COLOURS OF NOISE FRACTALS AND APPLICATIONS

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ABSTRACT
Due to uncertainty in it, noise prevents exact prediction of the future from the past. Noise is generally described by spectral densities of certain functional dependence on frequency. Years of research revealed relations between natural phenomena and noise spectral distributions of either man-made or natural sources of different spectral density signal content. However, since many random functions of noise appear in nature and in technology in power spectra and power law relations, certain categories of noise spectral density distributions are generally described as powers of frequency¸ being grouped in such a way that each one represents certain specific natural and man-made phenomena. On the other hand, most of the natural phenomena have fractal dimensions that combine together spectral behaviour that occurs in reality as can be seen by measurements results. The paper shows these functional descriptions of noise in terms of colours and their combination with fractals theories, which enable development of advanced technologies
Keywords: Frequency dependent noise, stochastic fractals and noise, applications.

'I ascribe to nature neither beauty, deformity, order, nor confusion. It is only from the view point of our imagination that we say that things are beautiful or unsightly, orderly or chaotic'.

Baruch Spinoza, 1665

1 DETERMINISM, UNCERTAINTY AND LIFE

Noise is a basic entity in the essence of life as explained in [1]. 'Deterministic life' may become impossible for human beings, since it means clear knowledge of the future, including unavoidable disasters, with no hope for a change. On the other hand, uncertainty encourages going on with life, trying to improve and maintain current states, accomplish targets and keep an amount of optimism.

Noise appears in all areas of science and technology, and it is involved with everything in life. It can be described by suitable mathematical formulation that illustrates different topics in an abstract way and leads to general conclusions. An important feature is that there are many kinds of noise which differ by their dependence on frequency. Each such dependence leads to different phenomena in nature and different applications.

2 EXAMPLES OF FREQUENCY DEPENDENCE OF NOISE

2.1 Ambient noise in the sea

While signals are desirable sounds, in the case of ambient sounds, noise is undesired. Different kinds of noise in the sea, with different spectra, have been formulated in the past as a consequence of a huge amount of research. This formulation includes functional relations such as white noise, pink noise and tonal components.

A pioneering research was done by Knudsen *et al.* [2]; see Fig. 1. They obtained the following relation for sea surface agitation at 1–100 kHz:

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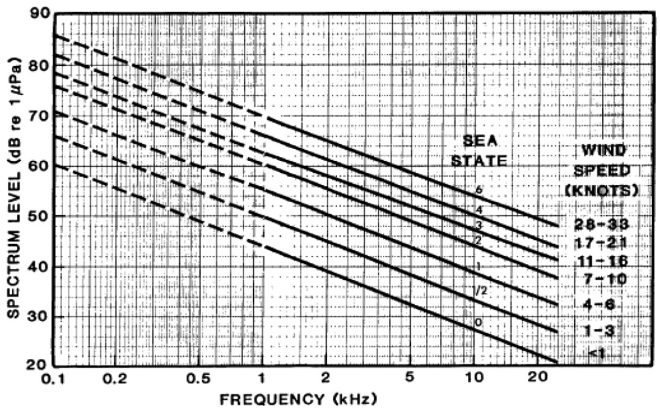


Figure 1: Knudsen *et al*. [2] curves of underwater noise.

$$SPL_{surf} = \begin{cases} SPL_{1k} & for\ f < 1000\,Hz \\ SPL_{1k} - 17\,\lg_{10}\left\{\dfrac{f}{1000}\right\} & for\ f > 1000\,Hz \end{cases} \tag{1}$$

SPL_{1k} is a constant that depends on the sea state. f – frequency (Hz)

Mellen [3] obtained through classical statistical mechanics, theoretical formulae for the ambient ocean noise at the ultrasonic band f > 50 kHz, which is due to thermal noise. The result for the thermal noise is:

$$NL = -15 + 20\,\lg_{10}(f);\ dB\ re\ 1\mu Pa;\ The\ frequency\ f\ in\ kHz. \tag{2}$$

In general, as Wenz [4] has shown, the ambient noise consists of at least three constituents: turbulent pressure fluctuations (1–100 Hz), wind dependent noise due to bubbles and spray that result from surface agitation (50 Hz to 20 kHz) and density-dependent ocean traffic (100–100 Hz). See Fig. 2. Using the graph, the relevant components can be added. Yet, the overall dependence of noise on frequency is clearly observed. The 'rule of fives' of Wenz considers both frequency and wind speed on noise level is:

$$NL(f,U) = 25 - \frac{5}{3}\times 10\,[\lg_{10}(f) - \lg_{10}(\frac{U}{5})] \tag{3}$$

It is very important to note that the amalgamated observation of the ambient noise reveals a similarity structure, in both the acoustical and wind dependency, as Kerman [5] has shown.

This topic is reviewed, for example, by Urick [6] and Carey & Richard [7].

2.2 Illustration of the effect of frequency shift of noise spectra on annoyance; see Rosenhouse [8]

Annoyance by noise depends strongly on its informative, spectral contents and individual effect on people. Yet, standards dictate certain formal limitations, ignoring such details. In practice it happens in many cases of recreational areas, industrial premises and other kinds of activities, that even when

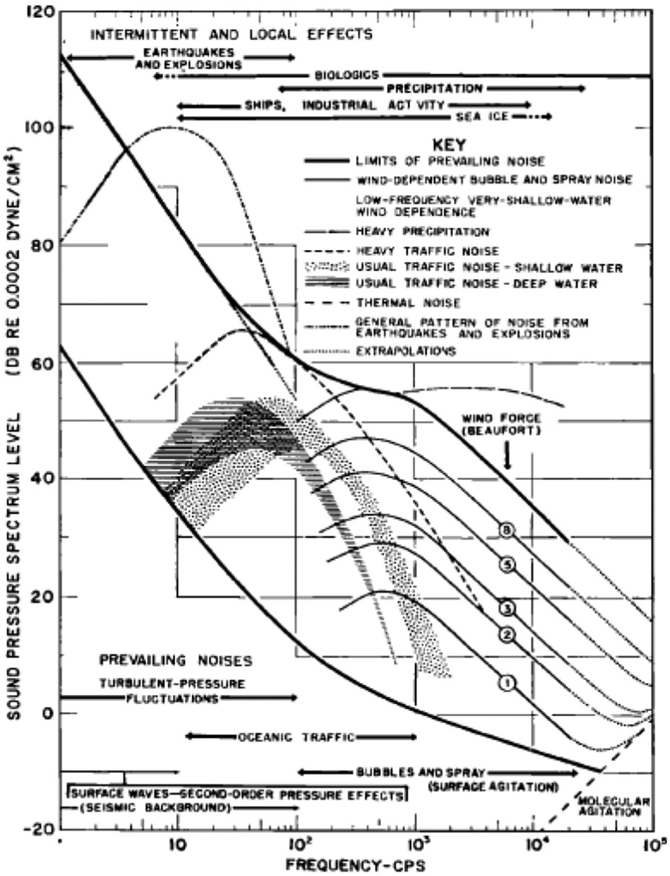
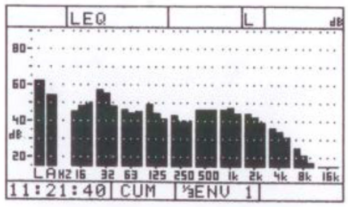


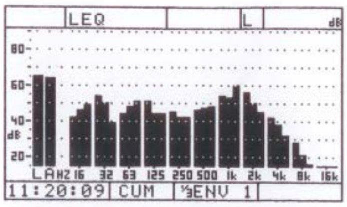
Figure 2: Composite ambient noise spectra — Wenz [4].

results of measurements satisfy the standards limits, complaints do not stop, often involving threats of legal acts. The following case study of the effect, based on actual acoustic measurements, shows factors that cause extreme sensitivity to certain noise patterns, even if the total amount of noise remains unchanged. The effect of noise colour difference is enhanced if the added noise has a certain periodicity, located where the background noise has lower masking effect. Since in many cases the background noise has less annoyance effect or resembles white or pink noise, other noise sources of different spectra can be clearly heard, if they include local amplitudes in the frequency domain that are higher than the background noise.

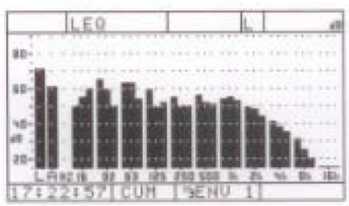
The following example is of neighbours' complaints about intruding noise coming from a complex of swimming pools. The spectra of different cases of noise are given in Fig. 3. The top figure in Fig. 3 is a spectrum of noise without children's shouts from a nearby swimming pool, where lower frequencies are dominant. The neighbours did not complain about that. Almost the same happens when transportation noise is included but the lower frequencies became more dominant, but neighbours did not complain about that. See bottom figure in Fig. 3. The spectrum in the middle figure is for the case where children's shouts were added. Those shouts caused a dominant effect at higher frequencies and this was the problem that needed to be solved.



OVERALL	->	Lin	62.6dB	'A'	54.6dB
Hz	- Lin	Hz	- Lin	Hz	- Lin
12	45.5dB	16	48.6dB	20	50.9dB
25	57.5dB	32	55.1dB	40	48.8dB
50	46.0dB	63	44.9dB	80	45.7dB
100	49.6dB	125	44.1dB	160	41.8dB
200	43.9dB	250	40.8dB	315	40.7dB
400	46.6dB	500	46.5dB	630	46.6dB
800	46.7dB	1k	47.6dB	1k25	44.0dB
1k6	44.9dB	2k	42.8dB	2k5	39.9dB
3k15	36.6dB	4k	34.7dB	5k	31.4dB
6k3	25.8dB	8k	21.6dB	10k	17.1dB
12k5	---.-dB	16k	---.-dB	20k	---.-dB



OVERALL	->	Lin	65.4dB	'A'	64.1dB
Hz	- Lin	Hz	- Lin	Hz	- Lin
12	42.0dB	16	46.3dB	20	49.8dB
25	54.6dB	32	50.7dB	40	39.8dB
50	44.1dB	63	48.3dB	80	51.9dB
100	51.0dB	125	44.3dB	160	44.8dB
200	45.7dB	250	42.0dB	315	42.8dB
400	46.8dB	500	47.1dB	630	48.0dB
800	54.8dB	1k	53.8dB	1k25	60.5dB
1k6	56.9dB	2k	50.0dB	2k5	45.0dB
3k15	42.2dB	4k	38.6dB	5k	32.4dB
6k3	28.1dB	8k	21.7dB	10k	16.5dB
12k5	---.-dB	16k	---.-dB	20k	---.-dB



OVERALL	->	Lin	71.5dB	'A'	61.7dB
Hz	- Lin	Hz	- Lin	Hz	- Lin
12	50.1dB	16	55.6dB	20	60.4dB
25	65.3dB	32	59.9dB	40	50.4dB
50	63.1dB	63	63.3dB	80	54.2dB
100	59.5dB	125	49.7dB	160	52.2dB
200	55.4dB	250	50.9dB	315	50.1dB
400	56.2dB	500	52.5dB	630	51.7dB
800	54.6dB	1k	55.6dB	1k25	53.6dB
1k6	49.5dB	2k	48.5dB	2k5	44.8dB
3k15	41.5dB	4k	38.6dB	5k	35.8dB
6k3	31.9dB	8k	25.7dB	10k	20.1dB
12k5	---.-dB	16k	---.-dB	20k	---.-dB

Figure 3: The effect of noise spectra of a swimming pools complex on residential areas. Top part: No children's shouts and no transportation noise. Middle part: Children's shouts added. Lower part: Transportation noise added.

Acoustic solutions include means for undesirable noise reduction to levels much below the background noise, by as much as 9 decibels, to allow background noise masking of disturbing sources. Such reduction alters the intruding noise status from being strongly heard at the privacy zone.

3 COLOURS OF NOISE

The noise scales were discovered by Johnson [9], where the random process S (f) is given by the following general dependence on frequency.

$$S(f) = \frac{C}{f^{\alpha}}; \text{ C is a constant. f is the frequency in Hertz.} \tag{4}$$

Following this definition, research has shown later that many phenomena in physics (including astrophysics and geophysics), biology, technology, speech and music, economics and psychology are involved with the noise range $0.5 \le \alpha \le 1.5$ as will be shown later.

A form of a white noise is Johnson's noise (Johnson [10]) which results from a thermal excitation of electrical components, such as a resistor. The noise is caused by the motion of the charge of atoms from which the resistor is built. The thermal excitation is caused by heat, and thus, when the resistor

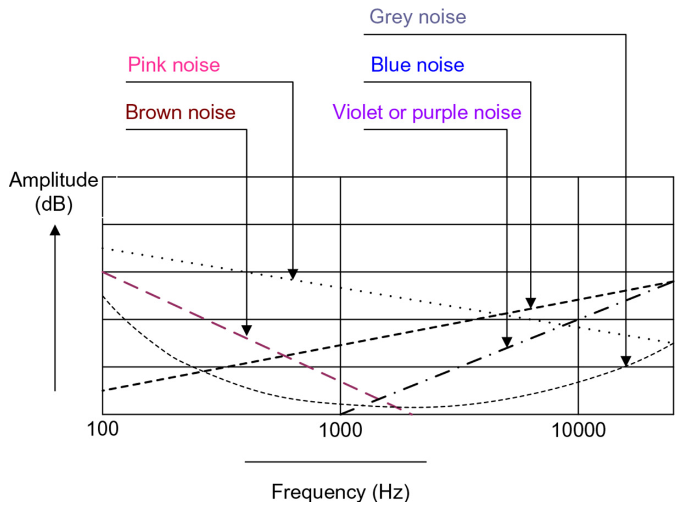


Figure 4: A scheme of common colours of noise. White noise is represented by a horizontal line.

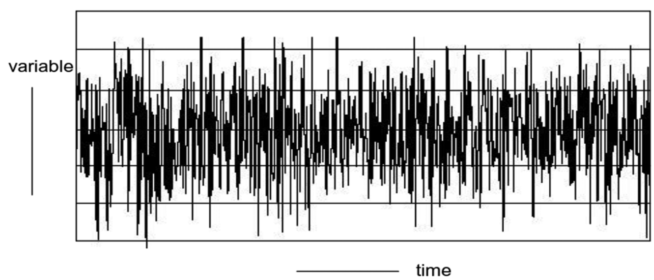


Figure 5: A typical plot of white noise.

gets hotter it becomes noisier. Many tests of electrical equipment and components are using several colours: white noise, pink or brown, and researchers and laboratories have, for many decades, been applying various standard noise colours for different acoustic measurements. The different colours (white which is proportional to $1/f^0$, pink which is proportional to $1/f^1$, brown which is proportional to $1/f^2$ and others) have different coefficients, α, as will be discussed later, and the power spectra being a plot of a square magnitude of the Fourier transform against log frequency as shown in Fig. 4.

3.1 White noise

White noise is proportional to one. See Fig. 5.

$$S(f) = \frac{C}{f^{\alpha}};\ \alpha = 0 \therefore S(f) = \frac{C}{f^{0}},\ \text{C is a constant} \tag{5}$$

'White noise' is a random signal of no correlation at two difference times. Its power spectral density is flat over the whole range of frequency bands. Theoretically, the frequency band of white noise is infinite and as a result carries an infinite power which, also, does not depend on time. Acoustic white noise is analogous to optical 'white light', which also consists of a flat power spectrum that has equal quantities of intensity at all the frequencies within the visible light. Analogously, acoustic white noise contains equal quantities of intensity at all the audible frequencies. In this case, each amplitude has the same chance to be at a certain point as any other amplitude has. Thus, it is not difficult to create white noise by using a program that generates stochastic signals of the size −0.5 to 0.5.

White light includes all the visible colours equally, and as a result it is practically a broad band noise. White noise does not necessarily represent each phenomenon that occurs in nature. Frequently, there is interest in Gaussian distribution of the acoustic power, where most of its values are close to zero. Frequently, there is also interest in values that range between −0.5 and 0.5, as the Gaussian distribution gives.

While a Gaussian process is a stochastic process, for which any finite combination of samples will be normally distributed, many other kinds of possible probability density distributions that are available suit other applications, such as Laplace and Cauchy probability density functions. A major difference between distributions is the thickness of their 'tails'. There is a chance that a thicker tail will be responsible for more extreme events (Rosenhouse [11]) and the larger the Gaussian bell for the same average of noise distribution (e.g. the noise of aircrafts take off measured at a control point near an airport) the noisier the extreme cases.

Gaussian White Noise (GWN)–The Gaussian distribution has a mean value (m) of a certain value that is statistically calculated, as well as the standard deviation (σ). In the case of Gaussian white noise the mean or the expected value is zero: $m = E(x) = \int_{-\infty}^{\infty} \left[x\ p(x) \right] dx = 0$, and the mean standard deviation, σ of the pseudo random sequence becomes: $\sigma = \sqrt{E(x-m)^2} = \sqrt{E(x)^2}$, for m=0. x is a random variable defined on a probability space. p (x) is the probability density function (pdf). If x is a discrete random variable, then a probability mass function (pmf), f(x) is used and:

$$E(x) = \sum_i \left[x\ p(x) \right] dx = 0;\ \sigma = \sqrt{E(x-m)^2} = \sqrt{E(x)^2}\text{, for m = 0.} \tag{6}$$

GWN provides simulation of real situations in our world. It is often used as a source of numbers generator due to its independent statistical features.

An additive white Gaussian noise (AWGN) channel is regularly used in communication.

The input signal s (t) is contaminated by noise, n (t), that can be AWGN, during its transmission from the source location to the receiver, and it becomes distorted in amplitude and phase, including a varying time delay. The result is the output:

$$y(t) = s(t) + n(t) \tag{7}$$

3.2 Pink noise

Pink noise depends on the ratio 1/f. Its bandwidth ranges up to 20 kHz. See Fig. 6:

$$S(f) = \frac{C}{f^a};\ a = 1$$

$$\therefore S(f) = \frac{C}{f}\ \text{ C is a constant} \tag{8}$$

Examples in nature include brain EEG, brain MEG, human heart sound, human time estimation, squid giant axon, vacuum tube and semiconductor noise, music and natural sounds.

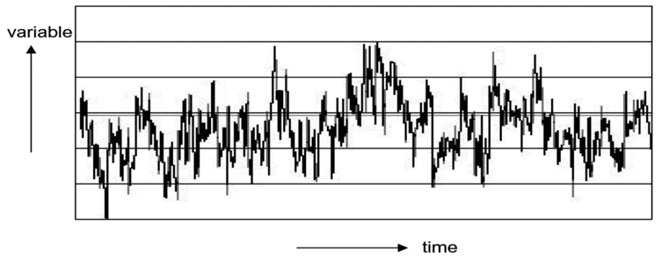


Figure 6: A typical plot of pink noise.

As a result of this dependence, pink noise has a uniform distribution along the logarithmic frequency scale, which means a flat spectral density as a function of the percentage of the band width–or an equal power for each octave band. It means that pink noise is a frequency spectrum with an intensity that decays approximately at the rate of 3 dB per octave (or 10 dB per each decade). Practically, passing white noise through a filter of 3 dB per octave intensity decay yields pink noise.

Pink noise has applications in sound and audio systems and tests. Since pink noise is perceived as more natural for the human ear, it is very popular in investigation of tests in building and environmental acoustics. This application has advantages, since 1/3rd octave bands suit humans' ability to discriminate irregularities of the frequency response, and also since measurement of 1/3rd octave bands smoothens many very narrow peaks.

The flicker noise, which is 1/f noise, is very close to pink noise. The difference is that the flicker noise is limited to 2 kHz. It is used in solid state physics and it describes noise radiated from defects along the wave-guide channel, as a result of base currents in conductors (Barnes and Allan [12]).

3.3 Brown or Brownian noise

Brown noise is the integral of white noise. See Fig. 7. Its definition is:

$$S(f) = \frac{C}{f^{\alpha}};\ \alpha = 2$$

$$\therefore S(f) = \frac{C}{f^{2}} \text{ C is a constant} \tag{9}$$

If H = 0.5 then Brownian motion occurs, with independent and uncorrelated increments. H > 0.5 yields smooth curves and positive correlated increments. H < 0.5 results in erratic rough curves and increments with negative correlation.

Each point along the route of Brownian motion depends on the value of the previous point. Each new point is moved stochastically a little bit from the previous state due to a 'random walk', a term originally coined by Pearson [13]. Brown noise is generated by addition of a random number to the previous value.

Equation 9 shows that brown noise expresses a drop of 20 dB per decade or 6 dB per octave band. That drop means more energy at the lower frequencies and a roar of low frequencies. It is heard also as the fall of water in water cascades or heavy rain.

As it is, Brownian noise can be heard if it is acoustic, but in general it carries a mathematical statistical form of many aspects which makes it universal, including thermal fluctuations that have proven the existence of molecules, life, earth, and environmental sciences, as well as stock market behaviour, being in general responsible for dynamical life sustaining processes under the same title of Brownian noise.

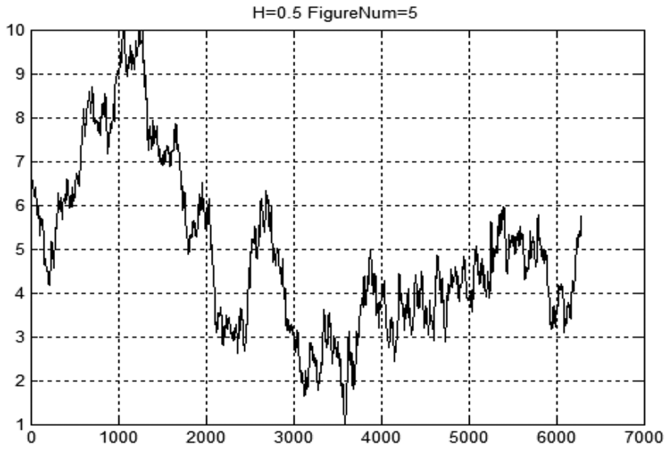


Figure 7: An example of Brownian noise–Hurst parameter = 0.5–performed by Daphne Sobolev.

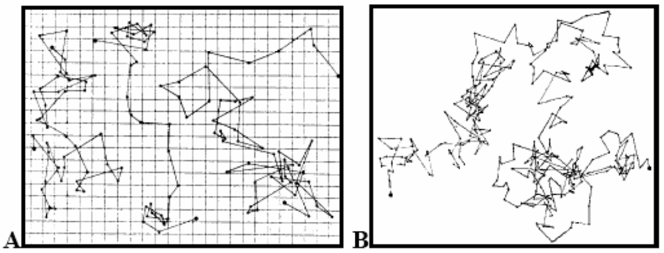


Figure 8: Brownian motion as measured by Perrin (1909).

The situation where stochastic events, such as pollen movement in fluids (see Fig. 8) and their collective behaviour can be quantitatively given in terms of probability and statistics, leads to the general mathematical theory of the 'random walk'. This theory has uses in many areas, such as molecular biology, wireless nets and changes in the stock market.

The mathematical formality of describing Brownian motion (BM) can be modelled by a simple description as follows:

The position of a particle **x**(t) at the time t is a result of a random process. The change of location in each step leads to a new location as follows:

$$\mathrm{x}(t+\Delta t)=\mathrm{x}(t)+\mathrm{v}\,\Delta t^{0.5}N(0,1) \tag{10}$$

v is the average speed of a particle and N(0,1) is a normal randomly distributed variable. The increment $\mathbf{x}(t_2)-\mathbf{x}(t_1)$ has a Gaussian distribution, with the average E and the variance = σ^2 (σ – standard deviation) properties as follows:

$$E\left[x(t_2)-x(t_1)\right]=0;\,Var\left[x(t_2)-x(t_1)\right]\propto\left|t_2-t_1\right|. \tag{11}$$

The position of a particle $\mathbf{x}(t)$ is continuous, but not differentiable. The increments: $x(t_0+t)-x(t_0)$ and $\frac{\left[x(t_0+rt)-x(t_0)\right]}{r^{0.5}}$ are statistical self-similar. Also: If $t_0 = 0$, $x(t_0) = 0$, then, $x(t)$ and $\frac{x(rt)}{r^{0.5}}$ are statistically equivalent. These last assumptions allow for development of the algorithm that generates 1D Brownian motion.

The fractional Brownian motion (fBm) is a continuous Gaussian process $B_H(t)$, on [0,T]. It starts at zero and has zero expectation for all t's in this domain. In fact it is an extension of the Brownian motion (Bm), which carries the form:

$$\mathrm{var}\left[x(t_2)-x(t_1)\right]\propto\left|t_2-t_1\right|^{2H} \tag{12}$$

$x(t)\ and\ r^H\ x(rt)$ are statistically self-similar with respect to the Hurst parameter (or index), which is a real number associated with fBm. Thus, the fBm is properly rescaled by dividing the amplitudes by r^H.

$0\le H\le 1$ determines the roughness of the curve. H, which is associated with fBm, is a real number named after Hurst (Hurst, [1951]), and it was introduced by Mandelbrot and van Ness [1968]. The increment $\Delta B_H(t)=B_H(t-1)-B_H(t)$ is the fractional Brownian noise.

The fBm satisfies the following covariance function:

$$E\left[B_H(t)B_H(t_1)\right]=\frac{1}{2}\left[\left|t\right|^{2H}+\left|t_1\right|^{2H}-\left|t-t_1\right|^{2H}\right] \tag{13}$$

The fBm of the Weyl type (see in Mandelbrot and van Ness [1968], is defined as

$$B_H(t)-B_H(0)=$$
$$\frac{1}{\Gamma\left(H+\frac{1}{2}\right)}\left\{\int_{-\infty}^{0}[(t-t_1)^{H-0.5}-(-t_1)^{H-0.5}]dB(t_1)+\int_{0}^{t}\left[(t-t_1)^{H-0.5}dB(t_1)\right]\right\}$$

for $t > 0$ and for $t < 0$. (14)

Both fBm of the Riemanian – Liouville type and the one of Weyl type are self-similar, having the property:

$$B_H(at)\equiv a^H B_H(t);\ a>0; \tag{15}$$

The symbol ≡ in eqn (15) is equality in the stochastic sense. Hence, Weyl integral is a fractional integral of white noise, used to define the Brownian motion process. The random walk is based on the mutual dependence between two random variables–e.g. Gaussian distribution:

$$p(x_i,m,\sigma)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left[\frac{-\left(x_i-m\right)^2}{2\sigma^2}\right] \tag{16}$$

This link was very important for the future development of tools to forecast stock market behaviour as the wave principle suggested by Elliott (1871–1948) in a book he published in 1940.

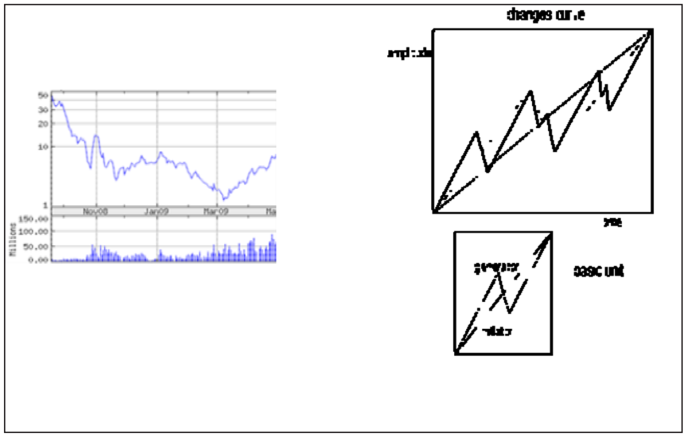


Figure 9: An example of price changes of a stock, self-similar and self-affine systems.

3.4 Black noise

It is an anti-phased white noise, which as such it cancels the primary white noise, by a negative overlapping the original signal shape. By definition, it is an 'anti-sound', which has a specific application of cancelling undesirable noise. This is the meaning of active noise control.

Black noise is also defined as noise whose spectrum varies as:

$$S(f) = \frac{C}{f^{a}};\ a > 2$$

$$\therefore S(f) < \frac{C}{f^{2}} \quad \text{C is a constant} \tag{17}$$

Black noise is used to describe various environmental processes as natural and unnatural catastrophes–as floods, drought, bear-like financial markets and population persistence problems because of environmental changes, all backed up by scientific evidence. Such disasters tend to appear in groups.

4 SUMMARY

Noises can be classified in accordance with the specific spectral shapes and specific parameters of each topic involved. Some basic ideas about noise, out of a huge amount of topics in science, art, medicine and technologies involved with noise were presented here. Noise is colourful. It exists everywhere, carrying endless shapes.

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