LH-DM_{L_2}}} - F_{2q}^{-^{LH-DM_{L_2}}}}\right)^p\right)^{1/p} \quad (36)$$

Construct distance functions $d_p^{PIS^{HH-DM_2}}$ and $d_p^{NIS^{HH-DM_{L_2}}}$:

$$d_p^{PIS^{HH-DM_{L_2}}} = \left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{*^{HH-DM_{L_1}}} - F_{1q}^{HH-DM_{L_1}}(X)}{F_{1q}^{*^{HH-DM_{L_1}}} - F_{1q}^{-^{HH-DM_{L_1}}}}\right)^p + \sum_{q=1}^{k_2} w_{2q} \left(\frac{F_{2q}^{*^{HH-DM_{L_2}}} - F_{2q}^{HH-DM_{L_2}}(X)}{F_{2q}^{*^{HH-DM_{L_2}}} - F_{2q}^{-^{HH-DM_{L_2}}}}\right)^p\right)^{1/p} \quad (37)$$

and

$$d_p^{NIS^{HH-DM_{L_2}}} = \left(\sum_{q=1}^{k_1} w_{1q} \left(\frac{F_{1q}^{HH-DM_{L_1}}(X) - F_{1q}^{-HH-DM_{L_1}}}{F_{1q}^{*HH-DM_{L_1}} - F_{1q}^{-HH-DM_{L_1}}} \right)^p + \right.$$
$$\left. \sum_{q=1}^{k_2} w_{2q} \left(\frac{F_{2q}^{HH-DM_{L_2}}(X) - F_{2q}^{-HH-DM_{L_2}}}{F_{2q}^{*HH-DM_{L_2}} - F_{2q}^{-HH-DM_{L_2}}} \right)^p \right)^{1/p} \quad (38)$$

Step 16:

Construct the following bi-objective problem with two commensurable (but conflicting) objectives, using the distance functions $d_p^{PIS^{LL-DM_{L_2}}}$ and $d_p^{NIS^{LL-DM_{L_2}}}$:

$Minimize\ d_p^{PIS^{LL-DM_{L_2}}}(X)$

$Maximize\ d_p^{NIS^{LL-DM_{L_2}}}(X)$

subject to   (39)

$X \in \mathbb{M}$

where

$p = 1,2, \ldots, \infty.$

Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{HL-DM_{L_2}}}$ and $d_p^{NIS^{HL-DM_{L_2}}}$:

$Minimize\ d_p^{PIS^{HL-DM_{L_2}}}(X)$

$Maximize\ d_p^{NIS^{HL-DM_{L_2}}}(X)$

subject to   (40)

$X \in \mathbb{M}$

where

$p = 1,2, \ldots, \infty.$

Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{LH-DM_{L_2}}}$ and $d_p^{NIS^{LH-DM_{L_2}}}$:

$Minimize\ d_p^{PIS^{LH-DM_{L_2}}}(X)$

$Maximize\ d_p^{NIS^{LH-DM_{L_2}}}(X)$

subject to   (41)

$X \in \mathbb{M}$
where

$p = 1,2, \ldots, \infty.$

Construct the following bi-objective problem with two commensurable (but conflicting) objectives using the distance functions $d_p^{PIS^{HH-DM_{L_2}}}$ and $d_p^{NIS^{HH-DM_{L_2}}}$:

$Minimized_p^{PIS^{HH-DM_{L_2}}}(X)$

$Maximize\ d_p^{NIS^{HH-DM_{L_2}}}(X)$

subject to   (42)

$X \in \mathbb{M}$

where

$p = 1,2, \ldots, \infty.$

Step 17:

Construct PIS Payoff table for problem (39) and obtain:

$d_p^{-LL-DM_{L_2}} = \left(\left(d_p^{PIS^{LL-DM_{L_2}}} \right)^-, \left(d_p^{NIS^{LL-DM_{L_2}}} \right)^- \right),$

$d_p^{*LL-DM_{L_2}} = \left(\left(d_p^{PIS^{LL-DM_{L_2}}} \right)^*, \left(d_p^{NIS^{LL-DM_{L_2}}} \right)^* \right).$

where

$\left(d_p^{PIS^{LL-DM_{L_2}}} \right)^* =$
$\underset{X \in \mathbb{M}}{Minimize}\, d_p^{PIS^{LL-DM_{L_2}}}(X)$ and the solution is $X^{PIS^{LL-DM_{L_2}}}$,

$\left(d_p^{NIS^{LL-DM_{L_2}}} \right)^* =$
$\underset{X \in \mathbb{M}}{Maximize}\, d_p^{NIS^{LL-DM_{L_2}}}(X)$ and the solution is $X^{NIS^{LL-DM_{L_2}}}$,

$\left(d_p^{PIS^{LL-DM_{L_2}}} \right)^- = d_p^{PIS^{LL-DM_{L_2}}} \left(X^{NIS^{LL-DM_{L_2}}} \right)$ and

$\left(d_p^{NIS^{LL-DM_{L_2}}} \right)^- = d_p^{NIS^{LL-DM_{L_2}}} \left(X^{PIS^{LL-DM_{L_2}}} \right).$

Construct PIS Payoff table for problem (40) and obtain:

$d_p^{-HL-DM_{L_2}} = \left(\left(d_p^{PIS^{HL-DM_{L_2}}} \right)^-, \left(d_p^{NIS^{HL-DM_{L_2}}} \right)^- \right),$

$d_p^{*HL-DM_{L_2}} = \left(\left(d_p^{PIS^{HL-DM_{L_2}}} \right)^*, \left(d_p^{NIS^{HL-DM_{L_2}}} \right)^* \right).$

Construct PIS Payoff table for problem (41) and obtain:

$d_p^{-LH-DM_{L_2}} = \left(\left(d_p^{PIS^{LH-DM_{L_2}}} \right)^-, \left(d_p^{NIS^{LH-DM_{L_2}}} \right)^- \right),$

$d_p^{*LH-DM_{L_2}} = \left(\left(d_p^{PIS^{LH-DM_{L_2}}} \right)^*, \left(d_p^{NIS^{LH-DM_{L_2}}} \right)^* \right).$

Construct PIS Payoff table for problem (42) and obtain:

$d_p^{-HH-DM_{L_2}} = \left(\left(d_p^{PIS^{HH-DM_{L_2}}} \right)^-, \left(d_p^{NIS^{HH-DM_{L_2}}} \right)^- \right),$

$d_p^{*HH-DM_{L_2}} = \left(\left(d_p^{PIS^{HH-DM_{L_2}}} \right)^*, \left(d_p^{NIS^{HH-DM_{L_2}}} \right)^* \right).$

Step 18:

Construct the following satisfactory level model, [11-13], (for finite value of p) for problem (39):

$Maximize\ \delta^{\mathrm{LL}-DM_{L_2}}$

subject to $\hspace{6cm}$ (43)

$$\left(\frac{d_p^{PIS}{}^{\mathrm{LL}-DM_{L_2}}(X)-(d_p^{PIS}{}^{\mathrm{LL}-DM_{L_2}})^*}{\left(d_p^{PIS}{}^{\mathrm{LL}-DM_{L_2}}\right)^- - \left(d_p^{PIS}{}^{\mathrm{LL}-DM_{L_2}}\right)^*}\right) \geq \delta^{\mathrm{LL}-DM_{L_2}},$$

$$\left(\frac{\left(d_p^{NIS}{}^{\mathrm{LL}-DM_{L_2}}\right)^* - d_p^{NIS}{}^{\mathrm{LL}-DM_{L_2}}(X)}{\left(d_p^{NIS}{}^{\mathrm{LL}-DM_{L_2}}\right)^* - \left(d_p^{NIS}{}^{\mathrm{LL}-DM_{L_2}}\right)^-}\right) \geq \delta^{\mathrm{LL}-DM_{L_2}},$$

$$\frac{x_{1i}-\left(X_{1i}^{*}{}^{\mathrm{LL}-DM_{L_1}} - \tau_i^{\mathrm{LL}-DM_{L_1}-L}\right)}{\tau_i^{\mathrm{LL}-DM_{L_1}-L}} \geq \delta^{\mathrm{LL}-DM_{L_2}}, i = 1,2,\dots,n_1,$$

$$\frac{\left(X_{1i}^{*}{}^{\mathrm{LL}-DM_{L_1}} + \tau_i^{\mathrm{LL}-DM_{L_1}-R}\right)-x_{1i}}{\tau_i^{\mathrm{LL}-DM_{L_1}-R}} \geq \delta^{\mathrm{LL}-DM_{L_2}}, i = 1,2,\dots,n_1$$

$X \in \mathbb{M}, \delta^{\mathrm{LL}-DM_{L_2}} \in [0,1].$

Construct the following satisfactory level model (for finite value of p) for problem (40):

$Maximize\ \delta^{\mathrm{HL}-DM_{L_2}}$

subject to $\hspace{6cm}$ (44)

$$\left(\frac{d_p^{PIS}{}^{\mathrm{HL}-DM_{L_2}}(X)-(d_p^{PIS}{}^{\mathrm{HL}-DM_{L_2}})^*}{\left(d_p^{PIS}{}^{\mathrm{HL}-DM_{L_2}}\right)^- - \left(d_p^{PIS}{}^{\mathrm{HL}-DM_{L_2}}\right)^*}\right) \geq \delta^{\mathrm{HL}-DM_{L_2}},$$

$$\left(\frac{\left(d_p^{NIS}{}^{\mathrm{HL}-DM_{L_2}}\right)^* - d_p^{NIS}{}^{\mathrm{HL}-DM_{L_2}}(X)}{\left(d_p^{NIS}{}^{\mathrm{HL}-DM_{L_2}}\right)^* - \left(d_p^{NIS}{}^{\mathrm{HL}-DM_{L_2}}\right)^-}\right) \geq \delta^{\mathrm{HL}-DM_{L_2}},$$

$$\frac{x_{1i}-\left(X_{1i}^{*}{}^{\mathrm{HL}-DM_{L_1}} - \tau_i^{\mathrm{HL}-DM_{L_1}-L}\right)}{\tau_i^{\mathrm{HL}-DM_{L_1}-L}} \geq \delta^{\mathrm{HL}-DM_{L_2}}, i = 1,2,\dots,n_1,$$

$$\frac{\left(X_{1i}^{*}{}^{\mathrm{HL}-DM_{L_1}} + \tau_i^{\mathrm{HL}-DM_{L_1}-R}\right)-x_{1i}}{\tau_i^{\mathrm{HL}-DM_{L_1}-R}} \geq \delta^{\mathrm{HL}-DM_{L_2}}, i = 1,2,\dots,n_1$$

$X \in \mathbb{M}, \delta^{\mathrm{HL}-DM_{L_2}} \in [0,1].$

Construct the following satisfactory level model (for finite value of p) for problem (41):

$Maximize\ \delta^{\mathrm{LH}-DM_{L_2}}$

subject to $\hspace{6cm}$ (45)

$$\left(\frac{d_p^{PIS}{}^{\mathrm{LH}-DM_{L_2}}(X)-(d_p^{PIS}{}^{\mathrm{LH}-DM_{L_2}})^*}{\left(d_p^{PIS}{}^{\mathrm{LH}-DM_{L_2}}\right)^- - \left(d_p^{PIS}{}^{\mathrm{LH}-DM_{L_2}}\right)^*}\right) \geq \delta^{\mathrm{LH}-DM_{L_2}},$$

$$\left(\frac{\left(d_p^{NIS}{}^{\mathrm{LH}-DM_{L_2}}\right)^* - d_p^{NIS}{}^{\mathrm{LH}-DM_{L_2}}(X)}{\left(d_p^{NIS}{}^{\mathrm{LH}-DM_{L_2}}\right)^* - \left(d_p^{NIS}{}^{\mathrm{LH}-DM_{L_2}}\right)^-}\right) \geq \delta^{\mathrm{LH}-DM_{L_2}},$$

$$\frac{x_{1i}-\left(X_{1i}^{*}{}^{\mathrm{LH}-DM_{L_1}} - \tau_i^{\mathrm{LH}-DM_{L_1}-L}\right)}{\tau_i^{\mathrm{LH}-DM_{L_1}-L}} \geq \delta^{\mathrm{LH}-DM_{L_2}}, i = 1,2,\dots,n_1,$$

$$\frac{\left(X_{1i}^{*}{}^{\mathrm{LH}-DM_{L_1}} + \tau_i^{\mathrm{LH}-DM_{L_1}-R}\right)-x_{1i}}{\tau_i^{\mathrm{LH}-DM_{L_1}-R}} \geq \delta^{\mathrm{LH}-DM_{L_2}}, i = 1,2,\dots,n_1,$$

$X \in \mathbb{M}, \delta^{\mathrm{LH}-DM_{L_2}} \in [0,1].$

Construct the following satisfactory level model (for finite value of p) for problem (42):

$Maximize\ \delta^{\mathrm{HH}-DM_{L_2}}$

subject to $\hspace{6cm}$ (46)

$$\left(\frac{d_p^{PIS}{}^{\mathrm{HH}-DM_{L_2}}(X)-(d_p^{PIS}{}^{\mathrm{HH}-DM_{L_2}})^*}{\left(d_p^{PIS}{}^{\mathrm{HH}-DM_{L_2}}\right)^- - \left(d_p^{PIS}{}^{\mathrm{HH}-DM_{L_2}}\right)^*}\right) \geq \delta^{\mathrm{HH}-DM_{L_2}},$$

$$\left(\frac{\left(d_p^{NIS}{}^{\mathrm{HH}-DM_{L_2}}\right)^* - d_p^{NIS}{}^{\mathrm{HH}-DM_{L_2}}(X)}{\left(d_p^{NIS}{}^{\mathrm{HH}-DM_{L_1}}\right)^* - \left(d_p^{NIS}{}^{\mathrm{HH}-DM_{L_1}}\right)^-}\right) \geq \delta^{\mathrm{HH}-DM_{L_2}},$$

$$\frac{x_{1i}-\left(X_{1i}^{*}{}^{\mathrm{HH}-DM_{L_1}} - \tau_i^{\mathrm{HH}-DM_{L_1}-L}\right)}{\tau_i^{\mathrm{HH}-DM_{L_1}-L}} \geq \delta^{\mathrm{HH}-DM_{L_2}}, i = 1,2,\dots,n_1,$$

$$\frac{\left(X_{1i}^{*}{}^{\mathrm{HH}-DM_{L_1}} + \tau_i^{\mathrm{HH}-DM_{L_1}-R}\right)-x_{1i}}{\tau_i^{\mathrm{HH}-DM_{L_1}-R}} \geq \delta^{\mathrm{HH}-DM_{L_2}}, i = 1,2,\dots,n_1,$$

$X \in \mathbb{M}, \delta^{\mathrm{HH}-DM_{L_2}} \in [0,1].$

Step 19:

Solve problems (43-468) to obtain the satisfactory levels $\delta^{*}{}^{\mathrm{LL}-DM_{L_2}}$, $\delta^{*}{}^{\mathrm{HL}-DM_{L_2}}$, $\delta^{*}{}^{\mathrm{LH}-DM_{L_2}}$ and $\delta^{*}{}^{\mathrm{HH}-DM_{L_2}}$ for the compromise solutions $X^{*}{}^{\mathrm{LL}-DM_{L_2}}$, $X^{*}{}^{\mathrm{HL}-DM_{L_2}}$, $X^{*}{}^{\mathrm{LH}-DM_{L_2}}$ and $X^{*}{}^{\mathrm{HH}-DM_{L_2}}$. If the DM_{L_2} is satisfied with the solutions, then go to step (20). Otherwise, go to step (14).

Step 20:

Ask the DM_{L_2} to select the maximum acceptable negative and positive tolerance (relaxation) values:

$$\tau_i^{\mathrm{LL}-DM_{L_2}-L}, \tau_i^{\mathrm{LL}-DM_{L_2}-R}, \tau_i^{\mathrm{HL}-DM_{L_2}-L}, \tau_i^{\mathrm{HL}-DM_{L_2}-R},$$
$$\tau_i^{\mathrm{LH}-DM_{L_2}-L}, \tau_i^{\mathrm{LH}-DM_{L_2}-R}, \tau_i^{\mathrm{HH}-DM_{L_2}-L}\ and\ \tau_i^{\mathrm{HH}-DM_{L_2}-R}, i = 1,2,\dots,n_2$$

on the decision vectors:

$$X_2^{*}{}^{\mathrm{LL}-DM_{L_2}} = \left(x_{21}^{*}{}^{\mathrm{LL}-DM_{L_2}}, x_{22}^{*}{}^{\mathrm{LL}-DM_{L_2}}, \dots, x_{2n_2}^{*}{}^{\mathrm{LL}-DM_{L_2}}\right),$$

$$X_2^{*}{}^{\mathrm{HL}-DM_{L_2}} = \left(x_{21}^{*}{}^{\mathrm{HL}-DM_{L_2}}, x_{22}^{*}{}^{\mathrm{HL}-DM_{L_2}}, \dots, x_{2n_2}^{*}{}^{\mathrm{HL}-DM_{L_2}}\right),$$

$$X_2^{*}{}^{\mathrm{LH}-DM_{L_2}} = \left(x_{21}^{*}{}^{\mathrm{LH}-DM_{L_2}}, x_{22}^{*}{}^{\mathrm{LH}-DM_{L_2}}, \dots, x_{2n_2}^{*}{}^{\mathrm{LH}-DM_{L_2}}\right),$$

and

$$X_2^{*^{HH-DM_{L_2}}} = \left(x_{21}^{*^{HH-DM_{L_2}}}, x_{22}^{*^{HH-DM_{L_2}}}, \ldots, x_{2n_2}^{*^{HH-DM_{L_2}}}\right).$$

Set i = i+1, if i ≤h goes to the next phase. Otherwise, stop.

4. ILLUSTRATIVE NUMERICAL EXAMPLE

Consider the following linear multi-level multi-objective decision making (LMLMODM) problem with rough parameters in the objective functions

First Level:

Maximize $f_{11}(X) = 2([2,3],[1,5))x_1 + ([3,5],[2,7)) x_2 + ([2,3],[1,4)) x_3$

Maximize $f_{12}(X) = ([6,7],[5,9))x_1 - x_2 + ([1,3],[1,6)) x_3 + ([1,3],[0,5))$

Second Level:

Maximize $f_{21}(X) = 2x_1 + ([5,6],[3,8)) x_2 - 2([0,3],[0,6)) x_3 + ([5,6],[3,7))$

Maximize $f_{22}(X) = x_1 - ([3,4],[2,6)) x_2 + ([1,3],[1,7)) x_3 + ([3,4],[2,6))$

Third Level:

Maximize $f_{31}(X) = ([2,5],[1,8))x_1 + 2 x_2 + x_3 + ([4,5],[3,6))$

Maximize $f_{32}(X) = 5x_1 + 2([1,2],[1,4)) x_2 - x_3 + ([6,7],[5,8))$

Subject to:

$X \in \mathbb{M} = \{3x_1 + 5x_2 + x_3 \leq 35, 2x_1 - x_2 + 12x_3 \leq 20, 5x_2 + 6x_3 \leq 16, x_1 \geq 1, x_1, x_2, x_3 \geq 0\}$

Solution:
Phase (1):

P^{LL}:

First Level:
MaximizeF$_{11}^{LL}$ = $4x_1 + 3x_2 + x_3 + 2$

Maximize F$_{12}^{LL}$ = $6x_1 - x_2 + x_3 + 1$

Second Level:

MaximizeF$_{21}^{LL}$ = $2x_1 + 5x_2 + 5$

Maximize F$_{22}^{LL}$ = $x_1 - 3 x_2 + x_3 + 3$

Third Level:

MaximizeF$_{31}^{LL}$ = $2x_1 - 2x_2 + x_3 + 4$

Maximize F$_{32}^{LL}$ = $5x_1 + 2 x_2 - x_3 + 6$

Subject to

X ∈ M

P^{HL}:

First Level:

MaximizeF$_{11}^{HL}$ = $6x_1 + 5x_2 + x_3 + 3$

Maximize F$_{12}^{HL}$ = $7x_1 - x_2 + 3x_3 + 3$

Second Level:

MaximizeF$_{21}^{HL}$ = $2x_1 + 6x_2 + +6x_3 + 6$

Maximize F$_{22}^{HL}$ = $x_1 - 4 x_2 + 3x_3 + 4$

Third Level:

MaximizeF$_{31}^{HL}$ = $5x_1 - 2x_2 + x_3 + 5$

Maximize F$_{32}^{HL}$ = $5x_1 + 4 x_2 - x_3 + 7$

Subject to

X ∈ M

P^{LH}:

First Level:

MaximizeF$_{11}^{LH}$ = $2x_1 + 2x_2 + x_3 + 1$

Maximize F$_{12}^{LH}$ = $5x_1 - x_2 + x_3$

Second Level:

MaximizeF$_{21}^{LH}$ = $2x_1 + 3x_2 + 3$

Maximize F$_{22}^{LH}$ = $x_1 - 2 x_2 + x_3 + 2$

Third Level:

MaximizeF$_{31}^{HL}$ = $x_1 - 2x_2 + x_3 + 3$

Maximize F$_{32}^{HL}$ = $5x_1 + 2 x_2 - x_3 + 5$

Subject to

X ∈ M

P^{HH}:

First Level:

MaximizeF$_{11}^{HH}$ = $10x_1 + 7x_2 + x_3 + 4$

Maximize F$_{12}^{HH}$ = $9x_1 - x_2 + 6x_3 + 5$

Second Level:

MaximizeF$_{21}^{HH}$ = $2x_1 + 8x_2 - 12x_3 + 7$

Maximize F$_{22}^{HH}$ = $x_1 - 6 x_2 + 7x_3 + 6$

Third Level:

$\text{Maximize} F_{31}^{HH} = 8x_1 - 2x_2 + x_3 + 6$

$\text{Maximize } F_{32}^{HH} = 5x_1 + 8x_2 - x_3 + 8$

Subject to

$X \in \mathbb{M}$

P^{LL}:

(First Level):

Obtain PIS and NIS payoff tables for problem P^{LL}:

Table 1. PIS payoff table for problem P^{LL}

	$F_{11}^{LL}(X)$	$F_{12}^{LL}(X)$	x_1	x_2	x_3
$Max. F_{11}^{LL}(X)$	45.85*	62.54	10.38	0.77	0
$Max \, F_{12}^{LL}(X)$	45.85	62.54*	10.38	0.77	0

PIS: f^{*LL}=(45.84615, 62.53846)

Table 2. NIS payoff table for problem P^{LL}

	$F_1^{LL}(X)$	$F_2^{LL}(X)$	x_1	x_2	x_3
$Min. F_{11}^{LL}(X)$	2^-	1	0	0	0
$Min. F_{12}^{LL}(X)$	11.6	-2.2^-	0	3.2	0

NIS: f^{-LL}=(2,−2.2)

Next, construct equation and obtain the following equations:

$$d_p^{PIS^{LL-DM_{L_1}}} = \left[w_1^p \left(\frac{45.84615 - F_{11}^{LL}(X)}{45.84615 - 2} \right)^p + \right.$$
$$\left. w_2^p \left(\frac{62.53846 - F_{12}^{LL}(X)}{62.53846 - (-2.2)} \right)^p \right]^{1/p}$$

$$d_p^{NIS^{LL-DM_{L_1}}} = \left[w_1^p \left(\frac{F_{11}^{LL}(X) - (2)}{45.84615 - 2} \right)^p + w_2^p \left(\frac{F_{12}^{LL}(X) - (-2.2)}{62.53846 - (-2.2)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = 0.5$ and p=2

Table 3. PIS payoff table of problem (11) when p=2

	$d_2^{PIS^{LL}}$	$d_2^{NIS^{LL}}$	$F_{11}^{LL}(X)$	$F_{12}^{LL}(X)$	x_1	x_2	x_3
Min. $d_2^{PIS^{LL}}$	22.40*	0.71^-	45.9	62.5	10.4	0.8	0
Max. $d_2^{NIS^{LL}}$	0	0.56*	45.85	62.5	10.4	0.8	0

d_2^{*LL} = (22.40027, 0.5590170) , d_2^{-LL}= (0, 0.7071067812).

Now, it is easy to compute step (10):

$Maximize \, \delta^{LL-DM_{L_1}}$

Subject to

$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$$\left(\frac{d_1^{PIS^{LL}}(X) - 22.40027}{0.5590170 - 22.40027} \right) \ge \delta^{LL-DM_{L_1}},$$

$$\left(\frac{zero - d_1^{NIS^{LL}}(X)}{zero - 0.7071067812} \right) \ge \delta^{LL-DM_{L_1}}, \delta^{LL-DM_{L_1}} \in [0,1].$$

The maximum "satisfactory level" ($\delta^{LL-DM_{L_1}}$=0.9447279E-07) is achieved for the solution x_1^{*LL} =10.38462, x_2^{*LL}=0.7692308, x_3^{*LL}= zero.

(Second Level)

Obtain PIS and NIS payoff tables for problem P^{LL}:

Table 4. PIS payoff table for problem P^{LL}

	$F_{21}^{LL}(X)$	$F_{22}^{LL}(X)$	x_1	x_2	x_3
$Max F_{21}^{LL}(X)$	33.6667*	- 0.2667	6.3333	3.2	0
$Max \, F_{22}^{LL}(X)$	25	13*	10	0	0

PIS: f^{*LL} = (33.6667, 13)

Table 5. NIS payoff table for problem P^{LL}

	$F_{21}^{LL}(X)$	$F_{22}^{LL}(X)$	x_1	x_2	x_3
$Min. F_{21}^{LL}(X)$	5^-	0	0	0	0
$Min. F_{22}^{LL}(X)$	21	-6.6^-	0	3.2	0

NIS: f^{-LL} = (5,−6.6)

Next, construct equation and obtain the following equations:

$$d_p^{PIS^{LL-DM_{L_2}}} = \left[w_1^p \left(\frac{45.84615 - F_{11}^{LL}(X)}{45.84615 - 2} \right)^p + \right.$$
$$w_2^p \left(\frac{62.53846 - F_{12}^{LL}(X)}{62.53846 - (-2.2)} \right)^p + w_3^p \left(\frac{33.6667 - F_{21}^{LL}(X)}{33.6667 - 5} \right)^p +$$
$$\left. w_4^p \left(\frac{13 - F_{22}^{LL}(X)}{13 - (-6.6)} \right)^p \right]^{1/p}$$

$$d_p^{NIS^{LL-DM_{L_2}}} = \left[w_1^p \left(\frac{F_{11}^{LL}(X) - (2)}{45.84615 - 2} \right)^p + w_2^p \left(\frac{F_{12}^{LL}(X) - (-2.2)}{62.53846 - (-2.2)} \right)^p + \right.$$
$$\left. w_3^p \left(\frac{F_{21}^{LL}(X) - (5)}{33.6667 - 5} \right)^p + w_4^p \left(\frac{F_{22}^{LL}(X) - (-6.6)}{13 - (-6.6)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = 0.25$ and p=2,

Table 6. PIS payoff table of problem (11) when p=2

	$d_2^{PIS^{LL}}$	$d_2^{NIS^{LL}}$	$F_{21}^{LL}(X)$	$F_{22}^{LL}(X)$	x_1	x_2	x_3
$Min. d_2^{PIS^{LL}}$	11.2*	0.47^-	29.62	11.1	10.4	0.77	0
$Max. d_2^{NIS^{LL}}$	0.04	0.17*	29.62	11.1	10.4	0.77	0

d_2^{*LL} =(11.20014 , 0.1688567), d_2^{-LL} =(0.04301006758, 0.4710896811).

Now, it is easy to compute step (10):

$Maximize \, \delta^{LL-DM_{L_2}}$

Subject to

$$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$$

$$\left(\frac{d_1^{PIS^{LL}}(X) - 11.20014}{0.1688567 - 11.20014} \right) \ge \delta^{LL-DM_{L_2}},$$

$$\left(\frac{0.04301006758 - d_1^{NIS^{LL}}(X)}{0.04301006758 - 0.4710896811} \right) \ge \delta^{LL-DM_{L_2}},$$

$$\delta^{LL-DM_{L_2}} \in [0,1].$$

$$\left(\frac{(10.38462 + 0.001) - x_1}{0.001} \right) \ge \delta^{LL-DM_{L_2}},$$

$$\left(\frac{x_1 - (10.38462 - 0.001)}{0.001} \right) \ge \delta^{LL-DM_{L_2}}$$

The maximum "satisfactory level" ($\delta^{LL-DM_{L_2}}$=0.7886892E-07) is achieved for the solution $x_1^{*^{LL}}$=10.38462, $x_2^{*^{LL}}$=0.7692308, $x_3^{*^{LL}}$= zero.

(Third Level):

Obtain PIS and NIS payoff tables for problem P^{LL}:

Table 7. PIS payoff table for problem P^{LL}

	$F_{31}^{LL}(X)$	$F_{32}^{LL}(X)$	x_1	x_2	x_3
$Max. F_{31}^{LL}(X)$	24*	56	10	0	0
$Max. F_{32}^{LL}(X)$	23.23	59.46*	10.38	0.77	0

PIS: $f^{*^{LL}}$= (24, 59.46154)

Table 8. NIS payoff table for problem **P^{LL}** Table 8. NIS payoff table for problem P^{LL}

	$F_{31}^{LL}(X)$	$F_{32}^{LL}(X)$	x_1	x_2	x_3
$Min. F_{31}^{LL}(X)$	−2.4⁻	12.4	0	3.2	0
$Min. F_{32}^{LL}(X)$	5.67	4.3⁻	0	0	1.67

NIS: $f^{-^{LL}}$= (−2.4, 4.3)

Next, construct equation and obtain the following equations:

$$d_p^{PIS^{LL-DM_{L_3}}} = \left[w_1^p \left(\frac{45.84615 - F_{11}^{LL}(X)}{45.84615 - 2} \right)^p + \right.$$
$$w_2^p \left(\frac{62.53846 - F_{12}^{LL}(X)}{62.53846 - (-2.2)} \right)^p + w_3^p \left(\frac{33.6667 - F_{21}^{LL}(X)}{33.6667 - 5} \right)^p +$$
$$w_4^p \left(\frac{13 - F_{22}^{LL}(X)}{13 - (-6.6)} \right)^p + w_5^p \left(\frac{24 - F_{31}^{LL}(X)}{24 - (-2.4)} \right)^p +$$
$$\left. w_6^p \left(\frac{59.46154 - F_{32}^{LL}(X)}{59.46154 - (4.3)} \right)^p \right]^{1/p}$$

$$d_p^{NIS^{LL-DM_{L_3}}} = \left[w_1^p \left(\frac{F_{11}^{LL}(X) - (2)}{45.84615 - 2} \right)^p + w_2^p \left(\frac{F_{12}^{LL}(X) - (-2.2)}{62.53846 - (-2.2)} \right)^p + \right.$$
$$w_3^p \left(\frac{F_{21}^{LL}(X) - (5)}{33.6667 - 5} \right)^p + w_4^p \left(\frac{F_{22}^{LL}(X) - (-6.6)}{13 - (-6.6)} \right)^p +$$
$$\left. w_5^p \left(\frac{F_{31}^{LL}(X) - (-2.4)}{24 - (-2.4)} \right)^p + w_6^p \left(\frac{F_{32}^{LL}(X) - (4.3)}{59.46154 - (4.3)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = w_5^p = w_6^p = 1/6$ and p=2,

Table 9. PIS payoff table of problem (11) when p=2

	$d_3^{PIS^{LL}}$	$d_3^{NIS^{LL}}$	$F_{31}^{LL}(X)$	$F_{32}^{LL}(X)$	x_1	x_2	x_3
$Min. d_3^{PIS^{LL}}$	7.47*	0.35⁻	23.23	59.46	10.4	0.77	0
$Max. d_3^{NIS^{LL}}$	0.03	0.10*	23.23	59.46	10.4	0.77	0

$d_3^{*^{LL}}$ = (7.466758 ,0.1035186), $d_3^{-^{LL}}$ =(0.02925962351, 0.3532933399).

Now, it is easy to compute step (10):

$Maximize\ \delta^{LL-DM_{L_3}}$

Subject to

$$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$$

$$\left(\frac{d_3^{PIS^{LL}}(X) - 7.466758}{0.1035186 - 7.466758} \right) \ge \delta^{LL-DM_{L_3}},$$

$$\left(\frac{0.02925962351 - d_3^{NIS^{LL}}(X)}{0.02925962351 - 0.3532933399} \right) \ge \delta^{LL-DM_{L_3}},$$

$$\delta^{LL-DM_{L_3}} \in [0,1].$$

$$\left(\frac{(0 + 0.001) - + x_3}{0.001} \right) \ge \delta^{LL-DM_{L_3}}, \left(\frac{x_3 - (0 - 0.001)}{0.001} \right) \ge \delta^{LL-DM_{L_3}}$$

The maximum "satisfactory level" ($\delta^{LL-DM_{L_3}}$=0.6082756E-07) is achieved for the solution $x_1^{*^{LL}}$ =10.38462, $x_2^{*^{LL}}$=0.7692308, $x_3^{*^{LL}}$=zero.

$\underline{P^{HL}}$:

(First Level):

Obtain PIS and NIS payoff tables for problem P^{HL}:

Table 10. PIS payoff table for problem P^{HL}

	$F_{11}^{HL}(X)$	$F_{12}^{HL}(X)$	x_1	x_2	x_3
$Max. F_{11}^{HL}(X)$	69.15*	74.92	10.38	0.77	0
$Max. F_{12}^{HL}(X)$	69.15	74.92*	10.38	0.77	0

PIS: $f^{*^{HL}}$=(69.15385, 74.92308)

Table 11. NIS payoff table for problem P^{HL}

	$F_1^{HL}(X)$	$F_2^{HL}(X)$	x_1	x_2	x_3
$Min. F_{11}^{HL}(X)$	3⁻	3	0	0	0
$Min. F_{12}^{HL}(X)$	19	−0.2⁻	0	3.2	0

NIS: $f^{-^{LL}}$=(3,−0.2)

Next, construct equation and obtain the following equations:

$$d_p^{PIS^{HL}-DM_{L_1}} = \left[w_1^p \left(\frac{69.15385 - F_{11}^{HL}(X)}{69.15385 - 3} \right)^p + \right.$$
$$\left. w_2^p \left(\frac{74.92308 - F_{12}^{HL}(X)}{74.92308 - (-0.2)} \right)^p \right]^{1/p}$$

$$d_p^{NIS^{HL}-DM_{L_1}} = \left[w_1^p \left(\frac{F_{11}^{HL}(X) - (3)}{69.15385 - 3} \right)^p + \right.$$
$$\left. w_2^p \left(\frac{F_{12}^{HL}(X) - (-0.2)}{74.92308 - (-0.2)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = 0.5$ and p=2,

Table 12. PIS payoff table of problem (11) when p=2

	$d_2^{PIS^{HL}}$	$d_2^{NIS^{HL}}$	F_{11}^{HL}	F_{12}^{HL}	x_1	x_2	x_3
Min. $d_2^{PIS^{HL}}$	34.58*	0.71$^-$	69.1 5	74.9 2	10.3 8	0.7 7	0
Max. $d_2^{NIS^{HL}}$	0	0.56*	69.1 5	74.9 2	10.3 8	0.7 7	0

$d_2^{*HL} = (34.57692, 0.559017)$, $d_2^{-HL} = (0, 0.7071067812)$.

Now, it is easy to compute step (10):

$Maximize\ \delta^{HL-DM_{L_1}}$

Subject to

$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$$\left(\frac{d_1^{PIS^{HL}}(X) - 34.57692}{0.5590170 - 34.57692} \right) \ge \delta^{HL-DM_{L_1}},$$

$$\left(\frac{zero - d_1^{NIS^{HL}}(X)}{zero - 0.7071067812} \right) \ge \delta^{HL-DM_{L_1}}, \delta^{HL-DM_{L_1}} \in [0,1].$$

The maximum "satisfactory level" ($\delta^{HL-DM_{L_1}}$ =0.1536454E-01) is achieved for the solution x_1^{*HL}=10.38462, x_2^{*HL}=0.7692308, x_3^{*HL}= zero.

(Second Level):

Obtain PIS and NIS payoff tables for problem P^{HL}:

Table 13. PIS payoff table for problem P^{HL}

	$F_{21}^{HL}(X)$	$F_{22}^{HL}(X)$	x_1	x_2	x_3
Max. $F_{21}^{HL}(X)$	39.23*	3.56	7.4	2.44	0.64
Max. $F_{22}^{HL}(X)$	26	14*	10	0	0

PIS: f^{*HL} = (39.2258, 14)

Table 14. NIS payoff table for problem P^{HL}

	$F_{21}^{HL}(X)$	$F_{22}^{HL}(X)$	x_1	x_2	x_3
Min. $F_{21}^{HL}(X)$	6$^-$	4	0	0	0
Min. $F_{22}^{HL}(X)$	25.2	-8.8$^-$	0	3.2	0

NIS: f^{-HL} = (6, -8.8)

Next, construct equation and obtain the following equations:

$$d_p^{PIS^{HL}-DM_{L_2}} = \left[w_1^p \left(\frac{69.15385 - F_{11}^{HL}(X)}{69.15385 - 3} \right)^p + \right.$$
$$w_2^p \left(\frac{74.92308 - F_{12}^{HL}(X)}{74.92308 - (-0.2)} \right)^p + w_3^p \left(\frac{39.2258 - F_{21}^{HL}(X)}{39.2258 - 6} \right)^p +$$
$$\left. w_4^p \left(\frac{14 - F_{22}^{HL}(X)}{14 - (-8.8)} \right)^p \right]^{1/p}$$

$$d_p^{NIS^{HL}-DM_{L_2}} = \left[w_1^p \left(\frac{F_{11}^{HL}(X) - (3)}{69.15385 - 3} \right)^p + w_2^p \left(\frac{F_{12}^{HL}(X) - (-0.2)}{74.92308 - (-0.2)} \right)^p + \right.$$
$$\left. w_3^p \left(\frac{F_{21}^{HL}(X) - (6)}{39.2258 - 6} \right)^p + w_4^p \left(\frac{F_{22}^{HL}(X) - (-8.8)}{14 - (-8.8)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = 0.25$ and p=2,

Table 15. PIS payoff table of problem (11) when p=2

	$d_2^{PIS^{HL}}$	$d_2^{NIS^{HL}}$	F_{21}^{HL}	F_{22}^{HL}	x_1	x_2	x_3
Min. $d_2^{PIS^{LL}}$	9.5*	0.6$^-$	39. 2	3.6	7.4	2.4 4	0.6 4
Max. $d_2^{NIS^{LL}}$	0.1	0.2*	31. 4	11. 3	10. 4	0.7 7	0

$d_2^{*HL} = (9.511326, 0.1645195)$, $d_2^{-HL} = (0.06597257621, 0.5708198897)$.

Now, it is easy to compute step (10):

$Maximize\ \delta^{HL-DM_{L_2}}$

Subject to

$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$$\left(\frac{d_1^{PIS^{HL}}(X) - 9.511326}{0.1645195 - 9.511326} \right) \ge$$
$$\delta^{HL-DM_{L_2}}, \left(\frac{0.06597257621 - d_1^{NIS^{HL}}(X)}{0.06597257621 - 0.5708198897} \right) \ge \delta^{HL-DM_{L_2}},$$

$\delta^{HL-DM_{L_2}} \in [0,1].$

$$\left(\frac{(10.38462 + 0.001) - x_1}{0.001} \right) \ge \delta^{HL-DM_{L_2}}, \left(\frac{x_1 - (10.38462 - 0.001)}{0.001} \right) \ge$$
$$\delta^{HL-DM_{L_2}}$$

The maximum "satisfactory level" ($\delta^{HL-DM_{L_2}}$ =zero) is achieved for the solution x_1^{*HL} =10.38462, x_2^{*HL}=0.7692308, x_3^{*HL}= 0.2121311e-03.

(Third Level):

Obtain PIS and NIS payoff tables for problem P^{HL}:

Table 16. PIS payoff table for problem P^{HL}

	$F_{31}^{HL}(X)$	$F_{32}^{HL}(X)$	x_1	x_2	x_3
Max. $F_{31}^{HL}(X)$	55.38*	62	10.38	0.77	0
Max. $F_{32}^{HL}(X)$	55.38	62*	10.38	0.77	0

PIS: f^{*HL} = (55.38462, 62)

Table 17. NIS payoff table for problem P^{HL}

	$F_{31}^{HL}(X)$	$F_{32}^{HL}(X)$	x_1	x_2	x_3
$Min.\,F_{31}^{HL}(X)$	-1.4^-	19.8	0	3.2	0
$Min.\,F_{32}^{HL}(X)$	6.67	5.33^-	0	0	1.67

NIS: $f^{-HL} = (-1.4, 5.3333)$

Next, construct equation and obtain the following equations:

$$d_p^{PIS^{HL-DM_{L_3}}} = \left[w_1^p \left(\frac{69.15385 - F_{11}^{HL}(X)}{69.15385 - 3} \right)^p + \right.$$
$$w_2^p \left(\frac{74.92308 - F_{12}^{HL}(X)}{74.92308 - (-0.2)} \right)^p + w_3^p \left(\frac{39.2258 - F_{21}^{HL}(X)}{39.2258 - 6} \right)^p +$$
$$w_4^p \left(\frac{14 - F_{22}^{HL}(X)}{14 - (-8.8)} \right)^p + w_5^p \left(\frac{55.38462 - F_{31}^{HL}(X)}{55.38462 - (-1.4)} \right)^p +$$
$$\left. w_6^p \left(\frac{62 - F_{32}^{HL}(X)}{62 - (5.3333)} \right)^p \right]^{1/p}$$

$$d_p^{NIS^{HL-DM_{L_3}}} = \left[w_1^p \left(\frac{F_{11}^{HL}(X) - (3)}{69.15385 - 3} \right)^p + w_2^p \left(\frac{F_{12}^{HL}(X) - (-0.2)}{74.92308 - (-0.2)} \right)^p + \right.$$
$$w_3^p \left(\frac{F_{21}^{HL}(X) - (6)}{39.2258 - 6} \right)^p + w_4^p \left(\frac{F_{22}^{HL}(X) - (-8.8)}{14 - (-8.8)} \right)^p +$$
$$\left. w_5^p \left(\frac{F_{31}^{HL}(X) - (-1.4)}{55.38462 - (-1.4)} \right)^p + w_6^p \left(\frac{F_{32}^{HL}(X) - (5.3333)}{62 - (5.33333)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = w_5^p = w_6^p = 1/6$ and p=2,

Table 18. PIS payoff table of problem (11) when p=2

	$d_3^{PIS^{HL}}$	$d_3^{NIS^{HL}}$	$F_{31}^{HL}(X)$	$F_{32}^{HL}(X)$	x_1	x_2	x_3
$Min.\,d_3^{PIS^{LL}}$	9.1^*	0.4^-	55.4	62	10.4	0.77	0
$Max.\,d_3^{NIS^{LL}}$	0.04	0.1^*	55.4	62	10.4	0.77	0

$d_3^{*HL} = (9.068215, 0.1009190)$, $d_3^{-HL} = (0.04398171747, 0.3859144257)$.

Now, it is easy to compute step (10):

$Maximize\ \delta^{HL-DM_{L_3}}$

Subject to

$3x_1 + 5x_2 + x_3 \leq 35, 2x_1 - x_2 + 12x_3 \leq 20, 5x_2 + 6x_3 \leq 16, x_1 \geq 1, x_1, x_2, x_3 \geq 0$

$\left(\frac{d_3^{PIS^{HL}}(X) - 9.068215}{0.1009190 - 9.068215} \right) \geq \delta^{HL-DM_{L_3}}$,

$\left(\frac{0.04398171747 - d_3^{NIS^{HL}}(X)}{0.04398171747 - 0.3859144257} \right) \geq \delta^{HL-DM_{L_3}}$

$\delta^{HL-DM_{L_3}} \in [0,1]$.

$\left(\frac{(0 + 0.001) - x_3}{0.001} \right) \geq \delta^{HL-DM_{L_3}}, \left(\frac{x_3 - (0 - 0.001)}{0.001} \right) \geq \delta^{HL-DM_{L_3}}$

The maximum "satisfactory level" ($\delta^{HL-DM_{L_3}}$ =zero) is achieved for the solution x_1^{*HL} =10.38462, x_2^{*HL}=0.7692308, x_3^{*HL}= zero.

P^{LH}:

(First Level):

Obtain PIS and NIS payoff tables for problem P^{LH}:

Table 19. PIS payoff table for problem P^{LH}

	$F_{11}^{LH}(X)$	$F_{12}^{LH}(X)$	x_1	x_2	x_3
$Max.\,F_{11}^{LH}(X)$	23.31^*	51.15	10.38	0.77	0
$Max.\,F_{12}^{LH}(X)$	23.31	51.15^*	10.38	0.77	0

PIS: $f^{*LH} = (23.30769, 51.15385)$

Table 20. NIS payoff table for problem P^{LH}

	$F_1^{LH}(X)$	$F_2^{LH}(X)$	x_1	x_2	x_3
$Min.\,F_{11}^{LH}(X)$	1^-	0	0	0	0
$Min.\,F_{12}^{LH}(X)$	7.4	-3.2^-	0	3.2	0

NIS: $f^{-LH} = (1, -3.2)$

Next, construct equation and obtain the following equations:

$$d_p^{PIS^{LH-DM_{L_1}}} = \left[w_1^p \left(\frac{23.30769 - F_{11}^{LH}(X)}{23.30769 - 1} \right)^p + \right.$$
$$\left. w_2^p \left(\frac{51.15385 - F_{12}^{LH}(X)}{51.15385 - (-3.2)} \right)^p \right]^{1/p}$$

$$d_p^{NIS^{LH-DM_{L_1}}} = \left[w_1^p \left(\frac{F_{11}^{LH}(X) - (1)}{23.30769 - 1} \right)^p + \right.$$
$$\left. w_2^p \left(\frac{F_{12}^{LH}(X) - (-3.2)}{51.15385 - (-3.2)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = 0.5$ and p=2,

Table 21. PIS payoff table of problem (11) when p=2

	$d_2^{PIS^{LH}}$	$d_2^{NIS^{LH}}$	$F_{11}^{LH}(X)$	$F_{12}^{LH}(X)$	x_1	x_2	x_3
$Min.\,d_2^{PIS^{LH}}$	11.13^*	0.71^-	23.31	51.2	10.4	0.8	0
$Max.\,d_2^{NIS^{LH}}$	0	0.56^*	23.31	51.2	10.4	0.8	0

$d_2^{*LH} = (11.13143, 0.5590170)$, $d_2^{-LH} = (0, 0.7071067812)$.

Now, it is easy to compute step (10):

$Maximize\ \delta^{LH-DM_{L_1}}$

Subject to

$3x_1 + 5x_2 + x_3 \leq 35, 2x_1 - x_2 + 12x_3 \leq 20, 5x_2 + 6x_3 \leq 16, x_1 \geq 1, x_1, x_2, x_3 \geq 0$

$\left(\frac{d_1^{PIS^{LH}}(X) - 11.13143}{0.5590170 - 11.13143} \right) \geq \delta^{LH-DM_{L_1}}$,

$\left(\frac{zero - d_1^{NIS^{LH}}(X)}{zero - 0.7071067812} \right) \geq \delta^{LH-DM_{L_1}}, \delta^{LH-DM_{L_1}} \in [0,1]$.

The maximum "satisfactory level"

($\delta^{LH-DM_{L_1}}$ =0.1536454E-01) is achieved for the solution x_1^{*LH} =10.38462, x_2^{*LH} =0.7692308, x_3^{*LH} = zero.

(Second Level):

Obtain PIS and NIS payoff tables for problem P^{LH}:

Table 22. PIS payoff table for problem P^{LH}

	$F_{21}^{LH}(X)$	$F_{22}^{LH}(X)$	x_1	x_2	x_3
$Max. F_{21}^{LH}(X)$	26.08*	10.85	10.38	0.77	0
$Max. F_{22}^{LH}(X)$	23	12*	10	0	0

PIS: f^{*LH} = (26.07692, 12)

Table 23. NIS payoff table for problem P^{LH}

	$F_{21}^{LH}(X)$	$F_{22}^{LH}(X)$	x_1	x_2	x_3
$Min. F_{21}^{LH}(X)$	3^-	2	0	0	0
$Min. F_{22}^{LH}(X)$	12.6	-4.4^-	0	3.2	0

NIS: f^{-LH} = (3, −4.4)

Next, construct equation and obtain the following equations:

$$d_p^{PIS\,LH-DM_{L_2}} = \left[w_1^p \left(\frac{23.30769 - F_{11}^{LH}(X)}{23.30769 - 1} \right)^p + \right.$$
$$w_2^p \left(\frac{51.15385 - F_{12}^{LH}(X)}{51.15385 - (-3.2)} \right)^p + w_3^p \left(\frac{26.07692 - F_{21}^{LH}(X)}{26.07692 - 3} \right)^p +$$
$$\left. w_4^p \left(\frac{12 - F_{22}^{LH}(X)}{12 - (-4.4)} \right)^p \right]^{1/p}$$

$$d_p^{NIS\,LH-DM_{L_2}} =$$

$$\left[w_1^p \left(\frac{F_{11}^{HL}(X) - (1)}{23.30769 - 1} \right)^p + w_2^p \left(\frac{F_{12}^{HL}(X) - (-3.2)}{51.15385 - (-3.2)} \right)^p + \right.$$
$$\left. w_3^p \left(\frac{F_{21}^{HL}(X) - (3)}{26.07692 - 3} \right)^p + w_4^p \left(\frac{F_{22}^{HL}(X) - (-4.4)}{12 - (-4.4)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = 0.25$ and p=2,

Table 24. PIS payoff table of problem (11) when p=2

	$d_2^{PIS^{LH}}$	$d_2^{NIS^{LH}}$	$F_{21}^{LH}(X)$	$F_{22}^{LH}(X)$	x_1	x_2	x_3
$Min. d_2^{PIS^{LL}}$	6.24*	0.49^-	26.1	10.9	10.4	0.8	0
$Max. d_2^{NIS^{LL}}$	0.03	0.19*	26.1	10.9	10.4	0.8	0

d_2^{*LH} = (6.236730, 0.1908760), d_2^{-LH} = (0.02462469512, 0.4914416041).

Now, it is easy to compute step (10):

$Maximize\ \delta^{LH-DM_{L_2}}$

Subject to

$3x_1 + 5x_2 + x_3 \leq 35, 2x_1 - x_2 + 12x_3 \leq 20, 5x_2 + 6x_3 \leq 16, x_1 \geq 1, x_1, x_2, x_3 \geq 0$

$$\left(\frac{d_1^{PIS^{LH}}(X) - 6.236730}{0.1908760 - 6.236730} \right) \geq \delta^{LH-DM_{L_2}},$$

$$\left(\frac{0.02462469512 - d_1^{NIS^{LH}}(X)}{0.02462469512 - 0.4914416041} \right) \geq \delta^{LH-DM_{L_2}},$$

$$\delta^{LH-DM_{L_2}} \in [0,1].$$

$$\left(\frac{(10.38462 + 0.001) - x_1}{0.001} \right) \geq \delta^{LH-DM_{L_2}}$$

$$\left(\frac{x_1 - (10.38462 - 0.001)}{0.001} \right) \geq \delta^{LH-DM_{L_2}}$$

The maximum "satisfactory level" ($\delta^{LH-DM_{L_2}}$=3561380) is achieved for the solution x_1^{*LH} =10.38462, x_2^{*LH} =0.7692308, x_3^{*LH} = zero.

(Third Level):

Obtain PIS and NIS payoff tables for problem P^{LH}:

Table 25. PIS payoff table for problem P^{LH}

	$F_{31}^{LH}(X)$	$F_{32}^{LH}(X)$	x_1	x_2	x_3
$Max. F_{31}^{HL}(X)$	13*	55	10	0	0
$Max. F_{32}^{LH}(X)$	11.85	58.46*	10.38	0.77	0

PIS: f^{*LH} = (13, 58.46154)

Table 26. NIS payoff table for problem P^{LH}

	$F_{31}^{LH}(X)$	$F_{32}^{LH}(X)$	x_1	x_2	x_3
$Min. F_{31}^{LH}(X)$	-3.4^-	11.4	0	3.2	0
$Min. F_{32}^{LH}(X)$	4.67	3.33^-	0	zero	1.67

NIS: f^{-LH} = (−3.4, 3.3333)

Next, construct equation and obtain the following equations:

$$d_p^{PIS\,LH-DM_{L_3}} = \left[w_1^p \left(\frac{23.30769 - F_{11}^{LH}(X)}{23.30769 - 1} \right)^p + \right.$$
$$w_2^p \left(\frac{51.15385 - F_{12}^{LH}(X)}{51.15385 - (-3.2)} \right)^p + w_3^p \left(\frac{26.07692 - F_{21}^{LH}(X)}{26.07692 - 3} \right)^p +$$
$$w_4^p \left(\frac{12 - F_{22}^{LH}(X)}{12 - (-4.4)} \right)^p + w_5^p \left(\frac{13 - F_{31}^{LH}(X)}{13 - (-3.4)} \right)^p +$$
$$\left. w_6^p \left(\frac{58.46154 - F_{32}^{LH}(X)}{58.46154 - (3.3333)} \right)^p \right]^{1/p}$$

$$d_p^{NIS\,LH-DM_{L_3}} = \left[w_1^p \left(\frac{F_{11}^{LH}(X) - (1)}{23.30769 - 1} \right)^p + w_2^p \left(\frac{F_{12}^{LH}(X) - (-3.2)}{51.15385 - (-3.2)} \right)^p + \right.$$
$$w_3^p \left(\frac{F_{21}^{LH}(X) - (3)}{26.07692 - 3} \right)^p + w_4^p \left(\frac{F_{22}^{LH}(X) - (-4.4)}{12 - (-4.4)} \right)^p +$$
$$\left. w_5^p \left(\frac{F_{31}^{LH}(X) - (-3.4)}{13 - (-3.4)} \right)^p + w_6^p \left(\frac{F_{32}^{LH}(X) - (3.3333)}{58.46154 - (3.33333)} \right)^p \right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = w_5^p = w_6^p = 1/6$ and p=2,

Table 27. PIS payoff table of problem (11) when p=2

	$d_3^{PIS^{LH}}$	$d_3^{NIS^{LH}}$	$F_{31}^{LH}(X)$	$F_{32}^{LH}(X)$	x_1	x_2	x_3
Min. $d_3^{PIS^{LL}}$	2.03*	0.4⁻	13	55	10	0	0
Max. $d_3^{NIS^{LL}}$	0.02	0.1*	11.85	58.46	10.4	0.77	0

d_3^{*LH} = (2.034558 ,.1087784), d_3^{-LH} = (0.01658311422, 0.386786411).

Now, it is easy to compute step (10):
$Maximize\ \delta^{LH-DM_{L_3}}$

Subject to

$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16\ , x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$\left(\dfrac{d_3^{PIS^{LH}}(X)-2.034558}{0.1087784-2.034558}\right) \ge \delta^{LH-DM_{L_3}},$

$\left(\dfrac{0.01658311422-d_3^{NIS^{LH}}(X)}{0.01658311422-0.386786411}\right) \ge \delta^{LH-DM_{L_3}},$

$\delta^{LH-DM_{L_3}} \in [0,1].$

$\left(\dfrac{(0+0.001)-x_3}{0.001}\right) \ge \delta^{LH-DM_{L_3}}, \left(\dfrac{x_3-(0-0.001)}{0.001}\right) \ge \delta^{LH-DM_{L_3}}$

The maximum "satisfactory level" ($\delta^{LH-DM_{L_3}}$ =zero) is achieved for the solution x_1^{*LH}=10, x_2^{*LH} = zero, x_3^{*LH} = zero.

$\underline{P^{HH}}$:
(First Level):

Obtain PIS and NIS payoff tables for problem P^{HH}:

Table 28. PIS payoff table for problem P^{HH}

	$F_{11}^{HH}(X)$	$F_{12}^{HH}(X)$	x_1	x_2	x_3
Max. $F_{11}^{HH}(X)$	113.23*	97.69	10.4	0.77	0
Max. $F_{12}^{HH}(X)$	113.23	97.69*	10.4	0.77	0

PIS: f^{*HH}= (113.2308, 97.69231)

Table 29. NIS payoff table for problem P^{HH}

	$F_1^{HH}(X)$	$F_2^{HH}(X)$	x_1	x_2	x_3
Min. $F_{11}^{HH}(X)$	4⁻	5	0	0	0
Min. $F_{12}^{HH}(X)$	26.4	1.8⁻	0	3.2	0

NIS: f^{-HH} = (4,1.8)

Next, construct equation and obtain the following equations:

$d_p^{PIS^{HH-DM_{L_1}}} = \left[w_1^p \left(\dfrac{113.2308-F_{11}^{HH}(X)}{113.2308-4}\right)^p + w_2^p \left(\dfrac{97.69231-F_{12}^{HH}(X)}{97.69231-(1.8)}\right)^p\right]^{1/p}$

$d_p^{NIS^{HH-DM_{L_1}}} = \left[w_1^p \left(\dfrac{F_{11}^{HH}(X)-(4)}{113.2308-4}\right)^p + w_2^p \left(\dfrac{F_{12}^{HH}(X)-(1.8)}{97.69231-(1.8)}\right)^p\right]^{1/p}$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p$=0.5 and p=2.

Table 30. PIS payoff table of problem (11) when p=2

	$d_2^{PIS^{HH}}$	$d_2^{NIS^{HH}}$	$F_{11}^{HH}(X)$	$F_{12}^{HH}(X)$	x_1	x_2	x_3
Min. $d_2^{PIS^{Hh}}$	56.1*	0.71⁻	113.23	97.69	10.4	0.8	0
Max. $d_2^{NIS^{HH}}$	0	0.56*	113.23	97.691	10.4	0.8	0

d_2^{*HH} = (56.09709 , 0.5590169) , d_2^{-HH} =(zero, 0.7071067812).

Now, it is easy to compute step (10):

$Maximize\ \delta^{HH-DM_{L_1}}$

Subject to

$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16\ , x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$\left(\dfrac{d_1^{PIS^{HH}}(X)-56.09709}{0.5590169-56.09709}\right) \ge \delta^{HH-DM_{L_1}},$

$\left(\dfrac{zero-d_1^{NIS^{HH}}(X)}{zero-0.7071067812}\right) \ge \delta^{HH-DM_{L_1}},$

$\delta^{HH-DM_{L_1}} \in [0,1].$

The maximum "satisfactory level" ($\delta^{HH-DM_{L_1}}$ =zero) is achieved for the solution x_1^{*HH} =10.38462, x_2^{*HH}=0.7692308, x_3^{*HH}= zero.

(Second Level):

Obtain PIS and NIS payoff tables for problem P^{HH}:

Table 31. PIS payoff table for problem P^{HH}

	$F_{21}^{HH}(X)$	$F_{22}^{HH}(X)$	x_1	x_2	x_3
Max. $F_{21}^{HH}(X)$	45.27*	-6.87	6.3	3.2	0
Max. $F_{22}^{HH}(X)$	−13	17.67*	0	0	1.67

PIS: f^{*HH}=(45.26667, 17.66667)

Table 32. NIS payoff table for problem P^{HH}

	$F_{21}^{HH}(X)$	$F_{22}^{HH}(X)$	x_1	x_2	x_3
Min. $F_{21}^{HH}(X)$	−13⁻	17.67	0	0	1.67
Min $F_{22}^{HH}(X)$	32.6	−13.2⁻	0	3.2	0

NIS: f^{-HL}= (-13,−13.2)

Next, construct equation and obtain the following equations:

$d_p^{PIS^{HH-DM_{L_2}}} = \left[w_1^p \left(\dfrac{113.2308-F_{11}^{HH}(X)}{113.2308-4}\right)^p + w_2^p \left(\dfrac{97.69231-F_{12}^{HH}(X)}{97.69231-(1.8)}\right)^p + w_3^p \left(\dfrac{45.26667-F_{21}^{HH}(X)}{45.26667-(-13)}\right)^p + w_4^p \left(\dfrac{17.66667-F_{22}^{HH}(X)}{17.66667-(-13.2)}\right)^p\right]^{1/p}$

$$d_p^{NIS^{HH-DM_{L_2}}} = \left[w_1^p \left(\frac{F_{11}^{HH}(X)-(4)}{113.2308-4}\right)^p + w_2^p \left(\frac{F_{12}^{HH}(X)-(1.8)}{97.69231-(1.8)}\right)^p + \right.$$
$$\left. w_3^p \left(\frac{F_{21}^{HH}(X)-(-13)}{45.26667-(-13)}\right)^p + w_4^p \left(\frac{F_{22}^{HH}(X)-(-13.2)}{17.66667-(-13.2)}\right)^p\right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = 0.25$ and p=2,

Table 33. PIS payoff table of problem (11) when p=2

	$d_2^{PIS^{HH}}$	$d_2^{NIS^{HH}}$	$F_{21}^{HH}(X)$	$F_{22}^{HH}(X)$	x_1	x_2	x_3
$Min.\ d_2^{PIS^{HH}}$	11.12*	0.35⁻	45.27	-6.87	6.3	3.2	0
$Max.\ d_2^{NIS^{HH}}$	0.04	0.17*	33.92	11.77	10.4	0.8	0

$d_2^{*HH} = (11.12249, 0.1680895)$, $d_2^{-HH} = (0.03948343031, 0.3545651373)$.

Now, it is easy to compute step (10):

Maximize $\delta^{HH-DM_{L_2}}$

Subject to

$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$\left(\frac{d_1^{PIS^{HH}}(X)-11.12249}{0.1680895-11.12249}\right) \ge \delta^{HH-DM_{L_2}}$,

$\left(\frac{0.03948343031-d_1^{NIS^{HH}}(X)}{0.03948343031-0.3545651373}\right) \ge \delta^{HH-DM_{L_2}}$,

$\delta^{HH-DM_{L_2}} \in [0,1]$.

$\left(\frac{(10.38462+0.001)-x_1}{0.001}\right) \ge \delta^{HH-DM_{L_2}}$
$\left(\frac{x_1-(10.38462-0.001)}{0.001}\right) \ge \delta^{HH-DM_{L_2}}$

The maximum "satisfactory level" ($\delta^{HH-DM_{L_2}}$ =zero) is achieved for the solution x_1^{*HH} =10.38462, x_2^{*HH} =0.7692308, x_3^{*HH} = zero.

(Third Level):

Obtain PIS and NIS payoff tables for problem P^{HH}:

Table 34. PIS payoff table for problem P^{HH}

	$F_{31}^{HH}(X)$	$F_{32}^{HH}(X)$	x_1	x_2	x_3
$Max.\ F_{31}^{HH}(X)$	87.54*	66.08	10.4	0.8	0
$Max.\ F_{32}^{HH}(X)$	87.54	66.08*	10.4	0.8	0

PIS: $f^{*HH} = (87.53846, 66.07692)$

Table 35. NIS payoff table for problem P^{HH}

	$F_{31}^{HH}(X)$	$F_{32}^{HH}(X)$	x_1	x_2	x_3
$Min.\ F_{31}^{HH}(X)$	−0.4⁻	33.6	0	3.2	0
$Min.\ F_{32}^{HH}(X)$	7.67	6.3⁻	0	0	1.67

NIS: $f^{-LH} = (-0.4, 6.3333)$

Next, construct equation and obtain the following equations:

$d_p^{PIS^{HH-DM_{L_3}}}$

$$= \left[w_1^p \left(\frac{113.2308-F_{11}^{HH}(X)}{113.2308-4}\right)^p + w_2^p \left(\frac{97.69231-F_{12}^{HH}(X)}{97.69231-(-3.2)}\right)^p + \right.$$
$$w_3^p \left(\frac{45.26667-F_{21}^{HH}(X)}{45.26667-(-13)}\right)^p + w_4^p \left(\frac{17.66667-F_{22}^{HH}(X)}{17.66667-(-13.2)}\right)^p +$$
$$\left. w_5^p \left(\frac{87.53846-F_{31}^{HH}(X)}{87.53846-(-0.4)}\right)^p + w_6^p \left(\frac{66.07692-F_{32}^{HH}(X)}{66.07692-(6.3333)}\right)^p\right]^{1/p}$$

$d_p^{NIS^{HH-DM_{L_3}}}$

$$= \left[w_1^p \left(\frac{F_{11}^{HH}(X)-(4)}{113.2308-4}\right)^p + w_2^p \left(\frac{F_{12}^{HH}(X)-(1.8)}{97.69231-(1.8)}\right)^p + \right.$$
$$w_3^p \left(\frac{F_{21}^{HH}(X)-(-13)}{45.26667-3}\right)^p + w_4^p \left(\frac{F_{22}^{HH}(X)-(-13.2)}{17.66667-(-13.2)}\right)^p +$$
$$\left. w_5^p \left(\frac{F_{31}^{HH}(X)-(-0.4)}{87.53846-(-0.4)}\right)^p + w_6^p \left(\frac{F_{32}^{HH}(X)-(6.3333)}{66.07692-(6.33333)}\right)^p\right]^{1/p}$$

Thus, problem (11) is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = w_5^p = w_6^p = 1/6$ and p=2,

Table 36. PIS payoff table of problem (11) when p=2

	$d_3^{PIS^{HH}}$	$d_3^{NIS^{HH}}$	$F_{31}^{HH}(X)$	$F_{32}^{HH}(X)$	x_1	x_2	x_3
$Min.\ d_3^{PIS^{HH}}$	14.42*	0.4⁻	87.54	66.08	10.4	0.77	0
$Max.\ d_3^{NIS^{HH}}$	0.05	0.9E−01*	87.54	66.08	10.4	0.77	0

$d_3^{*HH} = (14.42384, 0.8510848E-01)$,
$d_3^{-HH} = (0.04546254747, 0.3838009497)$.

Now, it is easy to compute step (10):

$Maximize\ \delta^{HL-DM_{L_3}}$

Subject to

$3x_1 + 5x_2 + x_3 \le 35, 2x_1 - x_2 + 12x_3 \le 20, 5x_2 + 6x_3 \le 16, x_1 \ge 1, x_1, x_2, x_3 \ge 0$

$\left(\frac{d_3^{PIS^{HH}}(X)-14.42384}{0.8510848E-01-14.42384}\right) \ge \delta^{HL-DM_{L_3}}$,

$\left(\frac{0.04546254747-d_3^{NIS^{HH}}(X)}{0.04546254747-0.3838009497}\right) \ge \delta^{HL-DM_{L_3}}$

$\delta^{HL-DM_{L_3}} \in [0,1]$.

$\left(\frac{(0+0.001)-x_3}{0.001}\right) \ge \delta^{HL-DM_{L_3}}$, $\left(\frac{x_3-(0-0.001)}{0.001}\right) \ge \delta^{HL-DM_{L_3}}$

The maximum "satisfactory level" ($\delta^{HL-DM_{L_3}}$ =0.2253846E-06) is achieved for the solution x_1^{*HH} =10.38461, x_2^{*HH} =0.7692309, x_3^{*HH} = 0.9999998E-07.

Note:
Numbers at Table (1) to Table (36) are approximated.
Table (37) presents a comparison among the proposed TOPSIS method, Global Criterion (GC) Method and the ideal objective vector (IOV). In general, the proposed TOPSIS algorithm is a good method to generate compromise solutions (at p=2).

Table 37. Comparison among the proposed algorithm, the GC method and the vector of ideal solutions

Objective		Proposed TOPSIS Algorithm method (p=2)	Global Criterion (GC) Method	Ideal Objective Vector	
				PIS	NIS
P^{LL}	F_1^{LL}	23.230778	5.6678893	24	-2.4
	F_2^{LL}	59.465616	1.0033662	59.4615	4.3
P^{HL}	F_1^{HL}	55.384638	13	55.38462	-1.4
	F_2^{HL}	62.0090232	15	62	5.333
P^{LH}	F_1^{LH}	13	3	13	-3.4
	F_2^{LH}	55	11	58.4615	3.3333
P^{HH}	F_1^{HH}	87.538418	13	87.5384	-0.4
	F_2^{HH}	66.076897	20	66.0769	6.3333

5. CONCLUSIONS

This paper extended TOPSIS approach to find compromise solutions for the multi-level multi-objective decision making problems with rough parameters in the objective functions (RMLMODM). A new hybrid algorithm based on modified TOPSIS method and the "Lower & Upper" approximations method for solving RMLMODM problems is proposed. Also, an illustrative numerical example is solved and compared the compromise solutions of the proposed algorithm with the vector of ideal solutions and the traditional global criterion method. The engineers and the scientists can apply the introduced hybrid algorithm to various practical RMLMODM problems to obtain numerical solutions

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