

Numerical investigation of pin-fin thermal performance for staggered and inline arrays at low Reynolds number

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ABSTRACT

Microprocessor pin fins are arguably one of the most ubiquitous cooling panaceas for a plethora of integrated chip (IC) electronics. The present study is an attempt to numerically analyze the thermal performance of microprocessor pin fins of different geometries for inline and staggered arrangements using finite volume method based solver Ansys Fluent. The effects of various parameters like Reynolds number, inter spacing ratio and the geometry of fins on the heat dissipation rate are explored. Inline and staggered arrangements for cylindrical and conical fins with same effective lengths are considered. Nusselt number for each arrangement with Reynolds number varying from 3423 to 34230 is calculated and considered as the selection criteria for heat transfer application of heat sinks with a constant wattage unit attached to it. Design and boundary conditions corresponding to different fins are taken pertaining to standard practices available through open literature. Results show a significant enhancement in heat transfer for staggered arrangement as compared to inline making it suitable for low Reynolds number micro heat transfer applications. An increment in Nusselt number is observed with increasing Reynolds Number for each of the arrangements and fin geometries.

1. INTRODUCTION

Fins or extended surfaces as so which aptly describes its main function, are used to increase the overall heat transfer rate of a typical heat generating specimen by increasing the effective surface area exposed to fluid flow. Thermal conductivity of a gas film is low resulting in a significant reduction in heat transfer coefficient. To compensate it, the heat flux can be increased by increasing the surface area available for heat dissipation, thus extended surfaces are used to serve the purpose [1]. Modern technology has seen a huge upsurge in the 21st century and the need for efficient technology has grown leaps and bounds in the past few decades. But most of the modern integrated chip (IC) electronics and microelectronics are accompanied by heating which is an undesirable by-product. As the processing capacity of a microchip or an IC increases heating become inevitable which often leads to damage of valuable circuitry.

Cooling systems involving pin fins are highly efficient and economical contingencies to tackle the overheating problem of electronics as it is air based and involves no moving parts which make it robust and maintenance free. Fins have widespread use in micro and macro electronic cooling such as IC engine cooling, trailing edges of gas-turbine blades and aerospace industry. Wide varieties of fins are used depending on the size of the substrate to be cooled and the amount of cooling to be provided. Pin fins are most suited to be used in Micro and Nano circuitry due to its compact size and optimum size-to-volume ratio as compared to longitudinal and radial fins. Parameters like arrangement and length of the fins affect its performance and efficiency greatly. Conventional practice dictates that the pin fins are used in an array of inline and staggered arrangement to obtain better cooling results in

comparison to any other types of arrangement. In existing studies, there have been investigations regarding the heat transfer and pressure drop of channels in pin fins considering various factors. Limbasiya and Roy [2] have compared and contrasted numerical results of micro channel heat sinks of different geometries and different arrangements establishing the superiority of staggered arrangement of heat sink over tandem arrangement in terms of cooling quality factors for Reynolds number in the range of 450-900 and beyond 900. Samarth and Sawankar [3] have conducted experiments to evaluate the dependence of friction factor and overall heat transfer rate on the various design parameters for perforated pin fin heat exchangers. Yun and Lee [4] applied the Taguchi method to symmetrically analyze the effect of various design parameters on the heat transfer and fluid flow characteristics of slit fins. The friction factor was found to be inversely proportional to clearance ratio and inter-fin spacing. Peng and Peterson [5] concluded that heat transfer performance of laminar flow can be augmented by increasing the ratio of hydraulic diameter and Centre-to-Centre distance of micro channel. Quite a few researchers have investigated on the heat and pressure loss characteristics of different combinations of staggered array of circular and elliptical pin fins (e.g. Brown et al. [6], Uzol and Camci [7], and Vanfossen [8]). Armstrong and Winstanley [9] researched on role of pin fins in turbine cooling applications which is relatively new avenue for the utility of fins. They reviewed the effect of various geometric parameters like length and pin inter-spacing on the heat flow and friction factor conducted experiment to explain the accelerating flow in converging pin fin channels. As the cross-section of the fins remain a dominant factor in deciding fin performance and efficiency experimentation on unorthodox cross-section of fins continue to extract maximum fin

performance. Uzol and Camci [10] have experimented on elliptical fins as a substitute for circular fins for gas turbine cooling blade applications. They have been able to accurately describe the physics behind pressure loss and heat transfer enhancements by using Wake flow field measurements and Particle Image Velocimetry (PIV) to create visualizations insides the wakes of elliptical and circular fins. Reverse engineering the fin arrangement to obtain higher heat transfer techniques causes an inevitable pressure loss in the confined space.

Wan [11] carried out experimental studies and optimization of pin fin shapes in flow boiling of micro pin fin heat sinks. Lampio and Karvinen [12] optimized convectively cooled micro heat sinks in their attempts to solve the age old complication. Heating of apparatus and equipment is a problem which has been troubling scientists and engineers all around the world since time immemorial so a vast aggregate of diverse and disparate research has been carried out in the areas of convective heat transfer and heat sinks. Tari and Mehertash [13] have carried out numerical and experimental studies on natural convection heat transfer on horizontal and slightly inclined plate fin heat sinks. Anoop et al. [14] worked on finding a characteristic correlation for heat transfer over serrated finned tubes in their experimental studies. Yu et al. [15] investigated analytic solutions of the friction factor and the Nusselt number for the low-Reynolds number flow between two wavy plate fins in 2017. Chen et al. [16] executed numerical and experimental studies of natural convection heat transfer characteristics for vertical annular finned tube heat exchangers. Composites are lightweight and sturdy materials with excellent physical and chemical properties which can be used in almost all mechanical mainstream applications. Wu et al. [17] carried out numerical simulation of heat transfer and fluid flow characteristics of fins made up of composite materials.

Bergles and co-workers [18-19] showed that when two independent or co-dependent heat transfer systems are placed under comparison then the possible merits and demerits of one system relative to the other depend on the nature of goals to be achieved and the set of constraints restricting them. Feasible goals may include increasing the heat transfer rate at the same time compacting the size of fins. Constraints may involve mass flow rate, wattage and pressure drop. Most of the IC electronics industry is still dependent on forced convection cooling of chips by using an ergonomic arrangement of fins. Present ways to tackle heating problems have scope of improvement and optimization in future leading to better and more compact design. Bensaci et al. [20] have studied the effect of natural convection on an inclined enclosure. Thangadurai et al. [21] have presented a preliminary study on thermal performance of shrouded heat-sinks in the range of moderate Reynolds numbers. The study dissents from the present study in aspects such as size and area of application, amount of heat generation and range of Reynolds number. Shrouded heat sinks are more suitable for high wattage applications like bipolar junction transistors whereas pin fins are mostly concentrated in chip cooling applications where low wattage is generated. Also the Reynolds number range varies likewise as it is directly dependent on the pragmatic aspect of velocity which can be reached by the particular application. As electronic chip cooling is investigated in the present study, low Reynolds number range is explored.

In the present paper, the above mentioned arrangements are used with varying length-to-diameter ratio and Reynolds

number to obtain a comparative plot of Nusselt number for cylindrical and conical fins. The Reynolds number is varied in the range of 3400-34000 which is standard for a source of forced convection used in CPUs and other electronic gadgets. Plots of maximum temperature and pressure loss in the control volume provide ample data to single out a particular type of fin arrangement at a particular Reynolds number. The number of fins in a particular arrangement is a variable and it is a function of the length-to-diameter ratio, which leads to increase in the base of research and diversification of the results.

2. MATERIALS AND METHODS

2.1 Design specifications

Parallel and conical frustum geometries are used to evaluate the Nusselt number and pressure loss in the control volume in which flow field is present. Inline and staggered arrangement of fins are used with two fin inter-spacing ratios are used vis-à-vis 2 and 3. Fins are attached to a fin base which has a hole at the centre to house the chip or the heat generating unit. Overall dimensions and specifications of the two geometries are given in Table. 1.

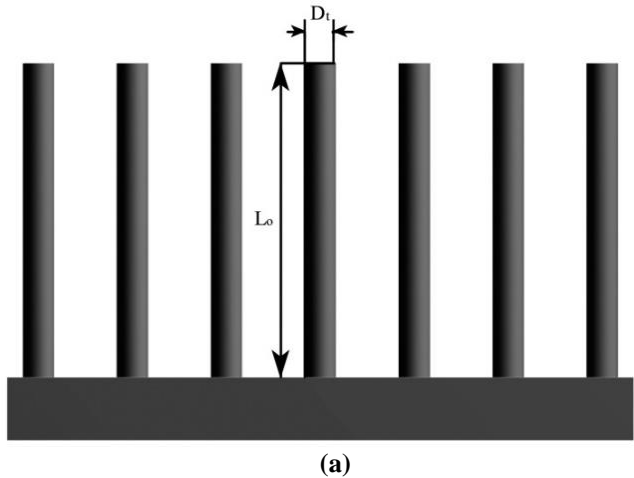
Table 1. Design specifications of fin geometries

Parameters (mm)	Length Of fin (L_o)	Diameter at fin tip (D_t)	Diameter at fin base (D_b)	Dimensions of chip ($L \times B \times H$)
Cylindrical fins	50	5	5	15x8x13
Conical fins	50	2.5	5	15x8x13

2.2 Engineering materials

Table 2. Specification of solid and fluid domains

Materials	Aluminium	Silicon	Air
Symbol	Al	Si	-
Density (ρ)	2719	2329	1.225
Specific Heat (C_p)	871	700	1006.43
Thermal Conductivity (K)	202.4	148	0.0242
Viscosity (μ)	-	-	1.789×10^{-5}



(a)

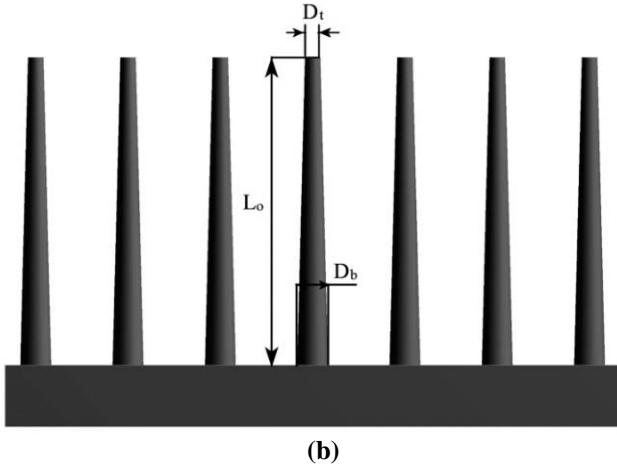


Figure 1. (a) Schematic of cylindrical fins; (b) Schematic of conical fins

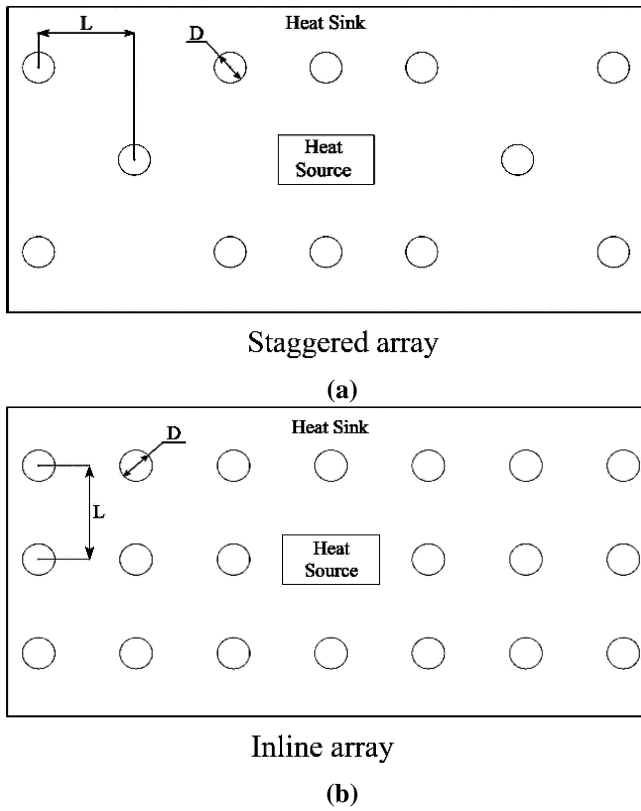


Figure 2. (a) Staggered arrangement of fins; (b) Inline arrangement of fins

Efficient heat dissipation through a fin depends on various properties of the fin material and fluid used in the domain. Thermal conductivity of the fin material plays a huge role in efficiency of the fin, so aluminum (Al) is chosen as the material for the fin considering its light weight, resistance to corrosion, high thermal conductivity and affordable cost. Modern day integrated chips use silicon (Si) as the main material due to its various advantageous properties like semiconductor properties, easy deposition of silicon on chip surface and affordable costs. Fluid domain consists of air flowing at various Reynolds number through the control volume.

3. THEORY

3.1 Mathematical formulations

The control volume used to analyze the fluid through fins is a cuboidal enclosure with fin shaped cavity engraved in it. The simulations are conducted in accordance with the set of partial differential equations defined under Reynolds Averaged Navier-Stokes (RANS) model combined with Shear stress transport (SST) $k-\omega$ model in commercially available Ansys Fluent [22]. The governing equations of continuity, momentum and energy for 3-D Cartesian coordinate system are shown below:

(a) Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (1)$$

(b) Momentum equations

$$\rho \frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \nabla \cdot (\mu \nabla u) \quad (2)$$

$$\rho \frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \nabla \cdot (\mu \nabla v) \quad (3)$$

$$\rho \frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \nabla \cdot (\mu \nabla w) \quad (4)$$

(c) Energy equations

$$\rho \frac{Di}{Dt} = \nabla \cdot (k \nabla T) - p \nabla \cdot (\vec{V}) + \phi \quad (5)$$

The following Shear stress transport (SST) turbulence model is used in characterising the fluid flow.

$$\frac{\partial(\rho k)}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j k) = \frac{\partial}{\partial x_j} [(\mu + \mu_t / \sigma_k) \frac{\partial k}{\partial x_j}] + P_k - \beta' \rho k \omega + P_{kb} \quad (6)$$

$$\frac{\partial(\rho \omega)}{\partial t} + \frac{\partial}{\partial x_j} (\rho u_j \omega) = \frac{\partial}{\partial x_j} [(\mu + \frac{\mu_t}{\sigma_k}) \frac{\partial \omega}{\partial x_j}] + \alpha \frac{\omega}{k} P_k - \beta \rho \omega^2 + P_{ab} \quad (7)$$

The coefficients used in the above SST equations are

$$\beta' = 0.09; \alpha = 5/9; \beta = 0.075; \sigma_k = 2; \sigma_\omega = 2 \quad (8)$$

3.2 Computational domain and boundary conditions

Shear stress transport (SST) $k-\omega$ turbulence model is employed to compute the heat flow occurring through conduction and forced convection in the control volume. The design consists of a solid domain and a fluid domain. Solid domain comprises of fins, support base, and the chip or heat source. Fluid domain is basically an enclosure surrounding the fins in which air flows at various Reynolds number.

A velocity-inlet boundary condition is defined at the inlet with an ambient air temperature of 300 K and pressure-outlet

boundary condition is defined at the outlet. The chip generates heat at a constant wattage of 100 W which is a standard for a microchip or an integrated chip (IC). The chip walls have wall boundary condition and considering the chip dimensions (15 x 8 x 13 mm) heat generation rate is defined as $6.41 \times 10^7 \text{ Watt/m}^3$.

The Reynolds number at the inlet varies from 3423 to 34230 corresponding to nine velocities specified in x-direction. A turbulent intensity of 5% and a turbulent viscosity ratio of 10 are specified at the inlet of the fluid domain as the forced convection is produced with a fan in the actual model. Adiabatic wall boundary condition is used at the walls of fluid domain control volume to prevent the escape of heat from walls causing discrepancies in temperature characteristics. The convergence criterion for continuity is set as 10^{-4} in residual Root Mean Square (RMS) error. The gauge pressure at both inlet and outlet is set as zero Pascals. As the inlet is velocity-based and the outlet is pressure-based, pressure-velocity coupling solution along with coupled scheme and least squares cell based gradient is used for running simulations. The solution is initialized using standard initialization method where the reference frame is relative to cell zones.

3.3 Grid independence test and meshing

Grid independence test is a viable method to validate mesh settings used for simulation. First the boundary conditions are applied to a coarser mesh setting and then gradually the mesh setting is changed to finer with more number of elements and nodes. As the mesh becomes more and more finer due the increasing number of elements and nodes the results get more accurate and attain a constant value after a particular mesh setting. The plot of Nusselt number with increasing number of nodes for cylindrical fins arranged in staggered form with an interspacing ratio of 3 and Reynolds number of 17115 is shown in figure 3.

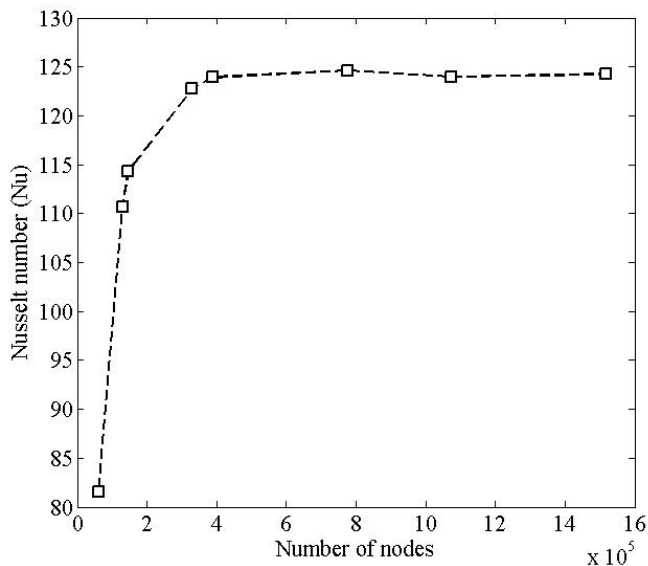


Figure 3. Grid independence test

From the above graph, it is inferred that the value of Nusselt number attains a constant value when the number of nodes is higher than 400000. Thus optimum meshing conditions are chosen for analysis of heat flow and pressure loss corresponding to that point. The mesh chosen for simulations has a multi-zone method setting and consists of hex dominant

elements. The average size meshing grids is 1.25 mm and maximum face size is 2.5 mm. The model has 736369 numbers of nodes and 2397418 numbers of elements. Above the corresponding mesh setting used the evaluated results such as Nusselt number and pressure drop coefficient tend to achieve a saturated state of values.

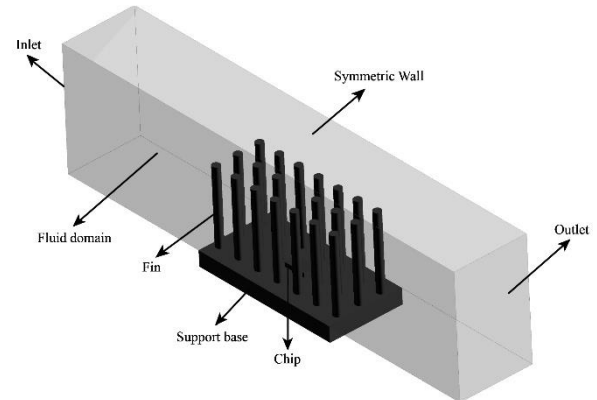
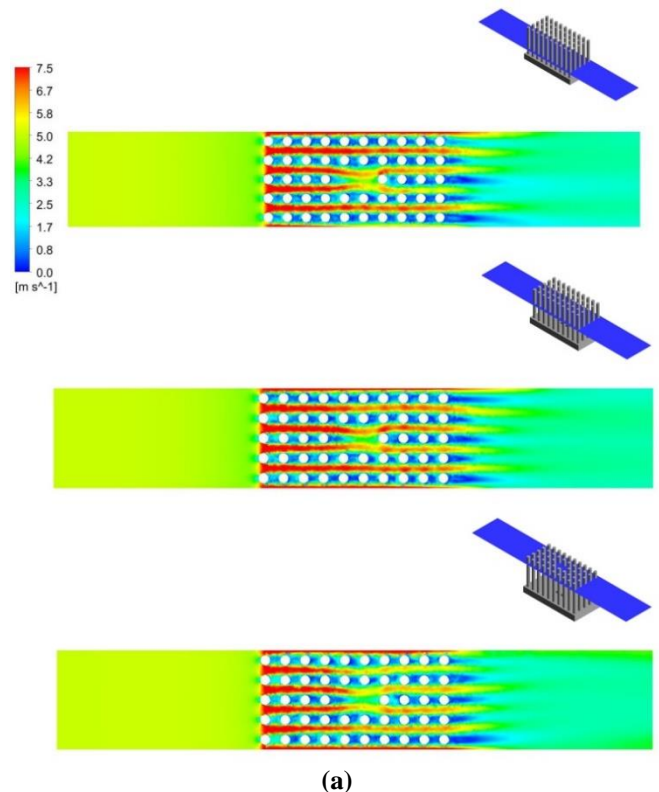


Figure 4. Labeled diagram of the work domain of the model

4. RESULTS

The thermal and kinematic behavior of the flow can be accurately estimated by analyzing the flow along different sections taken in lateral direction of flow domain. It can reveal the essence of velocity and temperature distributions along downstream direction of flow. The same results are shown in Fig. (4)- (5) corresponding to the cylindrical fins. The flow seems to be highly accelerating at all the section for Inline arrangement. This is due to high number density of fins and small inter-spacing between them.



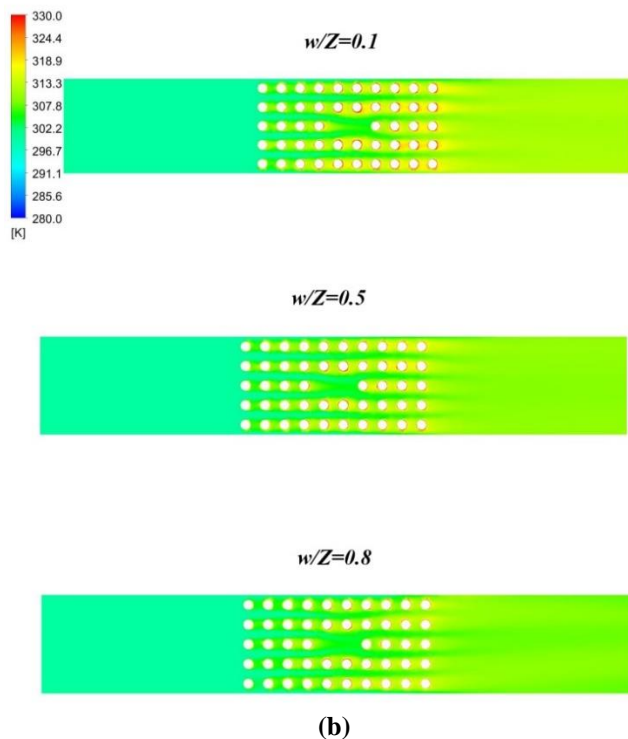


Figure 5. (a) Velocity contours for inline arrangement of cylindrical fins at $Re=17115$ and interspacing ratio = 2; (b) Temperature contours for inline arrangement of cylindrical fins at $Re=17115$ and interspacing ratio = 2

In the case of inline arrangement of fins as shown in figure-5, the temperature is higher in region nearer to the fin base ($w/Z = 0.1$) and decreases as sections farther than base are considered. The low inter-spacing for inline arrangement of fins degrades the quality of fluid flow due to formations of smaller eddies or vortices in the fin vicinity.

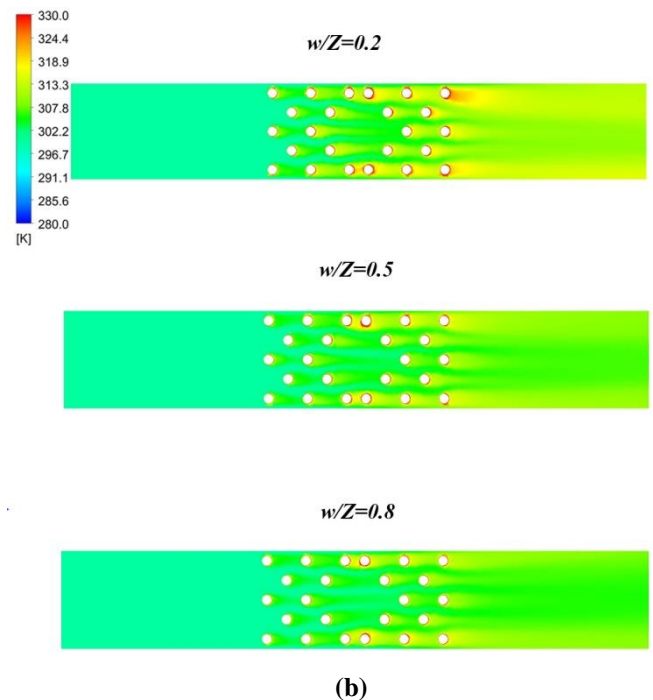
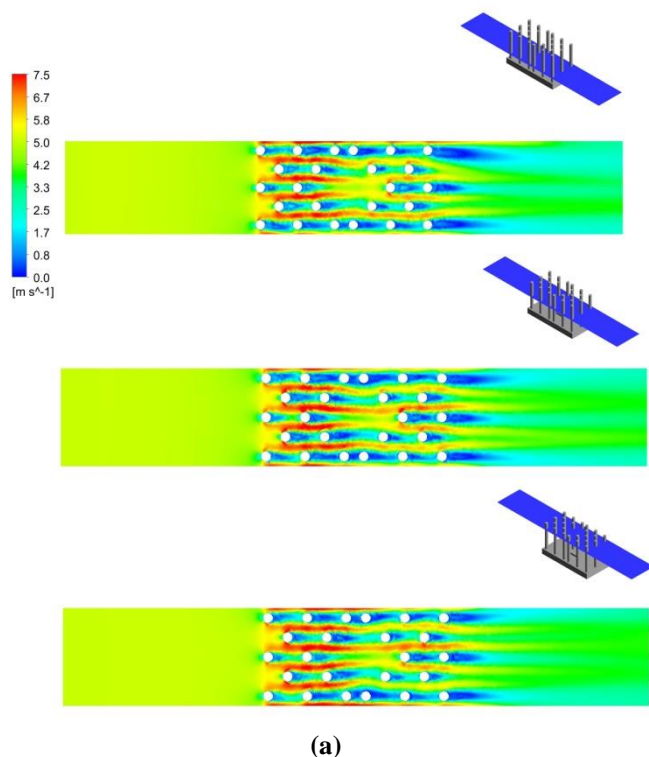
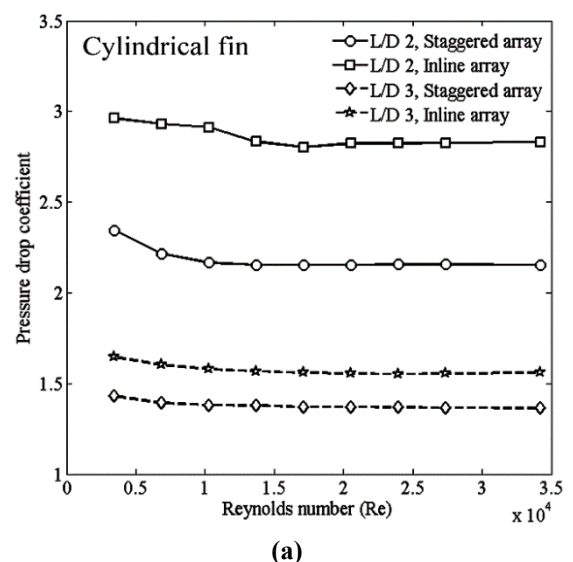


Figure 6. (a) Velocity contours for staggered arrangement of cylindrical fins at $Re=17115$ and interspacing ratio = 2; (b) Temperature contours for staggered arrangement of cylindrical fins at $Re=17115$ and interspacing ratio = 2

For staggered arrays as shown in figure-6, the temperature is high at almost all sections which insinuate a more efficient heat transfer to the fluid. As the staggered array receives much better quality of the flow as compared to inline array therefore the connective transport of thermal energy is more efficient. Here the size of eddies or vortices formed are much larger, causing more balanced turbulence mixing of the fluids.

The distributions of pressure drop with increasing Reynolds number are shown in fig. 7. The pressure drop coefficient is the parametric evaluation of total pressure loss with respect to the kinetic energy. It can be well seen that the pressure loss throughout the fin channel is almost invariant with the increasing Reynolds number. The flow across the conical fin faces variations in exposed area to the flow at any cross-section thus making the flow well adjusted.



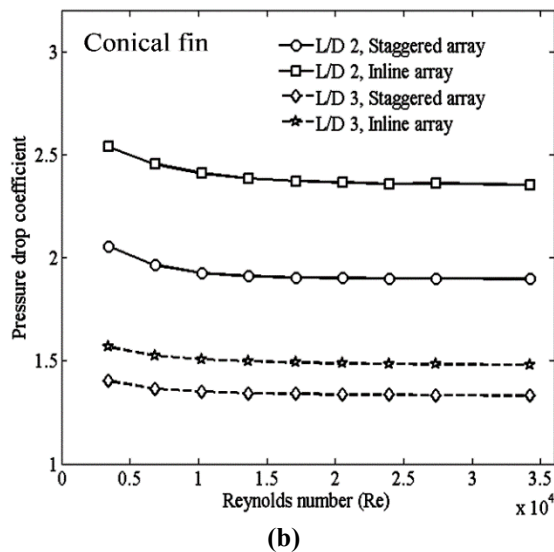


Figure 7. Plot for pressure drop coefficient for (a) cylindrical fin with increasing Reynolds number; Plot for pressure drop coefficient for (b) conical fin with increasing Reynolds number

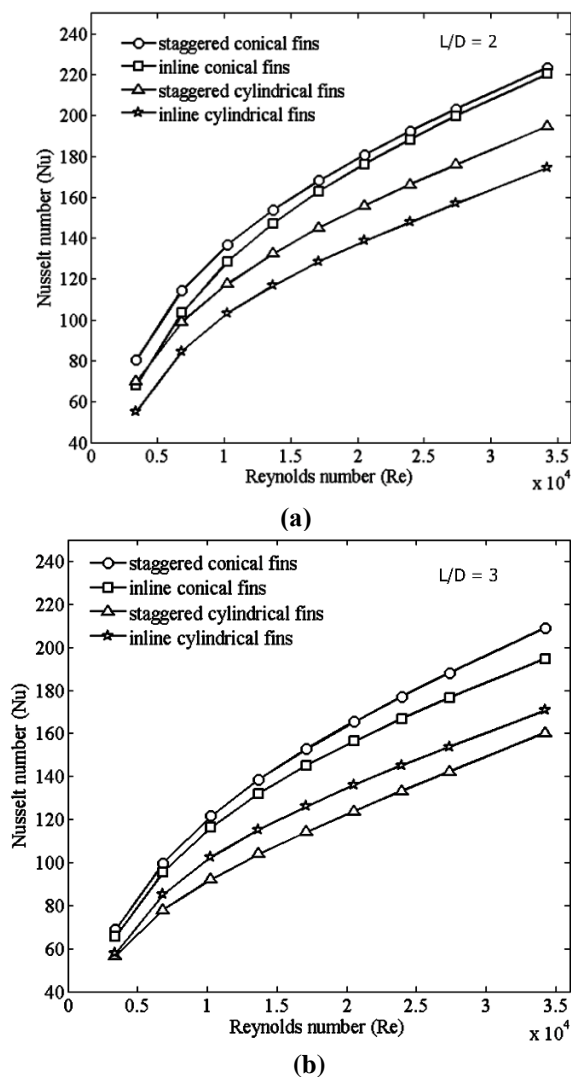


Figure 8. Plot for Nusselt number for (a) Inter-spacing ratio=2, with increasing Reynolds number; Plot for Nusselt number for (b) Inter-spacing ratio=3, with increasing Reynolds number

The graphs are showing the variations of Nusselt number for different arrangements of fins at different Reynolds number. Fig 8. (a) is showing the variation of Nusselt number for inter-spacing ratio=2 which increases with increasing Reynolds number for all fin arrangements. The staggered arrays are more efficient than inline arrays with conical fins being the most efficient. At higher Reynolds numbers the conical fins with both array type shows almost similar thermal performances.

The overall heat transfer depends on a number of factors like temperature difference, flow quality, fin shape, material, surface parameters etc. The convective heat transfer from the surface is driven mainly by the temperature difference at fluid solid interface which is further governed by the mixing of fluid. The staggered arrays have efficient mixing of the flow as well as less pressure drop coefficient which further increases the heat transfer and fin efficiency.

For inter-spacing ratio= 3, the variation of Nusselt number shows similar trends as compared to inter-spacing ratio= 2. The staggered array of cylindrical fins is less efficient than its inline array due to the reduced number of fins in case of the staggered arrangement. In this case the total surface area for convective heat transfer reduces due to the less number of fins hence showing significant reduction in Nusselt number.

5. CONCLUSIONS

The thermal performance of pin fins are numerically studied by varying governing parameters like Reynolds Number (3423 to 34230), Interspacing Ratio (L/D) and arrangement (inline and Staggered). The SST K- ω turbulence model is adopted to solve the discretized RANS equation with the help of finite volume method solver Fluent. The variation of the pressure drop coefficient with varying Reynolds number is shown for all arrangements and it is found out to be relatively independent of Reynolds number. Furthermore, the Nusselt number is calculated and it was found that the staggered arrangement is most suited to the efficient convective heat transfer for both types of fins (cylindrical and conical). The flow conditions and the shape of fins are the important parameters to show significant variation in Nusselt number. The conical fins are seen to be more helpful in enhancing thermal performance as well as fan efficiency by reducing the pressure drop. The applications and scope of work for fins are limitless and an extended study can be carried out by experimenting with shapes and sizes other than what are mentioned in the present study. The concept of fins can be easily extended to life scale models like engine radiators, turbine blades and supercomputer servers to provide cheap and durable cooling solutions.

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NOMENCLATURE

L_0	Length of fin[m]
D_t	Diameter of cross-section at fin tip[m]
D_b	Diameter of cross-section at fin base[m]
C_p	Specific heat capacity[J/kg-K]
K	Thermal conductivity[W/m-K]
V	Velocity vector[m/s]
x, y and z	Cartesian coordinates
u, v and w	velocities in x, y and z directions[m/s]
t	Time[s]
T	Temperature[K]
i	Internal Energy of fluid[J]
Re	Reynolds Number
Nu	Nusselt Number

Greek symbols

ρ	Density[kg/m ³]
μ	Viscosity[kg/m-s]
Φ	Dissipation function
$\beta, \beta', \alpha, \sigma_k, \sigma_w$	Constants