















$$L_5 = \frac{ARaGm(s + RaR)(1 - e^{-s})}{s^2(sPr - sSc + B)(sSc - s - MRa)}$$

$$L_6 = \frac{ARaGm(s + RaR)(1 - e^{-s})}{s^2(sPr - sSc + B)(sPr - s - MRa + B)}$$

$$A = ScSrPr, B = RaRPr$$

$$a_1 = -\frac{B}{Pr - Sc}, a_2 = \frac{RaGr}{Pr - 1}, a_3 = \frac{MRa - B}{Pr - 1}$$

$$a_4 = \frac{RaGm}{Sc - 1}, a_5 = \frac{RaM}{Sc - 1}, a_6 = \frac{A}{Pr - Sc}, a_7 = \frac{RaGm}{Pr - 1}$$

$$A_1 = -\frac{a_1 + RaR}{a_1^2}, A_2 = -\frac{RaR}{a_1}, A_3 = -A_1$$

$$A_4 = \frac{a_2}{a_3^2}, A_5 = -A_4, A_6 = -\frac{a_2}{a_3}, A_7 = -\frac{a_4}{a_5}$$

$$A_8 = \frac{a_4}{a_5}, A_9 = \frac{a_4 a_6 (a_1 + RaR)}{a_1^2 (a_1 - a_5)}, A_{10} = \frac{a_4 a_6 (a_5 + RaR)}{a_5^2 (a_5 - a_1)}$$

$$A_{11} = -(A_9 + A_{10}), A_{12} = \frac{a_4 a_6 RaR}{a_1 a_5}, A_{13} = \frac{a_6 a_7 (a_1 + RaR)}{a_1^2 (a_1 - a_5)}$$

$$A_{14} = \frac{a_6 a_7 (a_3 + RaR)}{a_3^2 (a_3 - a_1)}, A_{15} = -(A_{13} + A_{14}), A_{16} = \frac{a_6 a_7 RaR}{a_1 a_3}$$

$$\psi_1 = f(PrRa, RaR, y, t), \bar{\psi}_1 = f(PrRa, RaR, y, t - 1)H(t - 1)$$

$$\psi_2 = \operatorname{erfc}\left(\frac{y\sqrt{RaSc}}{2\sqrt{t}}\right), \bar{\psi}_2 = \operatorname{erfc}\left(\frac{y\sqrt{RaSc}}{2\sqrt{t - 1}}\right)H(t - 1)$$

$$\psi_3 = \lambda(ScRa, y, t), \bar{\psi}_3 = \lambda(ScRa, y, t - 1)H(t - 1)$$

$$\psi_4 = \psi(RaSc, a_1, y, t), \bar{\psi}_4 = \psi(RaSc, a_1, y, t - 1)H(t - 1)$$

$$\psi_5 = \psi(PrRa, RaR, y, t), \bar{\psi}_5 = \psi(PrRa, RaR, y, t - 1)H(t - 1)$$

$$\psi_6 = \psi(PrRa, a_1 + RaR, y, t), \bar{\psi}_6 = \psi(PrRa, a_1 + RaR, y, t - 1)H(t - 1)$$

$$\psi_7 = f(Ra, MRa, y, t), \bar{\psi}_7 = f(Ra, MRa, y, t - 1)H(t - 1)$$

$$\psi_8 = \psi(Ra, a_3 + MRa, y, t), \bar{\psi}_8 = \psi(Ra, a_3 + MRa, y, t - 1)H(t - 1)$$

$$\psi_9 = \psi(Ra, MRa, y, t), \bar{\psi}_9 = \psi(Ra, MRa, y, t - 1)H(t - 1)$$

$$\psi_{10} = \psi(Ra, a_5 + MRa, y, t), \bar{\psi}_{10} = \psi(Ra, a_5 + MRa, y, t - 1)H(t - 1)$$

$$\psi_{11} = \psi(Ra, a_1 + MRa, y, t), \bar{\psi}_{11} = \psi(Ra, a_1 + MRa, y, t - 1)H(t - 1)$$

$$\psi_{12} = \psi(PrRa, a_3 + RaR, y, t), \bar{\psi}_{12} = \psi(PrRa, a_3 + RaR, y, t - 1)H(t - 1)$$

$$\psi_{13} = \psi(RaSc, a_5, y, t), \bar{\psi}_{13} = \psi(RaSc, a_5, y, t - 1)H(t - 1)$$

$$\Omega_1 = F(Ra, RaM, t), \bar{\Omega}_1 = F(Ra, RaM, t - 1)H(t - 1)$$

$$\Omega_2 = \Omega(Ra, a_3 + MRa, t), \bar{\Omega}_2 = \Omega(Ra, a_3 + MRa, t - 1)H(t - 1)$$

$$\Omega_3 = \Omega(Ra, RaM, t), \bar{\Omega}_3 = \Omega(Ra, RaM, t - 1)H(t - 1)$$

$$\Omega_4 = \Omega(Ra, a_5 + MRa, t), \bar{\Omega}_4 = \Omega(Ra, a_5 + MRa, t - 1)H(t - 1)$$

$$\Omega_5 = \Omega(Ra, a_1 + MRa, t), \bar{\Omega}_5 = \Omega(Ra, a_1 + MRa, t - 1)H(t - 1)$$

$$\Omega_6 = \Omega(PrRa, a_3 + RaR, t), \bar{\Omega}_6 = \Omega(PrRa, a_3 + RaR, t - 1)H(t - 1)$$

$$\Omega_7 = \Omega(PrRa, RaR, t), \bar{\Omega}_7 = \Omega(PrRa, RaR, t - 1)H(t - 1)$$

$$\Omega_8 = F(PrRa, RaR, t), \bar{\Omega}_8 = F(PrRa, RaR, t - 1)H(t - 1)$$

$$\Omega_9 = -\sqrt{\frac{RaSc}{\pi t}}, \Omega_{10} = \Omega(RaSc, a_5, t), \bar{\Omega}_{10} = \Omega(RaSc, a_5, t - 1)H(t - 1)$$

$$\Omega_{11} = \Omega(RaSc, a_1, t), \bar{\Omega}_{11} = \Omega(RaSc, a_1, t - 1)H(t - 1)$$

$$\Omega_{12} = L(RaSc, t), \bar{\Omega}_{12} = L(RaSc, t - 1)H(t - 1)$$

$$\Omega_{13} = \Omega(PrRa, a_1 + RaR, t), \bar{\Omega}_{13} = \Omega(PrRa, a_1 + RaR, t - 1)H(t - 1)$$

$$\psi(\xi, \eta, y, t) = \frac{1}{2} \left[ e^{\sqrt{\xi\eta}y} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\xi}{t} + \sqrt{\eta t}}\right) + e^{-\sqrt{\xi\eta}y} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\xi}{t} - \sqrt{\eta t}}\right) \right]$$

$$f(\xi, \eta, y, t) = \left(\frac{t}{2} + \frac{y}{4}\sqrt{\frac{\xi}{\eta}}\right) e^{\sqrt{\xi\eta}y} \operatorname{erfc} +$$

$$\left(\frac{t}{2} - \frac{y}{4}\sqrt{\frac{\xi}{\eta}}\right) e^{-\sqrt{\xi\eta}y} \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\xi}{t} - \sqrt{\eta t}}\right)$$

$$\lambda(\xi, y, t) = \left(t + \frac{y^2\xi}{2}\right) \operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\xi}{t}}\right) - \frac{y\sqrt{\xi t}}{\sqrt{\pi}} e^{-\frac{\xi y^2}{4t}}$$

$$\Omega(\xi, \eta, t) = -\left[\sqrt{\frac{\xi}{\pi t}} e^{-\eta t} + \sqrt{\xi\eta} \operatorname{erf}(\sqrt{\eta t})\right]$$

$$F(\xi, \eta, t) = -\left[\frac{1}{2}\sqrt{\frac{\xi}{\eta}} \operatorname{erf}(\sqrt{\eta t}) + t\sqrt{\xi\eta} \operatorname{erf}(\sqrt{\eta t}) + \sqrt{\frac{\xi t}{\pi}} e^{-\eta t}\right]$$

$$L(\xi, t) = -2\sqrt{\frac{\xi t}{\pi}}$$

$$H(t - 1) = \begin{cases} 0, & t < 1 \\ 1, & t > 1 \end{cases} \text{ is the unit step function}$$