International Journal of Heat and Technology
Vol. 36, No. 2, June, 2018, pp. 397-405
Journal homepage: http://iieta.org/Journals/IJHT

Effect of conjugate heat transfer on flow of nanofluid in a rectangular enclosure

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https://doi.org/10.18280/ijht.360203

Received: 15 March 2018
Accepted: 2 May 2018

Keywords:
conjugate natural convection, nanofluid, finite element method, enclosure

ABSTRACT

An elaborate numerical study of developing a model regarding conjugate effect of fluid flow and heat transfer in a heat conducting vertical walled cavity filled with copper-water nanofluid has been presented in this paper. This model is mainly adopted for a cooling of electronic device and to control the fluid flow and heat transfer mechanism in an enclosure. The numerical results have been provided in graphical form showing effect of various relevant non-dimensional parameters. The relevant governing equations have been solved by using finite element method of Galerkin weighted residual approach. The analysis uses a two dimensional rectangular enclosure under conjugate convective conductive heat transfer conditions. The enclosure exposed to a constant and uniform heat flux at the left vertical thick wall generating a natural convection flow. The thicknesses of the remaining parts of the walls are assumed to be zero. The right wall is kept at a low constant temperature, while the horizontal walls are assumed to be adiabatic. A moveable divider is attached at the bottom wall of the cavity. The governing equations are derived for the conceptual model in the Cartesian coordinate system. The study has been carried out for the Rayleigh number Ra $=10^6$ and for the solid volume fraction $0 \leq \phi \leq 0.05$. The investigation is to be arrived out at different non-dimensional governing parameters. The effect of convective heat transfer coefficient, divider position and thickness of solid wall on the hydrodynamic and thermal characteristic of flow has been analyzed. Results are to be presented in terms of streamlines, isotherms and average Nusselt number of the nanofluid for different values of governing parameters.

1. INTRODUCTION

There are many situations in which natural convection is the preferred mechanism of heat transfer from highly packed electronic components due to its intrinsic reliability and noiseless operation. In such circumstances, employing traditional coolant such as water, ethylene glycol or mineral oils are not reliable due to their low thermal conductivity and hence poor cooling performance. Thus improvement of the thermal characteristics of the working fluid is a demanding task. Studies related with convective flow including wall of zero thickness in different enclosures can be found in the literatures [1-5]. A combined effect of free convection and conduction of a nanofluid flow is of special technical significance because of its frequent occurrence in many industrial applications such as geothermal reservoirs, cooling of nuclear reactors, thermal insulations and petroleum reservoirs [6-10]. The use of nanofluid as working media has been considered by many researchers [11-16] because the presence of nanoparticles enhances the thermal conductivity of the media. Conjugate natural convection in a rectangular enclosure surrounding by walls was firstly examined by Kim and Viskanta [17-18]. Their results show that wall conduction effects reduces the average temperature differences across the cavity, partially stabilize the flow, and decrease the heat transfer rate. Costa et al. [19] studied heat convection in an enclosure with rectangular partitions of finite thickness. The position, length and thermal conductivity of the partitions are varied. Conjugate natural convection in a square cavity with finite thickness horizontal walls studied by Mobedi [20]. Kuznetsov and Sheremet [21] and Al-Amin et al. [22] investigated the effect of thermal conductivity of the partitions on heat transfer inside the enclosure. Varol et al. [23] investigated conjugate heat transfer in porous triangular enclosures with thick bottom wall. However the effect of nanoparticles on heat transfer enhancement in natural convection was conducted by Aminossadati and Ghasemi [24], who considered natural convection cooling of a localized heat source at the bottom of a nanofluid filled enclosure. They found that the location of the heat sources provide to significantly affect the heat source maximum temperature. Oztop et al. [25] studied conjugate heat transfer in air filled tube inserted inside a cavity. The walls of the tube were considered thermally conductivity. In this study, the effect of arc shaped conductive baffles on natural convection heat transfer in a square enclosure is studied. Conjugate natural convection conduction heat transfer in a square enclosure with a finite wall thickness is studied numerically by Saleh and Hashim [26]. Alizadek and Dehghan [27] present a numerical investigation of conjugate natural convection of water bases nanofluid in a square cavity. They considered two different types of nanofluid as the working fluids. They also used a square volumetric heat generation source which is located within cavity, resembling a heat generating electronic device. Rahman and Alim [28] investigated MHD mixed convection flow in a vertical lid-driven square enclosure including a heat conducting horizontal circular cylinder with Joule heating. Zhang et al. [29] studied conjugate convection in an enclosure with time periodic sidewall temperature and inclination. Natural convection heat transfer inside a square cavity for different values of Rayleigh and Nusselt number was presented by Bhattacharya and Das [30]. Alrashid [31] numerically investigated conjugate heat transfer for cooling the circuit board. A 3D model of a flat circuit board with a heating electric chip mounted on it has been studied numerically. Gurturk et al. [32] investigated conjugate natural convection heat transfer in a cavity with Arc-Shaped partition with different materials. They observed that heat transfer temperature distribution and fluid flow are affected by changing the radius of the partition with the maximum heat transfer. Farhany and Abdulkadhim [33] performed a numerical simulation on conjugate natural convection heat transfer in a porous enclosure with partially heated wall. They found that heat transfer can be enhanced by increasing the Rayleigh number. Despite a number of numerical studies on rectangular enclosures reported in the literatures. The mentioned literature survey indicates that there is lack of information concerning the problem of combined convection flow and heat transfer enhancement of nanofluid in a rectangular enclosure with finite heat conducting solid wall. To the best of author's knowledge, investigation of conductive-convective heat transfer of the nanofluid in a thick walled rectangular enclosure with constant heat flux has not been under taken yet. In the present work, a numerical analysis has investigated on the problem of developing laminar conjugate heat transfer of copper water nanofluid in a rectangular enclosure submitted to uniform heat flux on its solid thick wall. It is also mentioned that a divider is placed on the bottom horizontal wall to control heat transfer especially in electronic devices. The effect of various parameters of this nanofluid on the flow of convective heat transfer performance are reported and analyzed. Results will be presented graphically in terms of parametric presentation of streamlines, isotherms and average Nusselt number of nanofluid.

2. PROBLEM FORMULATION

As shown in Figure 1, a two dimensional rectangular enclosure filled with electrically conducting fluid sides of width L and height H under conjugate conduction convection heat transfer condition has been considered for the present study. The enclosure was subject to a constant and uniform heat flux q at the left thick wall generating natural convection flow. The left wall has a thickness of $W_1 = \frac{w}{L} = 0.1$, while the thicknesses of the other boundaries of the wall are assumed to be zero. The right wall is kept at a low temperature T_c, while the horizontal walls are assumed adiabatic. A movable heat conducting divider of length 0.2 and width 0.1 is attached to the horizontal bottom wall of the cavity. The rectangular enclosure is filled with copper nanoparticles in water. The nanofluid used in the analysis is assumed to be Newtonian, incompressible and laminar.

The base fluid and nanoparticles (Cu) are in thermal equilibrium and there is no slip between them. The thermo-physical properties of the fluid and nanoparticles are given in Table 1. The thermophysical properties of the nanofluid are assumed constant with the base fluid. The density properties of nanoparticles are taken to be constant which determine base on Boussinesq approximation. The effect of viscous dissipation is neglected.

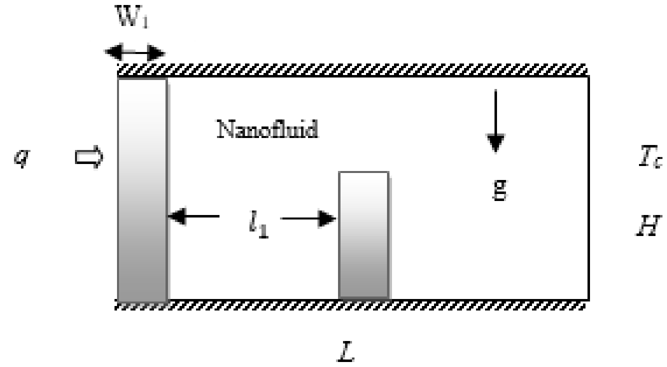

Figure 1. Physical Geometry of the Model

3. MATHEMATICAL EXPRESSION

The system of equations governing the flow using conservation of mass, momentum and energy equations for two-dimensional laminar and incompressible flow for nanofluid [24] and solid in non-dimensional form can be written as follows:

For Fluid:

Continuity Equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (1)$$

Momentum Equations:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right)$$

(2)

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial y} = -\frac{\partial P}{\partial Y} + \frac{\mu_{nf}}{\rho_{nf}\alpha_f}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{(\rho\beta)_{nf}}{\rho_{nf}\beta_f} Ra \Pr \theta$$

(3)

Energy Equation:

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{nf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \quad (4)$$

For Solid:

$$\frac{\partial^2 \theta_w}{\partial X^2} + \frac{\partial^2 \theta_w}{\partial Y^2} = 0 \quad (5)$$

The above equations are non-dimensionalized by using the following dimensionless quantities

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{uL}{\alpha_f}, V = \frac{vL}{\alpha_f}, P = \frac{\overline{p}L^2}{\rho_{nf}\alpha_f^2}, \theta = \frac{T - T_C}{\Delta T}$$
$$and\ \Delta T = \frac{q''L}{k_f} \quad (6)$$

The dimensionless parameter appearing in the equations (2-

5) are as follows: Rayleigh number $Ra = \frac{g\beta_f L^3 \Delta T}{\nu_f \alpha_f}$ and Prandlt number Pr $= \frac{\nu_f}{\alpha_f}$.

The non dimensional boundary conditions for the present problem are specified as follows:

For all rigid walls: $U = V = 0$

At the right vertical wall: $\theta = 0$

At the top and bottom walls: $\frac{\partial \theta}{\partial Y} = 0$

At the left side of thick wall: $q = -\frac{k_w}{k_{nf}} \frac{\partial \theta}{\partial X} |_w + \frac{H}{k_{nf}} h_\infty \theta_w = 1$

At the divider surface: $U = V = 0$.

At the fluid solid wall interfaces: $\frac{\partial \theta_{nf}}{\partial X} = K_r \frac{\partial \theta_w}{\partial X}$,

where, $K_r = \frac{K_w}{K_{nf}}$ is the solid fluid thermal conductivity ratio. The effective density of the nanofluid is given as

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_p \tag{7}$$

where ϕ is the solid volume fraction of nanoparticles. In addition the thermal diffusivity of the nanofluid is

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \tag{8}$$

where $(\rho C_p)_{nf}$ is the heat capacity of the nanofluid and expressed as

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_p \tag{9}$$

The thermal expansion coefficient of the nanofluid $(\rho\beta)_{nf}$ is expressed as

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_p \tag{10}$$

The effective dynamic viscosity of the nanofluid is given by Brinkmann [34] is,

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{11}$$

The thermal conductivity which for spherical nanoparticles, according to Maxwell [35] is

$$k_{nf} = k_f \left[\frac{(k_p + 2k_f) - 2\phi(k_f - k_p)}{(k_p + 2k_f) + \phi(k_f - k_p)} \right], \tag{12}$$

where k_p the thermal conductivity of is dispersed nanoparticles and k_f is the thermal conductivity of pure fluid.

The local Nusselt number at the thick wall on the enclosure base on the non dimensional variable can be express as, $Nu_l = -\frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial X} |_{x=0.1}$ and the average Nusselt number at the left thick wall as, $Nu_{av} = -\frac{k_{nf}}{k_f} \int_0^H \frac{\partial \theta}{\partial X} dy$.

Table 1. Thermo physical properties of fluid and nanoparticles [24]

Physical Properties	Fluid phase (Water)	Copper(Cu)
C_p(J/kgK)	4179	385
ρ (kg/m^3)	997.1	8933
k (W/mK)	0.6	401
β×10^{-5} (1/K)	21	1.67

4. NUMERICAL IMPLEMENTATION

The Galerkin finite element method [36, 37] is used to solve the the Eqs. (2) - (5) along with boundary conditions for the considered problem. The continuity equation is automatically fulfilled for large values of this constraint. Then the velocity components (U, V) and temperature (θ) are expanded using a basis set. The finite element method is recorded below the convergence criterion such that $\left|\psi^{n+1} - \psi^n\right| \le 10^{-4}$, where n is the number of iteration and Ψ is a function of U, V and θ. where the pressure P is eliminated by a constraint. The six node triangular element is used in this work for the development of the finite element equations. The non-linear residual equations are solved using Newton–Raphson method to determine the coefficients of the expansions.

4.1 Grid refinement check

An extensive mesh testing procedure is conducted to guarantee a grid-independent solution for Ra =10^6, $\phi = 0.05$, $l_1 = 0.4$,Pr= 6.2, $h_\infty = 100W/m^2K$,K$_r$ =10 and $W_1 = 0.1$ through a rectangular enclosure. Table 2 shows the Nusselt quantities on the grid size. The results show that a Nusselt number for 3258 elements shows a little difference with the results obtained for the other elements. Hence, considering the non-uniform grid system of 3258 elements is preferred for the computation.

Table 2. Grid sensitivity check at Ra =10^6, $\phi = 0.05$, $l_1 = 0.4$, Pr=6.2, $h_\infty = 100W/m^2K$, K$_r$ =10 and $W_1 = 0.1$

Nodes (elements)	6181 (394)	10062 (533)	13922 (1058)	16181 (3258)	28926 (13020)
Nu nanofluid	3.16853	3.20314	3.36192	3.49042	3.49057
Nu basefluid	2.49469	2.98212	3.14750	3.28430	3.28576
Time [s]	66.265	96.594	122.157	196.328	310.377

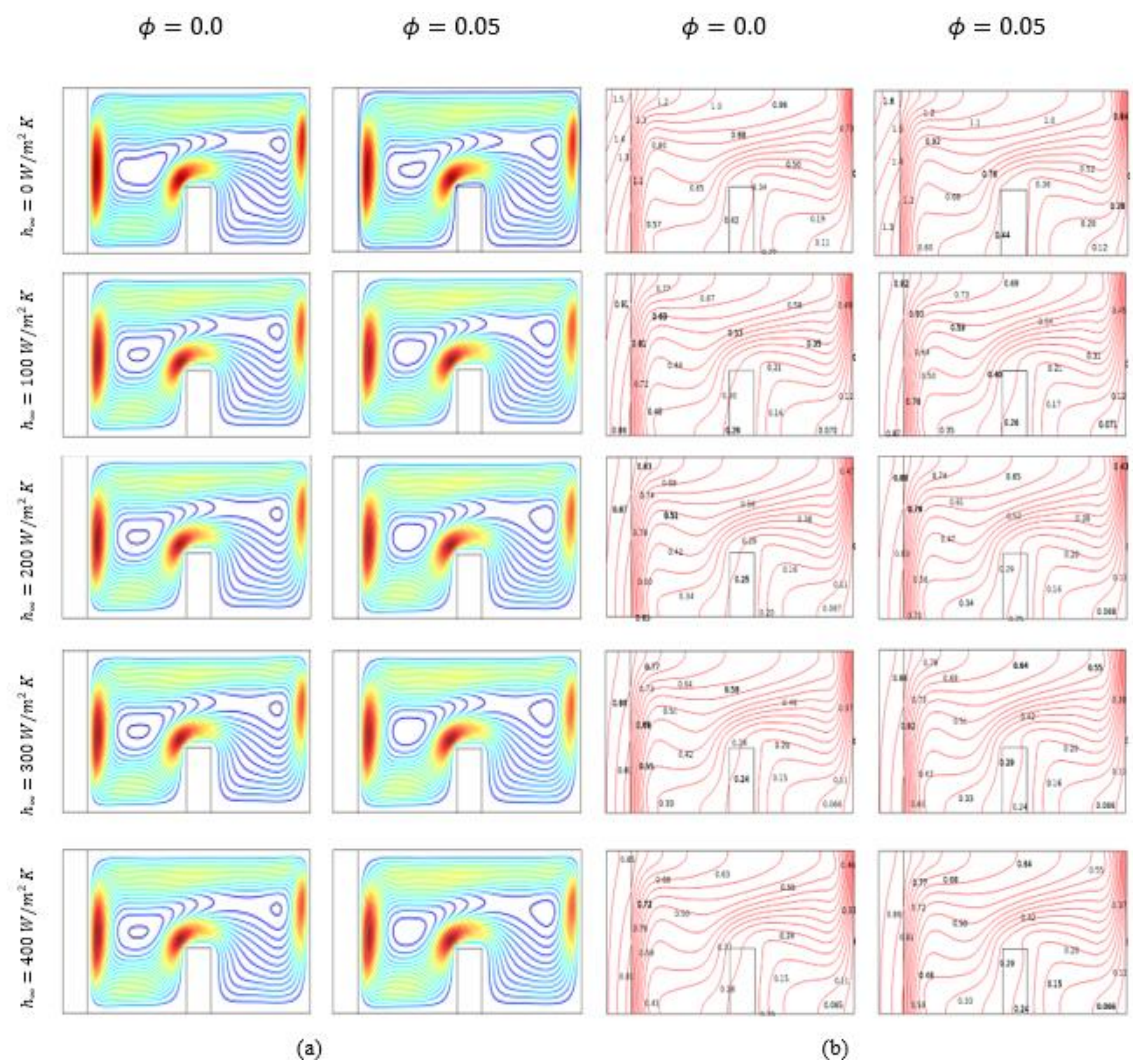

Figure 2. (a) The streamlines and (b) Isotherms at various convective heat transfer coefficient

5. RESULTS AND DISCUSSION

Finite element simulation is applied to perform the analysis on the conjugate natural convection flow and heat transfer in a rectangular enclosure filled with copper-water nanofluid. Effects of the parameters such as convective heat transfer coefficient (h_∞), divider position (l_1) and thickness of solid wall (W_1) on heat transfer and fluid flow inside the cavity have been studied for the range of solid volume fraction (ϕ) of 0 to 0.05. The range of h_∞, l_1 and W_1 for this investigation vary from 0 W/m^2 K to 400 W/m^2 K, 0.1 to 0.7 and 0.1to 0.3 respectively. The outcomes for the different cases are presented in the following sections.

5.1 Effect of convective heat transfer coefficient

Figures 2 (a) and (b) provide the information about the influence of convective heat transfer coefficient (h_∞) on the flow and temperature field for Pr =6.2, Ra=10^6, $l_1 = 0.4$, K$_r$ =10 and $W_1 = 0.1$ for both the pure fluid and nanofluid. The submitted heat is completely transferred from the left wall to the inner flow for $h_\infty = 0$. So, in the case of $h_\infty = 400\ W/m^2K$ the maximum value of the stream function decrease with respect to the case $h_\infty = 100W/m^2K$.

The buoyancy force and flow strength are more pronounced for higher values of convective heat transfer coefficient (h_∞) The thick wall temperature reduces with the increase of convective heat transfer coefficient (h_∞) , so heat is less absorbed by the inner flow which leads to decrease the maximum temperature. We also see in the case $h_\infty = 0$, the isotherms near the vertical side walls are more clustered; However the increase in convective heat transfer coefficient (h_∞) cause the decrease in the temperature gradient in the region. We notice here that isotherms are almost parallel to the vertical walls, which indicates the reduction of heat transfer. This point can be explained as when the ambient convective heat transfer coefficient (h_∞) is increased, the heat flux is absorbed by ambient rather than inner flow. However the increase in the value h_∞ causes decreases in the temperature gradients in the inner flow, which is also an indication of enhancing of cooling performance.

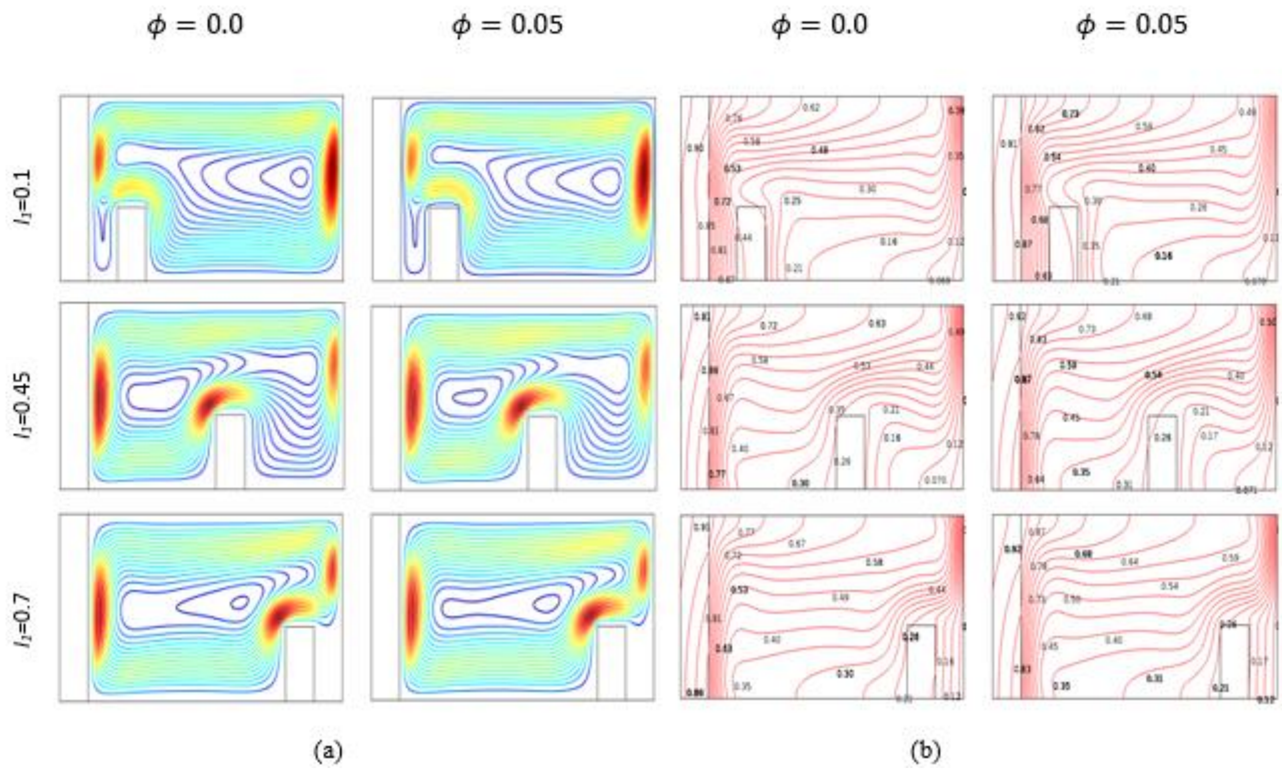

Figure 3. (a) The streamlines and (b) Isotherms at various divider position

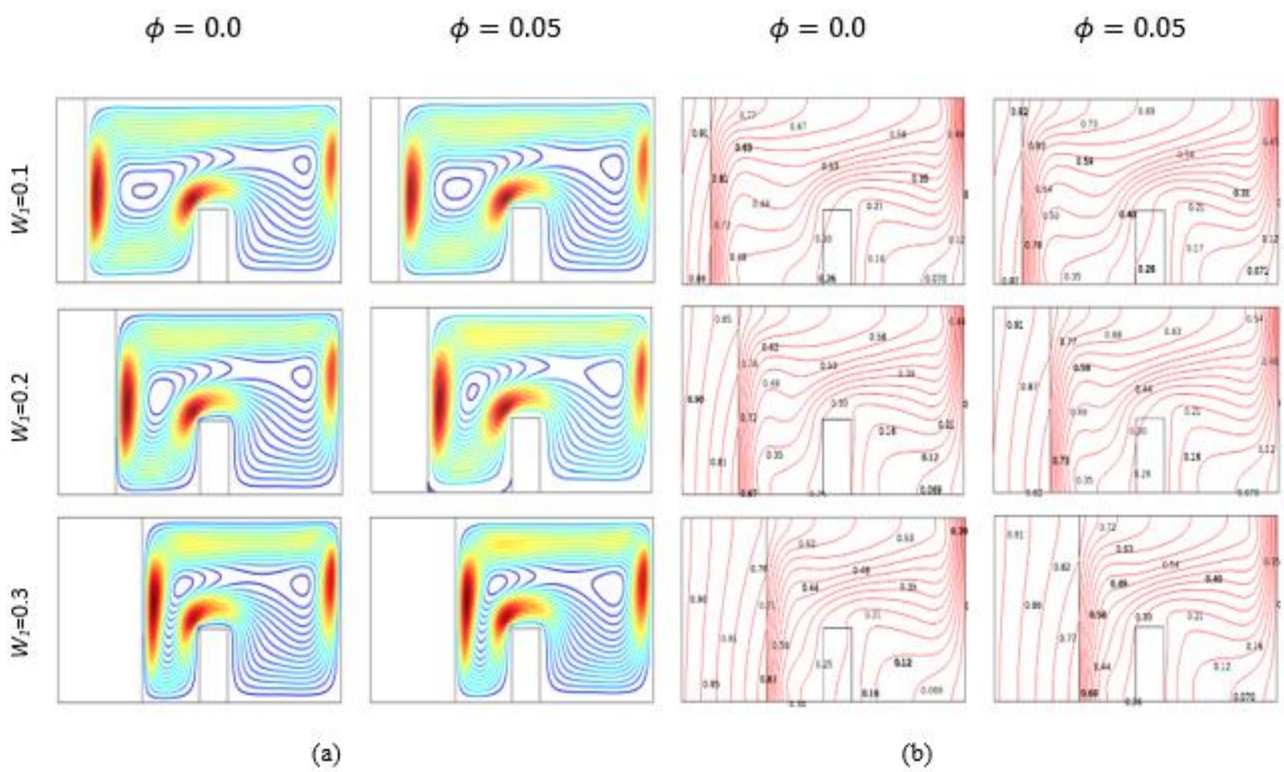

Figure 4. (a) The streamlines and (b) Isotherms at various solid wall thickness

5.2 Effect of divider position

The effect of divider position (l_1) on streamlines and isotherms are shown in Figures 3 (a) and (b) for Pr=6.2, Ra=10^6, h_∞ =100 W/m^{2}K, K_r =10 and $W_1 = 0.1$ for both the pure fluid and nanofluid with 5% concentration. We see in Figure 3 (a), when l_1 =0.1 a clockwise circulation is observed with one core which covers the whole cavity. In this case we notice that there is a weak flow between the divider and the left wall. So, the temperature of this region increases which cause from the reduction of the convective heat transfer coefficient. As the divider position (l_1) increases from the left wall the resistance of circulating flow decrease, that leads to increase in the flow strength. For $l_1 = 0.4$ we see again a clockwise circulation with one cores. With further increase of position of divider i.e. for $l_1 = 0.7$ we observe that the flow strength increases between the left wall and the divider. It is also noticeable that the center of the core location changes with the divider positions. We also see decrease in the distance between the left wall and divider cause the cold flow down. Through the increase in l_1 enhanced the heat transfer, so large amount of energy absorption by the inner flow occurs, which indicate the fact that heat is more transferred from the hot wall to the cold region. So the temperature of the thick wall reduces. It is also observed from the Figure 3 (b) that for smaller l_1 the isotherms are clustered near the wall and with the increase in distance between the left wall and the divider isotherms is parallel to the horizontal walls.

5.3 Effect of solid wall thickness

The effect of wall thickness (W_1) is depicted in Figures 4(a) and (b) for Pr=6.2, Ra=10^6, $l_1 = 0.4$, h_∞ =100 W/m^{2}K, and K_r =10 for both pure fluid and nanofluid with ϕ = 5%. We see that the parameter wall thickness (W_1) affects the fluid and solid wall temperature as well as the flow characteristics. We notice here that the strength of the flow circulation is much higher for a thin solid wall.

For thick solid wall $(W_1 = 0.1)$ we observe that a circular main cell with two cores is formed within the enclosure, with the increasing of wall thickness the shape of the circulating cell becomes elliptical. This is because the fluid adjacent to the hotter wall has lower density than the fluid at the middle of the enclosure. As a result the fluid moves upwards from the middle section of hot wall, and reach at the upper part of the enclosure, and then it is cooled, so its density increase, then the fluid flows downwards at the right side, and finally it directed to the left wall. So a clockwise circulation is observed. We also notice here that for small wall thickness $(W_1 = 0.1)$ the strength of flow between the solid wall and the divider is higher than the larger value of wall thickness (W_1). On the other hand the effect of wall thermal resistance is directly proportional with wall thickness, so when wall thickness (W_1) increase from $W_1 = 0.1$ to $W_1 = 0.3$, the temperature (isotherm) gradient within the wall decrease with less convection amount within the enclosure. This is because with the increase of solid wall thickness (W_1), it behaves as an insulated material in this case. So we can say the thermal resistance of the wall is inversely proportional with its thermal conductivity and directly with its wall thickness.

The variation of average Nusselt number (Nu) is displayed in Figure 5 (a) along the hot wall against the solid volume fraction for different values of ambient convective heat transfer coefficient (h_∞). We see that when convective heat transfer coefficient (h_∞) increase the input heat flux is less absorbed by the inner flow, which point the reduction in the average Nusselt number (Nu).

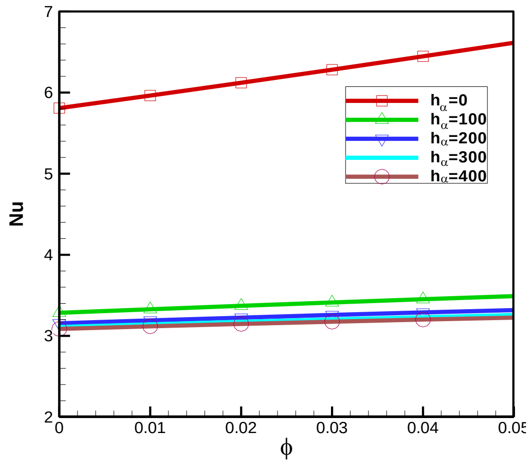

Figure 5 (a). Average Nusselt number at various convection heat transfer coefficient

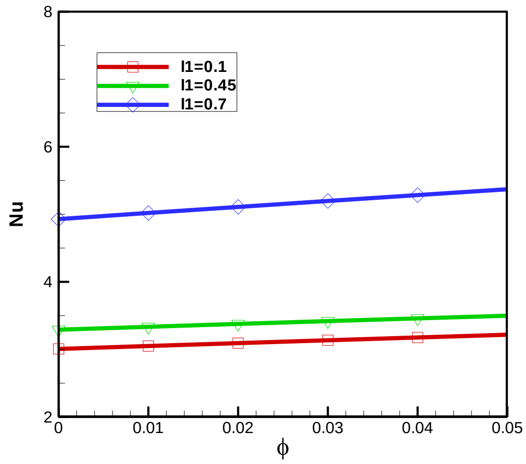

Figure 5 (b). Average Nusselt number at various divider positions

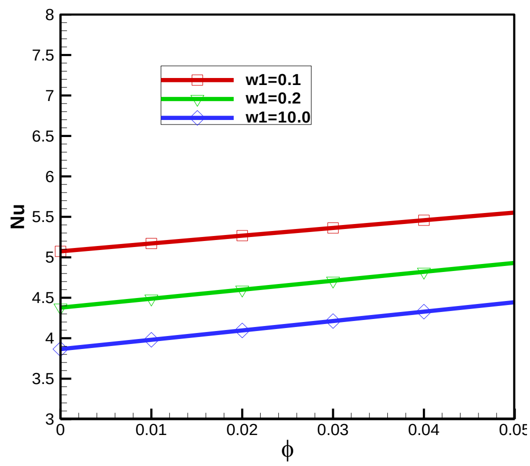

Figure 5 (c). Average Nusselt number at various solid wall thickness

Figure 5 (b) displays the variation of average Nusselt number (Nu) along the hot wall for various position of divider against the solid volume fraction. We see the increase in l_1, leads to the increase in the flow strength and hence the heat transfer rate enhances, so the average Nusselt number clearly increases with the increase of distance between the hot wall and the divider. When the divider moves from x= 0.1 to x=

0.45 the average Nusselt number increases faster because in this case convection dominate the conduction mode, but the rate of increasing average Nusselt number (Nu) decreases when the divider gets closure to the right wall, which is noticeable at ϕ=5% more effectively.

The effect of wall thickness (W_1) on Nusselt number (Nu) is depicted in Figure 5 (c) with the various values of solid volume fraction (ϕ).In general increasing wall (W_1) thickness decrease Nusselt number (Nu), because of the increased of thermal wall resistance which resist the heat transfer rate to the cavity.

6. CONCLUSION

The results of the numerical analysis lead to the following conclusions

- The heat transfer rate decrease by means of raising the convective heat transfer coefficient. It is also pragmatic that at a higher h_∞ the existence of the nanoparticles is more valuable.
- The location of the divider contributes to enhance the heat transfer rate. Moreover it has a significant effect on the thermal performance and flow pattern. One of the important finding in this regard is, the heat transfer is improved when the divider gets closure to the cold wall.
- The influence of the flow circulation of the fluid is much higher with thin wall. The strength of the circulation cell can be controlled by the thickness of the solid wall. The natural convection inside the nanofluid filled cavity decreases with increasing its wall thickness. The average Nuselt number trim down by increasing the wall thickness and Nu becomes constant for the highest values of the thickness parameter.

ACOKNOWLEDGEMENT

The work is supported by the department of mathematics, Bangladesh University of Engineering and Technology.

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NOMENCLATURES

C_p	Specific heat capacity (J/K)
B_0	magnetic field stregth (Wb/m^2)
g	gravitational acceleration (m/s^2)
Gr	Grashof number , $\beta g H^4 q'' / k\nu\alpha$
H	height of cavity (m)
h_∞	convective heat transfer coefficient (W/ m^{2}K)
Ha	Hartmann number
L	width of cavity (m)
l_1	distance between the wall and the divider(m)
K	thermal conductivity (W/mK)
K_r	conductivity ratio, $K_r = \frac{K_w}{K_{nf}}$
Nu	Nusselt number
Nu_l	local Nusselt number
Nu_{av}	average Nusselt number
p	pressure (N/ m^2)
P	dimensionless pressure
Pr	Prandtl number, ν_f / α_f
q''or q	heat generation per area,(W/ m^2)
T	temperature (K)
T_∞	ambient air flow
u,v	components of velocity (m/s)
U,V	dimensionless velocity components
x,y	Cartesian coordinates (m)

w	width of the thick wall (m)
w_1	dimensionless width of the thick wall
X,Y	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity (m^2s^{-1})
β	coefficient of thermal expansion (K^{-1})
ϕ	solid volume fraction
ΔT	Ref. temperature difference , $q''L/K_f$
θ	dimensionless fluid temperature
μ	dynamic viscosity of the fluid (m^2s^{-1})
ν	kinematic viscosity of the fluid (m^2s^{-1})
ρ	density of the fluid (kgm^{-3})
σ	fluid electrical conductivity ($\Omega^{-1}.m^{-1}$)
ψ	stream function

Subscripts

av	average
c	cold wall
f	pure fluid
nf	nanofluid
p	nanoparticles
s	solid
w	wall