

SHAPE OPTIMIZATION THE EASY WAY: THE ‘METHOD OF TENSILE TRIANGLES’

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ABSTRACT

A method based on the design rules of nature, which has been used previously for the reduction of notch stresses, is used here for the removal of underloaded parts in a mechanical component. A major advantage is that neither finite element analysis nor complex mathematics is necessary for this graphic method of shape optimization, which is demonstrated here using three examples.

Keywords: biomimetics, cave bear, graphic method, notch stress, shape optimization, tooth.

1 INTRODUCTION

Since the late 1980s, at the Forschungszentrum Karlsruhe GmbH (Research Center Karlsruhe), the computer simulation of load adaptive tree and bone growth has been used to shape-optimize mechanical components. Trees grow thicker tree rings at overloaded points to restore a disturbed uniform stress state. Bones also grow at overloaded points, but unlike trees underloaded parts of the structure are removed. These mechanisms have been modeled using finite element method (FEM)-based computer simulations called computer aided optimization (CAO) [1] and soft kill option (SKO) [2]. A summary of the conclusions drawn from the major mechanical design rules of nature, which we call ‘axiom of uniform stress’, has been given by Mattheck [3].

In 2003, a significant simplification of the CAO method was possible due to a new understanding of the nature of notch stresses, which are bending stresses [4, 5]. This process of notch shape optimization does not require the use of FEM and can be accomplished using a pocket calculator.

In late 2005, a pure graphic method, the ‘method of tensile triangles’, was derived by Mattheck. In a study of the shoulder fillet under tensile loading (Fig. 1), potential slip planes and shear stresses due to an obvious longitudinal shear result in equivalent tensile stresses. This results in principle tensile stresses that are tilted by about 45° to the beam axis near the notch. When we look at the foot of a tree’s trunk (see Fig. 2), a 45° tension rope is found to extend along the upper side of the root. When applied to technical components, an existing notch is bridged by a tensile triangle, as shown in Fig. 2; one notch creates two less dangerous notches of larger angles. Under a uniaxial stress state, as in Fig. 3A, there is only one notch, which can also be bridged by a tensile triangle. The repetition of this procedure is the ‘method of tensile triangles’. In the last step, the segments have to be rounded to obtain an optimized notch shape. Figure 3B illustrates the method proposed by Mattheck and was verified for a shoulder fillet with the same radial space [6–8]. This FEM calculation and all the other calculations in this paper are all done using two-dimensional plane stress elements with the same mesh density. The display variable is the von Mises stress, which is the comparison stress used for dimensioning in industries.

In [6, 7] Mattheck provides more examples of bi-axial loading to reduce notch stresses using the ‘method of tensile triangles’.

The goal of this paper is to demonstrate that the ‘method of tensile triangles’ can also be used to remove underloaded parts in a design proposal.

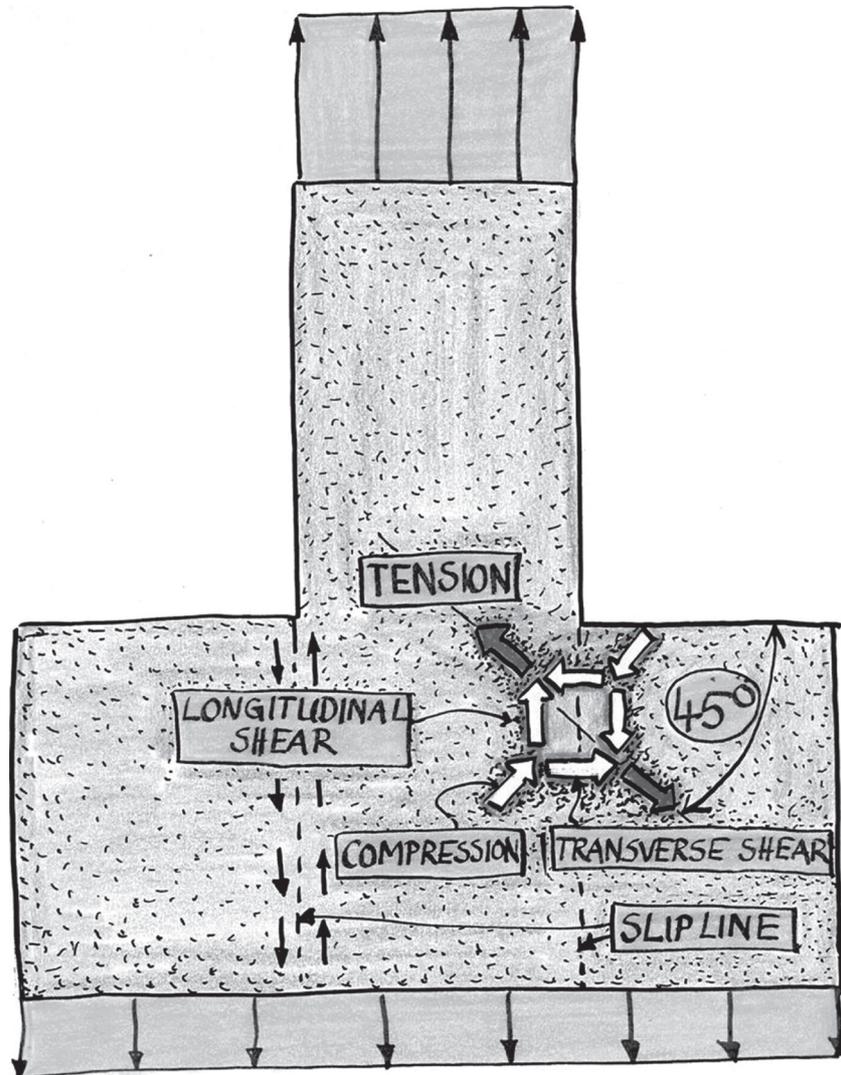


Figure 1: The two plausible counteracting pairs of shear forces and the related principle tensile stresses show that a 45° angle is optimum for the lower end of the notch contour line. If the undisturbed notch end is parallel to the force flow, no notch stress will be caused. Therefore, the lowest kink will not be smoothed in case of uniaxial tension.

2 LOAD-CONTROLLED GROWTH AND SHRINKING BY USE OF THE GRAPHIC 'METHOD OF TENSILE TRIANGLES'

2.1 Shoulder fillet

The shoulder fillet in Fig. 3B, optimized by means of the 'method of tensile triangles', is characterized by a very homogeneous stress along the notch contour. Now, assuming that there is no locally high stress maximum, the stress along the whole shoulder is not homogeneous, because the fillet corner

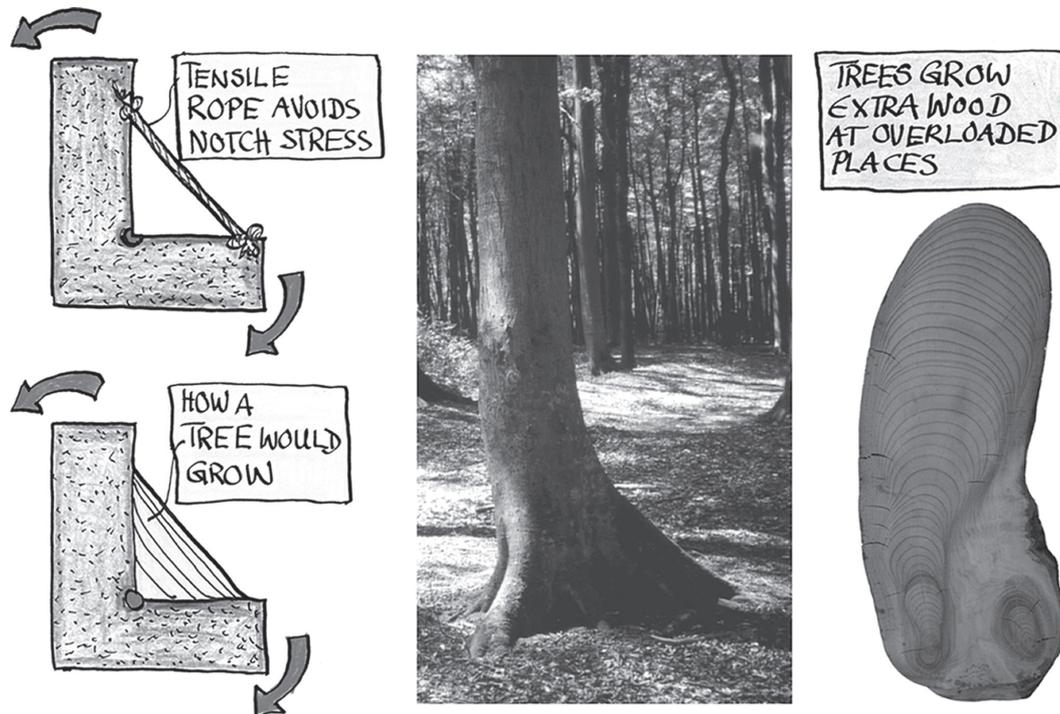


Figure 2: The buttress root bridging the kink at the base of a tree, like a rope.

is only subjected to low loads. So if there is no functional need for the corner, it can be removed to save weight. The transition point where growth switches to shrinking is, in engineering practice, given by functional boundary conditions. Figure 4 shows a drastic reduction in the notch stress peak as well as underloading in comparison with the non-optimized design. Thus, the ‘method of tensile triangles’ can be applied to minimize both local overloading and local underloading.

2.2 Bending bar of uniform stress

This example is known from many textbooks of mechanics [9]. Requiring a uniform distribution of bending stresses on the upper contour line, one obtains an analytical formula ($h(x) = k \times \sqrt{x/l}$). The height is a function of the bar length shown in Fig. 5B. Except the stress peaks due to clamping and single-point load, the stress plot is uniform along the surface of the ‘textbook’ shape. In Fig. 5C, the stress plot for a bar optimized by the ‘method of tensile triangles’ is shown. For the sake of comparison, we used a height to length ratio equivalent to three tensile triangles. The shape of the graphic solution is close to the square root shape. Good improvements are found for the mathematic and also for the graphic solution in comparison with the non-optimized bar in Fig. 5A. The diagram shows the stress over the shape contour. The stress is standardized to the theoretical bending stress (force \times arm/cross section).

2.3 Related concave and convex shapes

In Fig. 6A, a fork is optimized and the concave shape is found to be optimum in a similar way as in Fig. 5C for the related convex shape. This explains that, for example, the tooth of a cave bear has

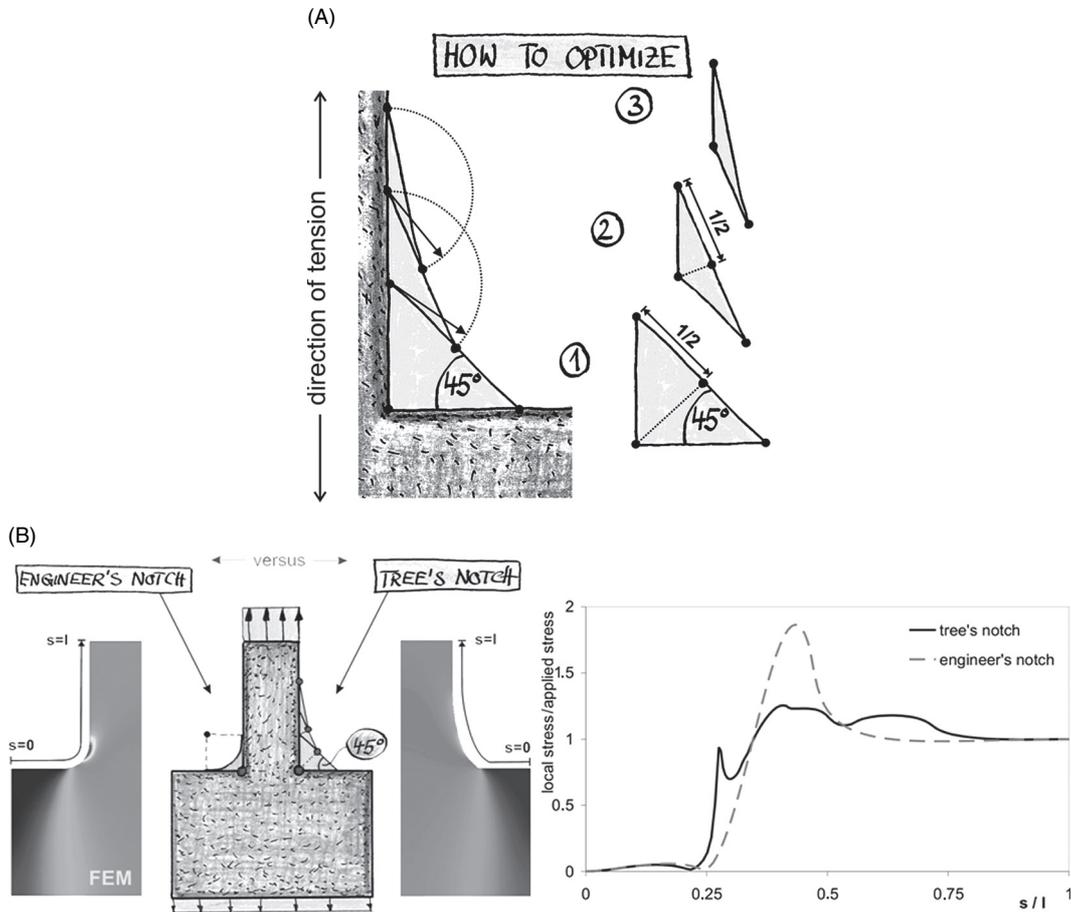


Figure 3: (A) Principle of the ‘method of tensile triangles’ for uniaxial tension. Finally, the remaining kinks are rounded by circles. (B) Stresses at the shoulder fillet rounding with a circular notch (left) and ‘method of tensile triangles’ (right) with the same radial design space.

a good shape fit in the surrounding bone, because its optimum convex shape is equal to the concave shape of the deepest part of the bony notch (Fig. 7). A portion more than the lower half of the bear’s tooth is fixed in the jaw. The tooth is loaded in many directions, but the main direction is assumed as shown in Fig. 7.

3 CONCLUSIONS

The ‘method of tensile triangles’ is an extremely simple graphic method to reduce notch stresses and enlarge the lifetime based on fatigue behavior. On the other hand, this new graphic method can also shrink away underloaded parts of a structure and save weight. Both options tend to provide a stress state according to the ‘*axiom of uniform stress*’, which is one of the most important design rules of nature [3].

The optimum shape designed using the ‘method of tensile triangles’ can be scaled up and down for different dimensions and loadings. It might be a ‘universal’ notch shape design bordered by

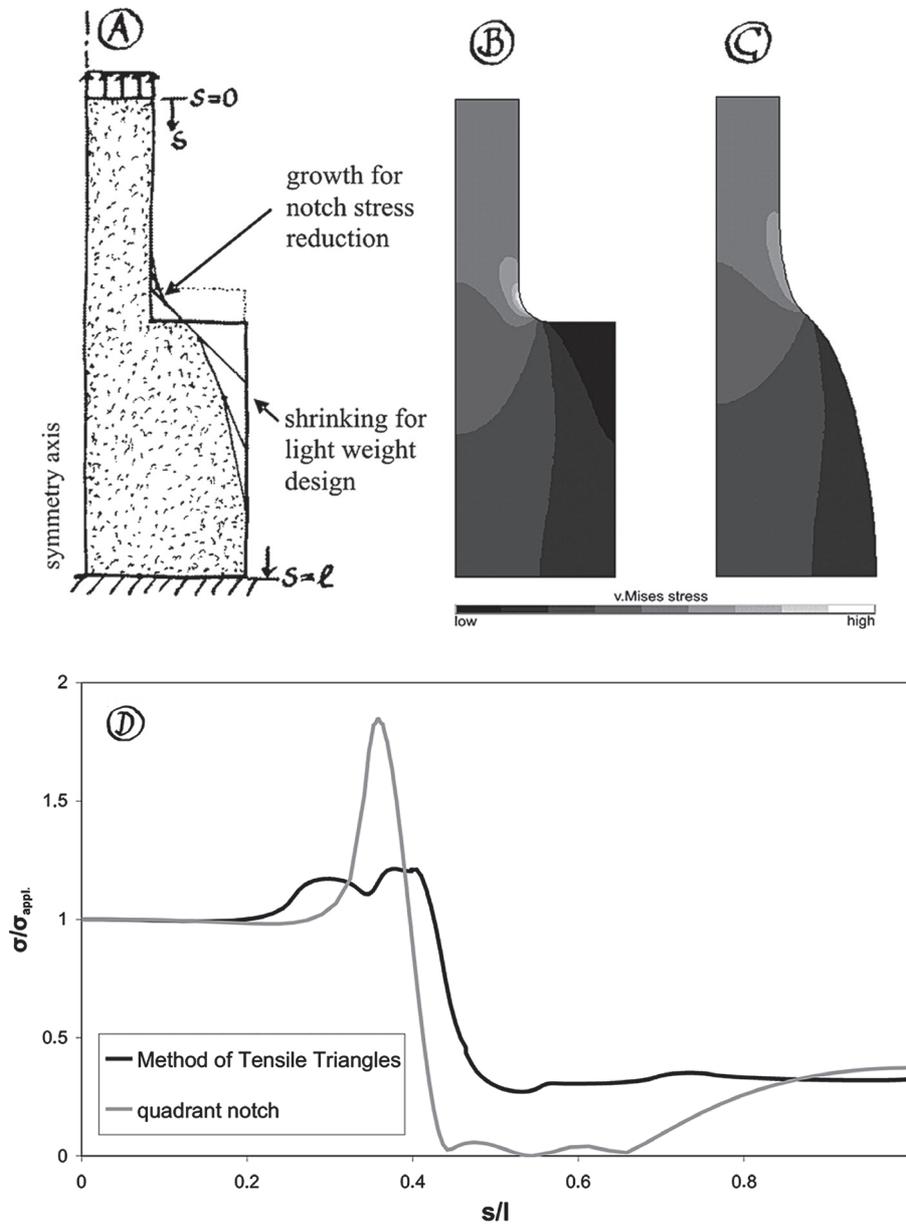


Figure 4: A shoulder fillet under tensile loading is optimized by use of the ‘method of tensile triangles’ in comparison with a classic circular engineering notch. (A) How to do it – the remaining kinks are smoothed by circles. (B) von Mises stress plot of non-optimized shape. (C) von Mises stress plot of smoothed optimized shape. (D) von Mises stress profile along contour.

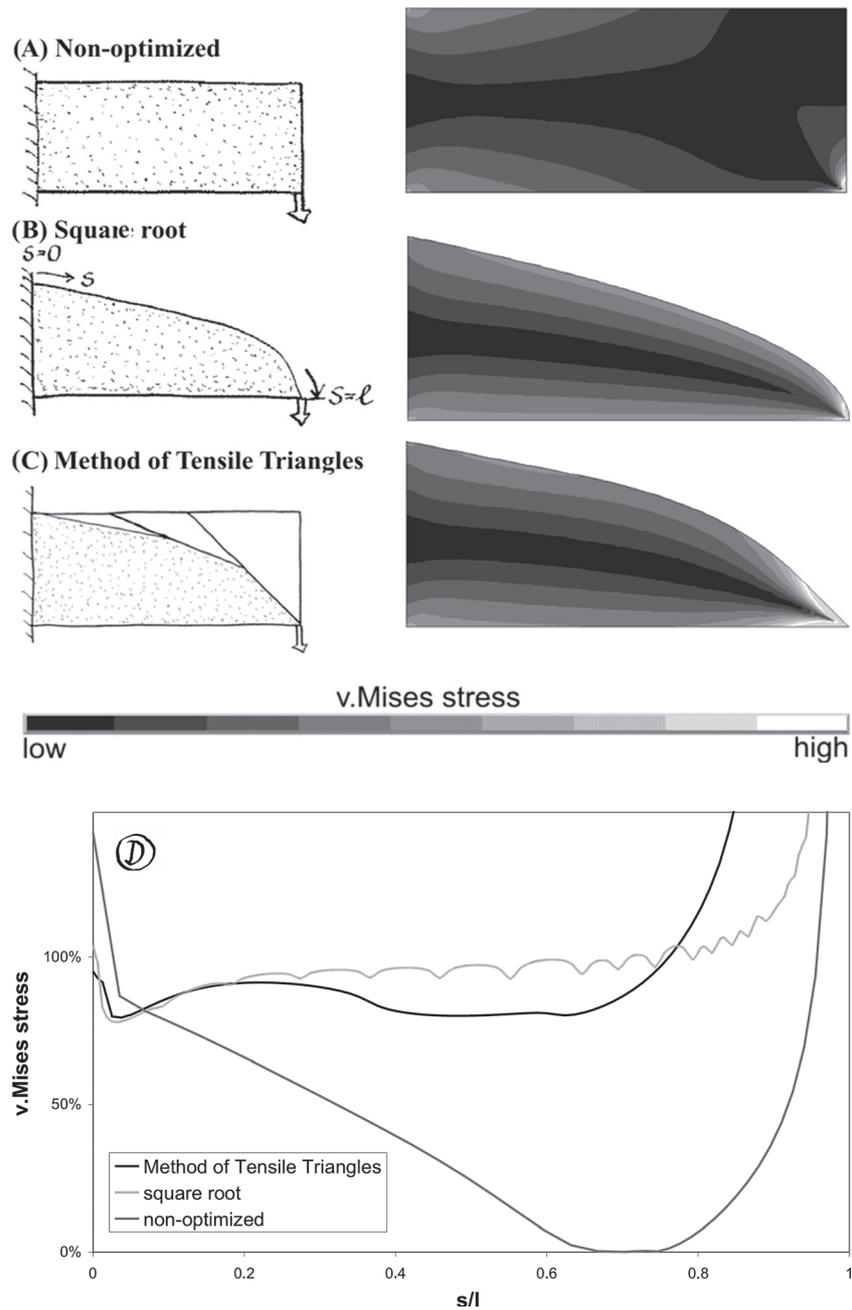


Figure 5: FEM–von Mises stress plots of a non-optimized rectangular bending bar, a square root bar, and a ‘method of tensile triangles’ bar. (A) High underloaded spots in the rectangular bar. (B) The square root shaped bending bar according to mechanical textbooks. (C) Bending bar obtained by application of the ‘method of tensile triangles’. The remaining kinks are smoothed. (D) von Mises stress profile along contour (stress peaks to the right are due to single load).

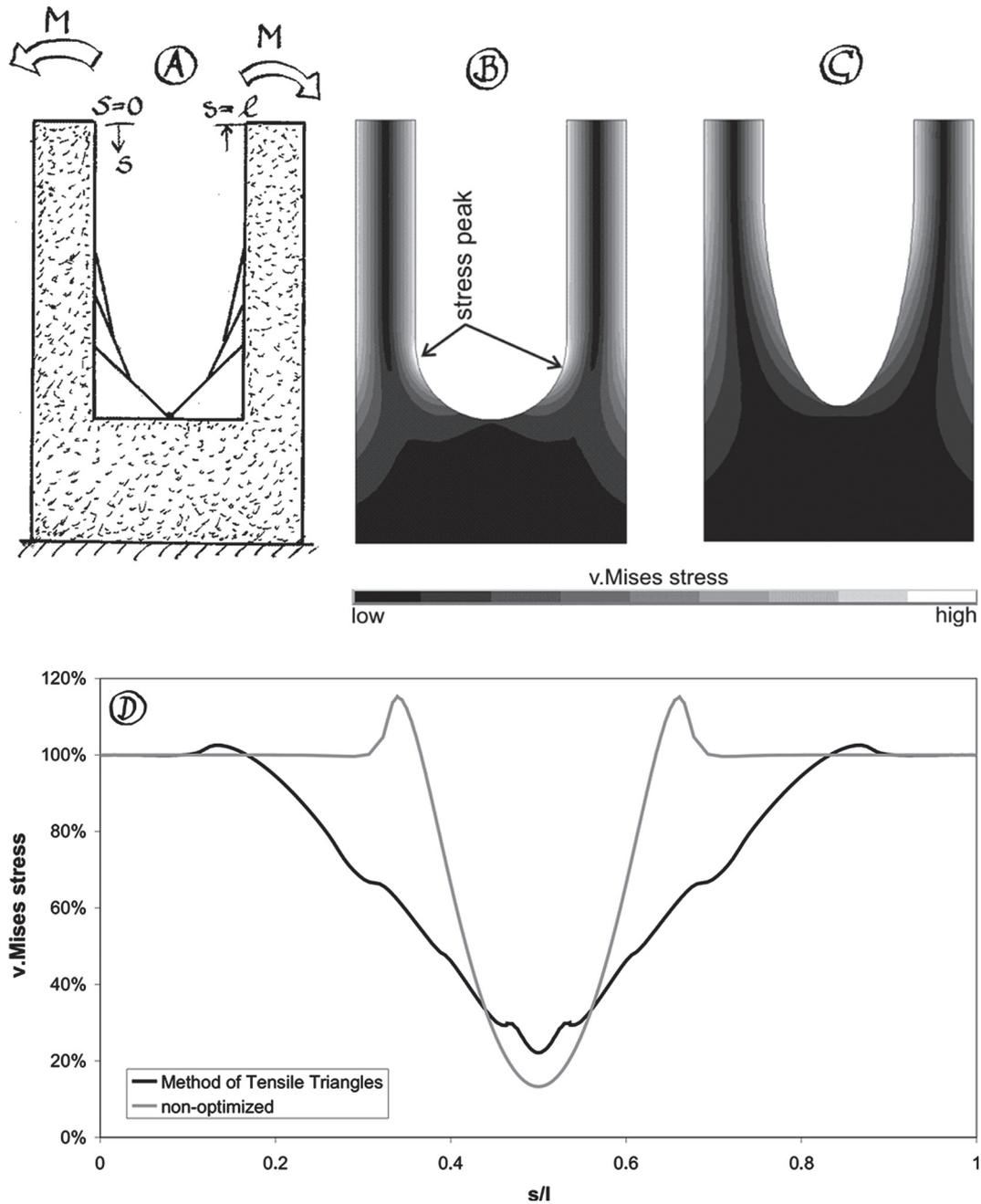


Figure 6: If this kind of fork is optimized using the ‘method of tensile triangles’, the concave internal shape has nearly no notch stresses anymore compared with the non-optimized design. (A) How to do it – the remaining kinks are smoothed by circles. (B) von Mises stress plot of the non-optimized shape. (C) von Mises stress plot of the smoothed optimized shape. (D) von Mises stress profile along contour.

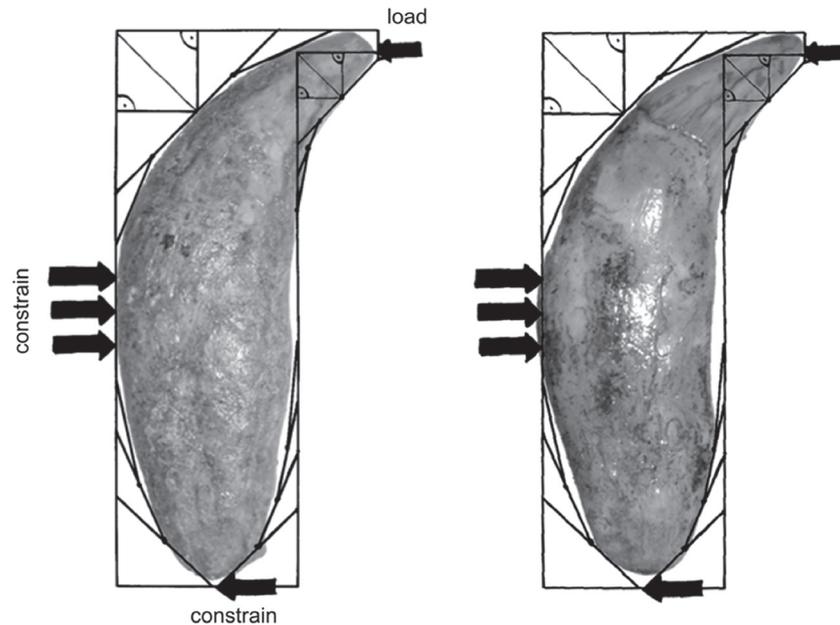


Figure 7: The tooth of a cave bear (*Ursus spelaeus*) can be modeled from a rectangular hook design proposal. Two teeth from different bears are shown. According to Fig. 6, the convex shape of the tooth is optimum in a similar way as the concave notch shape of the surrounding bone embedding.

limitations that are not known yet. Variations of the ‘method of tensile triangles’ have also been tested. For example, the next triangle does not start in the middle of the previous triangle’s baseline but at its quarter point. However, the authors feel that the method presented here is well balanced in terms of simplicity of the procedure and the benefits to design.

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