

## Multi Objective Optimization of a Power Distribution System Based on Mixed Integer Programming



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### ABSTRACT

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#### Keywords:

*multi objective optimization, power distribution system, reconfiguration, mixed integer programming*

This paper proposes a multi objective optimization of a power distribution system. The technique used here is network reconfiguration. The goal is to find an optimal configuration of the network which minimizes power losses while avoiding voltage drops and overloads online. To attain that goal, we minimize the active power losses and the voltage deviation at each node of the network. The minimization of the voltage deviation here consists in minimizing the reactive power losses online. The tool used is the GAMS software. The problem is solved by Mixed Integer Programming (MIP) which is a deterministic method. This method is implemented on two standard IEEE test power distribution networks 33-node and 69-node. The results obtained are satisfactory compared to other techniques in the literature.

## 1. INTRODUCTION

Electric energy is a product of everyday consumption which is characterized by a certain number of parameters such as: current, voltage, power, energy, consumption, etc. The role of power transmission and distribution lines is to transport energy from production sites to consumption sites, with steps to lower the voltage level in transformer stations.

In the literature, several studies have been carried out on minimizing online power losses in a power distribution system. In the field of optimization, there are metaheuristic or approximate methods and exact or deterministic methods.

Boum et al. [1] proposed a reconfiguration of the 33-node IEEE test network using the SOS algorithm. It turns out that the execution time is low and active power losses are minimized compared to other algorithms such as the genetic algorithm. They propose a study which is limited to the standard network IEEE 33-node and consequently the results are not reliable to conclude from the efficiency of the algorithm.

Wang and Cheng [2] proposed an optimization of a 33-node IEEE power distribution network by reconfiguration by applying the Plant Growth Simulation Algorithm (PGSA). They do not take into account the improvement in the voltage profile at the network nodes.

Nguyen and Thruong [3] proposed a CSA algorithm (Cuckoo Search algorithm). The authors presented a reconfiguration of the 33-node and 69-node IEEE networks minimizing active power losses online and improving the voltage profile. They propose a method which does not clearly explain the technique used to minimize the voltage deviation at the level of the nodes.

Rayapudi et al. [4] proposed an HSA (Harmony Search Algorithm) method and applied it on a 33-node network. The results show an optimal optimal reconfiguration.

Muttaqi et al. [5] proposes an algorithm which determines the power losses for the different combinations of possible switches and selects the one which has the least power losses.

He compares these results with those of other methods. They have only applied their algorithm on an IEEE 33- node network which is insufficient to conclude from the effectiveness of a method.

Niknam [6] proposed a reconfiguration based on an evolutionary algorithm which is the combination of Honey Bee Mating Optimization and the Discrete Particle Swarm Optimization (DPSO) called DPSO-MBMO. This algorithm is implemented and compared to other methods.

Zhen-kun et al. [7] proposed a Hybrid Particle Swarm Optimization which combines binary PSO and Discrete PSO. This technique is used for the problem of optimal reconfiguration. The authors first apply the binary PSO which selects a group of branches which should be opened. Second, they implement the second DPSO algorithm which selects the branch that should be opened in the branch group. This method converges quickly and has good stability.

Li et al. [8] proposed a PSO hybrid combining the binary PSO and the Discrete PSO. The results obtained show that this algorithm is robust and converges quickly.

Khodr et al. [9] propose a reconfiguration of the power distribution network formulated as a Mixed Integer Nonlinear Programming (MINLP) problem with a nonlinear objective function with binary decision variables. Difficulties encountered in solving nonlinear optimization problems with binary decision variables force the authors to use partition techniques such as Benders decomposition. Benders' partition algorithm is a two-level decomposition technique, master and slave, which defines an iterative procedure between two ordered levels for the search for the optimal solution. The master level in their work, represents the decision problem which is defined as a MINLP problem while the slave level deals with the operations problem, being a nonlinear OPF problem. This method allows us an appropriate treatment of the non-convexity associated with the binary variables and divides the total problem in two under problems easy to solve. The master problem determines the new network

configuration and the open switches. This problem is solved by the branch and bound method using the CPLEX solver. This solution obtained by CPLEX is transferred to the slave sub-problem which checks the technical feasibility of the solution of the master problem. The problem is solved using the CONOPT solver. They do not study the improvement of the tension profile.

The reconfiguration of the power distribution network aims to choose a switching combination of branches of the network which optimizes certain performance criteria of the system while maintaining certain constraints. The ability to automatically reconfigure the network quickly and reliably is a key to the requirement for Smart Grid [10].

Hyder and Mahata [10] presents a reconfiguration of the distribution network using the binary programming method. The mathematical formulation of the problem is a quadratic form and solved using the QMIP (Quadratic Mixed Integer Programming) algorithm. This technique applies to IEEE 32 bus, 70 bus, 135 bus standard distribution networks. The solver used for solving the mathematical problem is GUROBI.

Abdulaziz et al. [11] formulate a model for planning the distribution of power from an electrical network to solve the sizing, timing and location of the distribution station and the problems of expansion of the supply lines simultaneously. The objective function of the model represents the present value of the investment costs and energy losses of the system which occur throughout the planning period. The objective function (cost value) is minimized with the following constraints: power limits, avoiding voltage drops. The resolution algorithm used is MILP (Mixed Integer Linear Programming). They must propose results which must first be validated on a standard IEEE network.

Recent developments in optimal power flow (OPF) for radial networks open the promise of greater sophisticated management of power distribution networks. Such sophistication is necessary to effectively exploit the new dynamic energy resources. However, to be useful, optimization tools must also take into account the management of preexisting technologies which involve discrete decision variables [12].

Briglia et al. [12] propose methods to include discrete decision variables on transformer tracks and capacitor banks as well as ON / OFF loads. The algorithms used are MILP (Mixed Integer Linear Programming) with CIR (Convex Integer Relaxations). The simulation results obtained are applied to an electrical energy distribution network in Uruguay.

Krengel et al. [13] present an approach of examining the city's districts as a whole in order to create a basis for investment decisions relating to the supply of energy to residential housing on the basis of an optimized conversion of distributed energy. Using MILP (Mixed Integer Linear Programming), an optimal configuration of energy conversion units such as heat pumps, photovoltaic systems or conventional heating systems is determined taking into account the investment as well as their costs. Operation over a period of 20 years. In addition, it is possible to design a district heating network using graph theory to take advantage of load balancing effects and economies of scale. The functionality of the model is illustrated by the analysis of a small-town district in Germany.

Koutsoukis et al. [14] use a hybrid non-linear method. They propose a planning method for distribution networks which determines the optimal network reinforcement and expansion plan taking into account the service factors provided by

decentralized production units. The proposed method calculates network investments and operational costs in order to meet future demand load as well as the connection of new loads and decentralized production units. Results on a 21-bus power distribution system validates the performance of the method.

Borghetti [15] discusses minimum power losses, the problem of configuration of power distribution networks, including generation integrated by the MILP method. A model is proposed which takes into account the operating constraints typical of distribution networks, connection intensity limits and bus voltage requirements as well as the presence of integrated generation. The precision of the results and the calculation performance of the project are proposed.

In this paper, Mixed Integer Programming is proposed to find an optimal configuration of a power distribution system that minimizes the active and reactive power losses and therefore the voltage deviation at the nodes of the system.

The proposed method is very effective. It is a deterministic method which seeks the exact solution. This algorithm is implemented on two standard IEEE networks (33-node and 69-node), which justifies the reliability of the method. The results obtained are better compared to other algorithms in the literature.

## 2. PROBLEM FORMULATION

Consider the single-line diagram of a line shown in Figure 1.

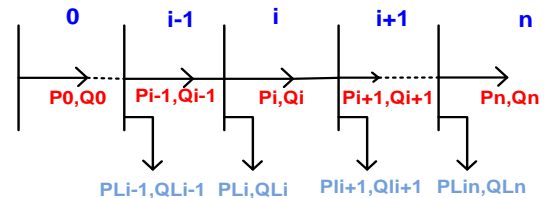


Figure 1. Single-line diagram of the line

Figure 1 shows a simplified single-line diagram of a part of an electrical power distribution network comprising branches and nodes. The customers are connected at the node level and are represented by their active and reactive powers.

The objective is to minimize line power losses by reconfiguring the distribution network. The minimization of the reactive power losses leads to the minimization of the voltage deviation at each node of the network.

The objective function of the problem is formulated as follows:

$$\min F = \min(f_1, f_2) \quad (1)$$

$f_1$ : Total active losses power ( $P_{loss}$ ).

$f_2$ : Total reactive losses power ( $Q_{loss}$ ).

$$f_1 = P_{loss}$$

$$f_2 = Q_{loss}$$

We then obtain the following function F:

$$\min F = \alpha f_1 + \beta f_2 \quad (2)$$

$\alpha$  and  $\beta$  are the weighting coefficients.

$$\min F = \alpha \sum_{i=0}^{n-1} \frac{P_i^2 + Q_i^2}{V_i^2} r_i + \beta \sum_{i=0}^{n-1} \frac{P_i^2 + Q_i^2}{V_i^2} x_i \quad (10)$$

## Constraints

During reconfiguration, it is necessary to avoid voltage drops and overloads at each node. So, you need the constraints for the objective function. They are:

$$V_{i\min} \leq V_i \leq V_{i\max}; i = 0, 1, 2, \dots, N_{bus} - 1 \quad (3)$$

$$0 \leq I_i \leq I_{i\max}; i = 0, 1, 2, \dots, N_{br} - 1 \quad (4)$$

With:

$V_i$ , min: Minimum value of the acceptable voltage at a node

$V_i$ , max: Maximum value of the acceptable voltage at a node

$I_i$ : Intensity of line current flowing through a branch

$I_i$ , max: Intensity of the maximum line current defined by the manufacturer

$N_{br}$ : Number of branch

$N_{bus}$ : Number of node

For the calculation of the power flow we use the following relationships:

$$P_{i+1} = P_i - r_i \frac{P_i^2 + Q_i^2}{V_i^2} - P_{Li+1} \quad (5)$$

$$Q_{i+1} = Q_i - x_i \frac{P_i^2 + Q_i^2}{V_i^2} - Q_{Li+1} \quad (6)$$

$$V_{i+1}^2 = V_i^2 - 2(r_i P_i + x_i Q_i) + (r_i^2 + x_i^2) \frac{P_i^2 + Q_i^2}{V_i^2} \quad (7)$$

With:

$P_i$ : Active power at node  $i$

$Q_i$ : réactive power at node  $i$

$P_{i+1}$ : Active power at node  $i+1$

$Q_{i+1}$ : Réactive power at node  $i+1$

$r_i$ : Resistance of the branch  $i$

$x_i$ : Reactance of the branch  $i$

$V_i$ : RMS value of the voltage at node  $i$

$V_{i+1}$ : RMS value of the voltage at node  $i+1$

$P_{Li+1}$ : Active power of the load connected at node  $i+1$

$Q_{Li+1}$ : Reactive power of the load connected at node  $i+1$

The total active power losses are expressed by the Eq. (8).

$$P_{loss} = \sum_{i=0}^{n-1} \frac{P_i^2 + Q_i^2}{V_i^2} r_i \quad (8)$$

$n = N_{br}$

The total reactive power losses are expressed by the Eq. (9).

$$Q_{loss} = \sum_{i=0}^{n-1} \frac{P_i^2 + Q_i^2}{V_i^2} x_i \quad (9)$$

The objective of the reconfiguration is to minimize the active and reactive power losses of during the power transit, the problem is formulated as follows:

The reconfiguration must obey the following rules:

- All loads must be supplied with electrical energy (if not the majority)
- The network configuration (power transit) must be radial.
- The network is linear.

In this study, the vector of decision variables giving the state of the switches is:

$$Y = [y_1 y_2 y_3 y_4 y_5 \dots y_{Nbr}]$$

After binary coding, the following state of the switches is obtained by branch:

$$S = [S_1 S_2 S_3 S_4 S_5 \dots S_{Nbr}]$$

## 3. PROBLEM SOLVING ALGORITHM

In this paper, the methodology for solving the problem involves calculating the power flow of the network.

### Description of the algorithm

#### Step 1: Initialization

This step defines the number of switches (branches) of the network and the objective functions.

#### Step 2: Declaration of parameters and decision variables

This step is used to declare the optimization parameters, variables and the binary branch connection decision variables.

#### Step 3: Network configuration

This step is used to configure the network and the characteristics of the lines and loads connected at the nodes are specified.

#### Step 4: Calculation of the voltages at the nodes of the initial configuration

After configuration, the voltages of each node are determined (initial configuration).

#### Step 5: Calculation of the active and reactive power losses of the initial configuration

Power losses are calculated using the recurrence equations for calculating power flow.

#### Step 6: Calculation of the voltage deviation at the nodes of the initial configuration

The highest voltage deviation of the network is determined.

#### Step 7: Minimize active power losses

This phase makes it possible to find the optimum of the objective function using Mixed Integer Programming (MIP).

#### Step 8: Minimize the losses of reactive power

This step allows you to find the optimum of the objective function using Mixed Integer Programming (MIP).

#### Step 9: Calculation of the voltage deviation of the new network configuration

The voltages are determined at the nodes of the configuration which minimizes the power active and reactive losses as well as the voltage deviation.

#### Step 10: Verification of constraints

#### Step 11: Show the result

#### 4. RESULTS AND DISCUSSION

The program is written in GAMS 23.5. The computer characteristics are: Processor: 1.70 GHz; RAM: 4.00 GB; OS: 64-bit WINDOWS 10.

In this paper, we implement our program on two standard IEEE test networks 33-node and 69-node.

##### 4.1 IEEE test network 33-node

###### Presentation of the structure:

The problem resolution flowchart is given in Figure 2.

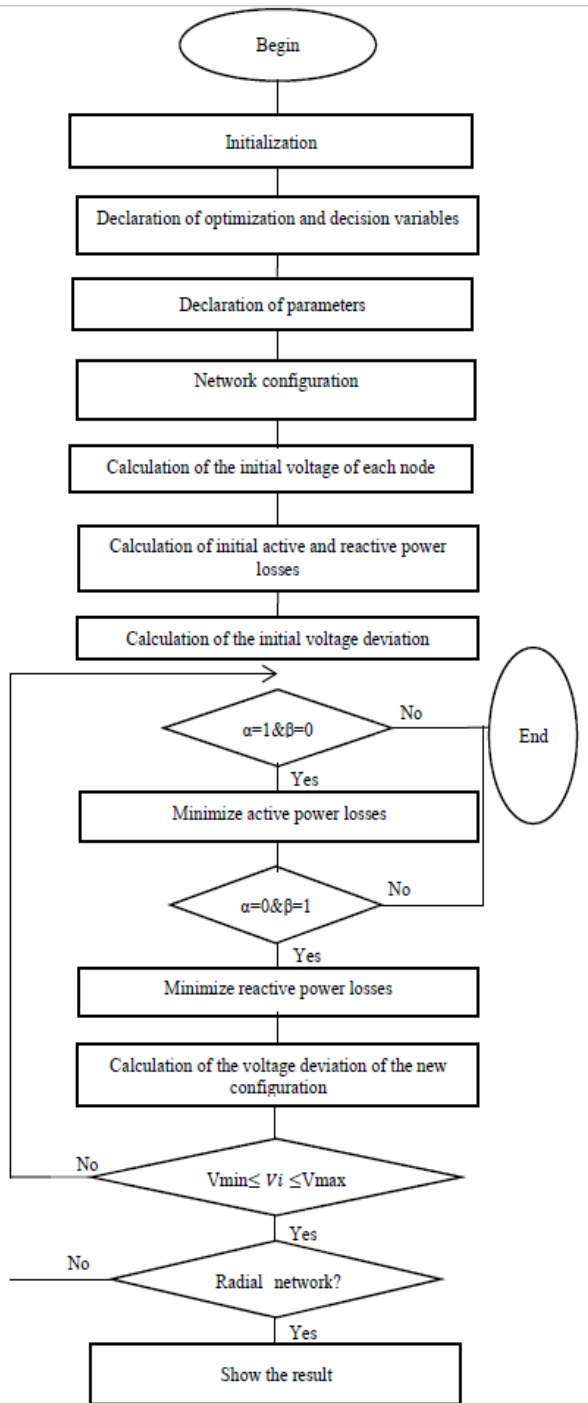


Figure 2. Problem resolution flowchart

The structure of the IEEE 33 node network used is given in Figure 3.

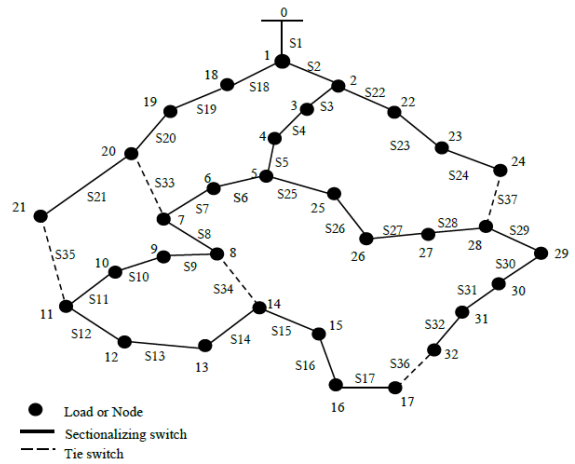


Figure 3. Structure of the IEEE 33 bus

###### Results

The RMS value of the voltage at node 0 of the network is 12.66 kV. The total active and reactive powers are 3715 kW and 2300 kVAr respectively. These data and the characteristics of this network as well as the powers of the charges are taken in work [2].

In this study, binary words are read from S1 to S37 and read from left to right.

The following binary code representing the state of the switches before reconfiguration is given in Figure 4.

In implementing the optimization program, the optimal binary reconfiguration code is given in Figure 5.

Table 1 presents the results obtained before and after reconfiguration.

Figure 6 presents the voltage profile before and after reconfiguration by applying the proposed algorithm for IEEE 33-node system.

Table 2 presents a comparative study between the results of the proposed algorithm and those found in the literature for an IEEE 33-node network.

###### Discussion

Table 1 presents the results before and after reconfiguration of the IEEE 33 bus network. Before the reconfiguration, the open switches are: S33, S34, S35, S36, S37. The active power losses are equal to 211.581 kW, the reactive power losses to 144.548 kVAr and the minimum node voltage is 0.9130 p.u which implies a network voltage deviation of 0.0870 p.u. After reconfiguration by the proposed algorithm, the open switches are S7, S9, S14, S32, S37 and the active power losses online are 136.765 kW, a reduction of 35.36%. The minimum voltage at the node of the network is 0.9348 p.u which implies a voltage deviation is 0.0652 p.u, a reduction of 25.05%.

Figure 6 shows an improvement of the tension profile of the nodes after reconfiguration using the proposed method. Before the reconfiguration, the minimum voltage was equal to 0.9130 p.u and after reconfiguration 0.9348 p.u. which implies a voltage deviation is 0.0652 p.u, a reduction of 25.05%.

These results are compared to those of the literature in Table 2. This comparative study shows that for the proposed algorithm, the minimum active power losses after reconfiguration are equal to 136.765 kW and the minimum voltage is 0.9348 p.u. These results are compared with those of the papers [3, 4] and are better. The results obtained by the HSA show that the minimum active power losses after reconfiguration are equal to 146.39 kW. The difference of

power losses with the proposed algorithm is 9.625 kW, a reduction of 6.57%. In addition the minimum voltage obtained is 0.9336 p.u which implies a difference of 0.0012 p.u with the proposed algorithm. The minimum active power losses obtained by the CSA method are equal to 138.87 kW. The

difference of power losses with the proposed technique is 2.105 kW, a reduction of 1.51%. The proposed method reduces the minimum losses by 6.57% compared to those of HSA and by 1.51% compared to those of CSA obtained.

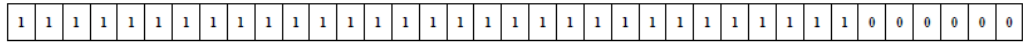


Figure 4. State of the switches for the 33-node system before reconfiguration

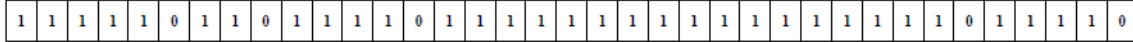


Figure 5. State of the switches for the 33-node system after reconfiguration

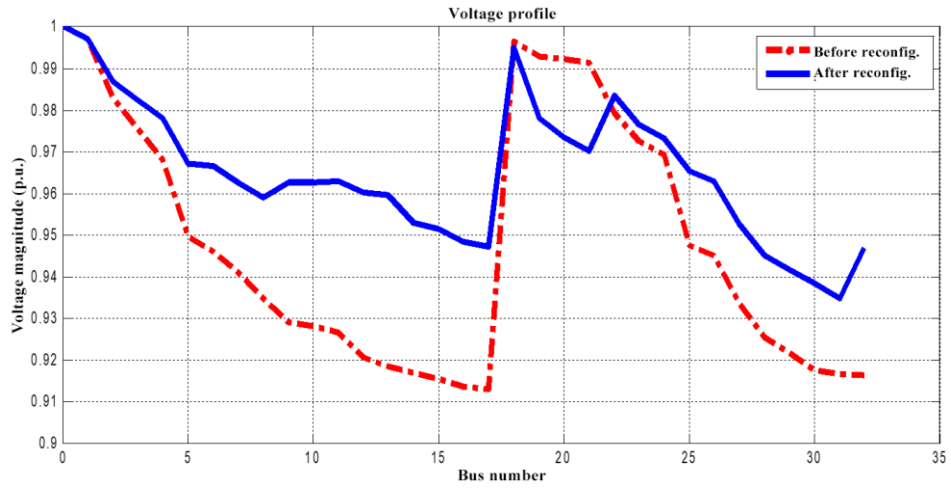


Figure 6. Voltage profile for the 33-node system before and after reconfiguration

Table 1. Reconfiguration of the IEEE 33 Bus network

	Before reconfiguration	After reconfiguration	Reduction(%)
Open switches	S33, S34, S35, S36, S37	S7, S9, S14, S32, S37	/
Real losses power (kW)	211.581	136.765	35.36
Reactive losses power (kVAr)	144.548	35.229	75.62
Vmin (p.u)	0.9130	0.9348	/
Vde(p.u)	0.0870	0.0652	25.05

Vmin: Minimal voltage at node; Vde: Voltage deviation

Table 2. Comparative study of the results of other algorithms for a network of 33 bus

Methods	Open switches	Power loss	Vde (p.u)(kW)	Vmin (p.u)
Initial	S33, S34, S35, S36, S37	211.581	0.0870	0.9130
Proposed method	S7, S9, S14, S32, S37	136.765	0.0652	0.9348
CSA [3]	S7, S9, S14, S32, S37	138.87	0.0576	0.94235
PSO [3]	S7, S9, S14, S32, S37	138.87	0.0576	0.94235
HSA [4]	S7, S10, S14, S28, S36	146.39	0.0664	0.9336

4.2 IEEE 69 bus test network

Presentation of the structure

The RMS value of the voltage at node 0 of the network is 12.66 kV. The total active and reactive powers are respectively 3802 kW and 3696 kVAr. The characteristics of this network are the same as those used in work [3].

Figure 7 shows the structure of the network. While table 3 shows the different switches associated with each branch.

Results

In this study, the binary words are from S1 to S73 and read from left to right. The following binary code representing the state of the switches before reconfiguration is given in Figure 8. In implementing the optimization program, the optimal binary reconfiguration code is given in Figure 9. Table 4 presents the results obtained before and after reconfiguration. Figure 10 presents the voltage profile before and after reconfiguration by applying the proposed algorithm for IEEE 33-node system. Table 5 presents a comparative study between the results of the proposed algorithm and those found in the literature for an IEEE 69-node network.

**Table 3.** Switches associated with each branch

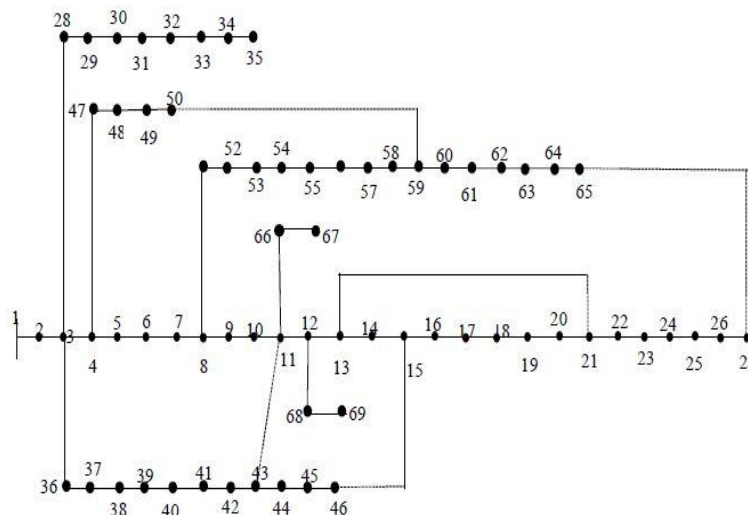
Bus to bus	Switches	Bus to bus	Switches	Bus to bus	Switches
1-2	S1	26-27	S26	51-52	S51
2-3	S2	3-28	S27	52-53	S52
3-4	S3	28-29	S28	53-54	S53
4-5	S4	29-30	S29	54-55	S54
5-6	S5	30-31	S30	55-56	S55
6-7	S6	31-32	S31	56-57	S56
7-8	S7	32-33	S32	57-58	S57
8-9	S8	33-34	S33	58-59	S58
9-10	S9	34-35	S34	59-60	S59
10-11	S10	3-36	S35	60-61	S60
11-12	S11	36-37	S36	61-62	S61
12-13	S12	37-38	S37	62-63	S62
13-14	S13	38-39	S38	63-64	S63
14-15	S14	39-40	S39	64-65	S64
15-16	S15	40-41	S40	11-66	S65
16-17	S16	41-42	S41	66-67	S66
17-18	S17	42-43	S42	12-68	S67
18-19	S18	43-44	S43	68-69	S68
19-20	S19	44-45	S44	11-43	S69
20-21	S20	45-46	S45	13-21	S70
21-22	S21	4-47	S46	15-46	S71
22-23	S22	47-48	S47	50-59	S72
23-24	S23	48-49	S48	27-65	S73
24-25	S24	49-50	S49		
25-26	S25	8-51	S50		

**Table 4.** Reconfiguration of the IEEE 69 Bus network

	Before reconfiguration	After reconfiguration	Reduction(%)
Open switches	S69, S70, S71, S72, S73	S14, S57, S61, S69, S70	/
Real losses power (kW)	224.95	97.917	56.47
Reactive losses power (kVAr)	37.538	23.256	38.04
Vmin (p.u)	0.9100	0.9500	/
Vde(p.u)	0.0900	0.0500	44.44

**Table 5.** Comparative study of the results of other algorithms for a network of 69 bus

Methods	Open switches	Power loss	Vde (p.u)(kW)	Vmin (p.u)
Initial	S69, S70, S71, S72, S73	224.95	0.0900	0.9100
Proposed method	S14, S57, S61, S69, S70	97.917	0.0500	0.9500
CSA [3]	S14, S57, S61, S69, S70	98.5680	0.0505	0.9495
PSO [3]	S14, S57, S61, S69, S70	98.5680	0.0505	0.9495
HSA [16]	S13, S18, S56, S61, S69	99.35	0.0572	0.9428



**Figure 7.** Structure of IEEE 69 bus network





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