4.2 Iteration Tikhonov regularization methods

Since the exact solution has good smoothness, so we may assume smoothness index $r=2$, so we can use iterative Tikhonov regularization method to solve this equation, the same rule to use Simpson left discretized integral equations obtained discrete iterative equation

$$x_{\alpha,\delta}^{m}=\sum_{k=0}^{m-1}\alpha^{k}\left[\left(\alpha I+A^{2}\right)^{-1}\right]^{k+1}Ay^{\delta},\ x_{\alpha}^{0,\delta}=0 \qquad (10)$$

where $y^{\delta}:=\left(y_i\right)^{\delta}$ is disturbed and discreted right hand side and satisfy $y_i=\exp(i/n)$

$$\left\|y-y^{\delta}\right\|=\sqrt{\frac{1}{n+1}\sum_{i=0}^{n}\left(y_i-y_i^{\delta}\right)^{2}}\leq\delta$$

The figure 6 will shows us the error figure when the regularization parameter $\alpha=0.001$ iteration parameter $m=1,2,3,4,5,6,7,8,9,100,200$, 300, 500, and the error $\delta=0.0001,0.001,0.01,0.1$.

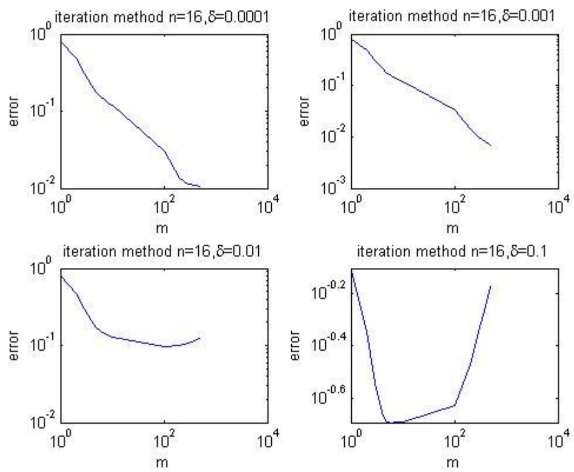

Figure 6. n=16 iteration methods error figure

From the figure 6, when the regularization parameter takes a fixed value, error estimation with the parameter m increases and then decreases to an optimal value and then increases, which fully reflects the positive effect of the iteration parameters. By comparing the obtained optimal convergence precision, iterative Tikhonov regularization method is equal to the higher than normal Tikhonov regularization method. This shows that the iterative Tikhonov regularization method selection parameter is more convenient to calculate faster, more accurate calculation, better stability advantages.

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