

MHD CONVECTIVE FLOW PAST A VERTICAL POROUS PLATE IN SLIP-FLOW REGIME WITH CHEMICAL REACTION, RADIATION, ABSORPTION AND VARIABLE VISCOSITY

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ABSTRACT

An attempt is made to investigate the laminar flow of a viscous, electrically conducting and heat generation/absorbing fluid over a porous plate in slip-flow regime on a continuously vertical permeable surface in the presence of a radiation, a first order homogeneous chemical reaction and variable viscosity. A magnetic field of uniform strength is assumed to be applied transversely to the direction of main flow. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. Expression for the velocity, temperature and concentration are obtained. The influences of the various parameter entering the problem namely, Grashof number, Prandtl number, Schmidt number, heat absorption parameter, radiation parameter, variable viscosity, rarefaction parameter and permeability of the porous medium on the velocity and skin friction are demonstrated graphically. These parameters have significant effect on dimensionless velocity and skin friction.

1. INTRODUCTION

In the slip-flow regime, the Navier-Stokes equations can only be employed provided tangential slip-velocity boundary conditions are implemented along the walls of the flow domain. In the region between the slip flow and free flow, the collisions between the molecules are equally significant as the collisions between the molecules and the surface. In fact, nearly two hundred years ago Navier proposed a general boundary condition that incorporates the possibility of fluid slip at a solid boundary. Navier's proposed condition assumes that the velocity, V_x , at a solid surface is proportional to the shear stress at the surface [Navier 1823, Goldstein 1965] and is given by $V_x = \gamma \frac{dv_x}{dy}$. Here, γ is the slip-strength or slip-

coefficient. If $\gamma = 0$ then the general assumed no-slip boundary condition is obtained. If γ is finite, fluid slip occurs at the wall but its effect depends upon the length scale of the flow. The above relation states that the velocity of the fluid at the plates is linearly proportional to the shear stress at the plate. Also, one could impose nonlinear slip boundary conditions (e.g. Yu & Amed [1]).

There is strong evidence to support the use of Navier-Stokes and energy equations to model slip flow problems in micro and nano-scales where the continuum assumption is no longer valid; while the boundary conditions are modified by including velocity slip and temperature jump at the channel walls to include the non-continuum effects. The most commonly used Navier model for the partial slip boundary conditions states that the liquid slip velocity is proportional to the rate of shear normal to the surface. The proportionality coefficient, the so-called slip length, is

defined as the extrapolated distance from the wall where the fluid tangential velocity component vanishes.

The fluid slippage phenomenon at the solid boundaries occur in several instances like flows through micro channels and nano channels, hydrophilic flows over hydrophobic boundaries at the micro and nano scales, applications where a thin film of light oil is attached to the moving plates or when the surface is coated with special coatings such as thick monolayer of hydrophobic octadecyltrichlorosilane (Derek et al.[2]), flows of fluids with concentrated suspensions, fluid slippage at the wall under vibrating conditions, flow of rarefied gases through nanopores, airflow in micro-electro-mechanical-systems (MEMS), velocity slip for lubrication of roller bearings, blunt body flow, rarefied subsonic air flow, hypersonic shock structure, high altitude and low density hypersonic flows in continuum transition regime, subsonic / supersonic flows in micro channels, single-phase flows and miscible and immiscible two-phase flows past homogenous surfaces and chemically patterned surfaces, applications in vacuum technology etc. Recently, several researchers have suggested that the no-slip boundary may not be suitable hydrophilic flows over hydrophobic boundaries at both the micro and nano scale. Sharma and Chaudhary [3] analyzed the effect of variable suction on transient free convection viscous incompressible flow past a vertical plate with periodic temperature variations in slip-flow regime. The closed form solution for steady periodic and transient velocity field under slip condition have been studied by Khaled and Vafai [4]. The slip-condition on MHD steady flow in a channel with permeable boundaries has been discussed by Makinde and Osalusi [5]. Ahmed and Kalita [6] investigated the effects of the transverse magnetic field and suction parameter on the oscillatory free convective flow past a vertical plate in slip – flow regime with variable suction and periodic plate

temperature. The analysis of an unsteady MHD free convection flow of a viscous incompressible electrically conducting fluid past on infinite vertical porous plate subjected to constant suction and heat sink was made by Sahoo et.al [7]. Jain and Sharma [8] have investigated the effect of viscous heating on flow past a vertical plate in slip-flow regime with periodic temperature variations. Moreover, Pal and Talukdar [9] reported perturbation analysis of unsteady magnetohydrodynamic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable with thermal radiation and chemical reaction neglecting the Soret effect and slip due to jump in temperature. Recently, Hakeem et.al [10] analysed the problem of Magneto convective heat and mass transfer over a porous plate with effects of chemical reaction, radiation absorption and variable viscosity.

The object of the present work is to investigate the effect of the rarefaction parameter, radiation absorption, mass diffusion, chemical reaction parameter and heat source parameter of heat generating fluid past a vertical porous plate in slip-flow regime subjected to variable suction in the presence of chemical reaction and variable viscosity. This work is an extension of the work done by Abdul Hakeem, Muthukumar, Sathiyathan and Ganga [10] to slip-flow regime case.

2. FORMATION OF THE PROBLEM

Let us consider a flow of an unsteady two dimensional convective flow of a viscous Incompressible, electrically conducting and heat absorbing fluid past an infinite vertical permeable moving plat in slip-flow regime Under the action of transverse magnetic field by making the following assumptions:

- 1) Reynolds numbers are so small that the induced magnetic field can be neglected.
- 2) All the fluid properties are isotropic and constant except that influence of density variation with temperature and viscosity μ which is assumed to vary as an inverse linear function of temperature \bar{T} .
- 3) The concentration of the diffusing species which are present are so less so that the Soret and Dufour effects are negligible.

The \bar{x} axis is taken along the upward vertical plate and \bar{y} axis normal to the plate. A uniform magnetic field is applied transversely in presence of thermal and concentration buoyancy effects with varying temperature dependent viscosity. The viscosity μ is assumed in the following form:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} \left[1 + \delta (\bar{T} - \bar{T}_\infty) \right]$$

$$= E (\bar{T} - T_r), \text{ where } E = \frac{\delta}{\mu_\infty} \text{ and } \bar{T}_r = \bar{T}_\infty - \frac{1}{\delta}$$

The governing equations are

$$\text{Continuity equation: } \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \dots \dots \dots (1)$$

Momentum equation:

$$\rho \left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = \frac{\partial}{\partial \bar{y}} \left(\mu \frac{\partial \bar{u}}{\partial \bar{y}} \right) - \nu \rho \frac{\bar{u}}{\bar{K}} + \nu \rho \frac{\bar{U}_\infty}{\bar{K}} - \sigma B_0^2 \bar{u} + \sigma B_0^2 \bar{U}_\infty + \rho g \beta_r (\bar{T} - \bar{T}_\infty) +$$

$$\rho g \beta_c (\bar{C} - \bar{C}_\infty) + \rho \frac{\partial \bar{U}_\infty}{\partial \bar{t}} \dots \dots \dots (2)$$

Energy equation:

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{k}{\rho C_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{Q_0}{\rho C_p} (\bar{T} - \bar{T}_\infty) + \bar{Q}_1 (\bar{C} - \bar{C}_\infty) \dots \dots \dots (3)$$

Mass diffusion equation:

$$\frac{\partial \bar{C}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} - C_l (\bar{C} - \bar{C}_\infty) \dots \dots \dots (4)$$

The relevant boundary conditions are

$$\bar{y} = 0 : \bar{u} = \bar{h} \left(\frac{\partial \bar{u}}{\partial \bar{y}} \right), \bar{T} = \bar{T}_w + \varepsilon (\bar{T} - \bar{T}_\infty) e^{\bar{n}\bar{t}},$$

$$\bar{C} = \bar{C}_w + \varepsilon (\bar{C}_w - \bar{C}_\infty) e^{\bar{n}\bar{t}} \dots \dots \dots (5)$$

$$\bar{y} \rightarrow \infty : \bar{u} \rightarrow \bar{U}_\infty = U_0 (1 + \varepsilon e^{\bar{n}\bar{t}}), \bar{T} \rightarrow \bar{T}_\infty, \bar{C} \rightarrow \bar{C}_\infty \dots \dots (6)$$

Suction velocity normal to the plate is assumed in the form

$$\bar{v} = -V_0 (1 + \varepsilon A e^{\bar{n}\bar{t}}) \dots \dots \dots (7)$$

where A is a positive constant, ε and εA are small values less than unity and V_0 is scale of suction velocity.

We introduce the following non-dimensional quantities

$$y = \frac{V_0}{U_0} \bar{y}, t = \frac{V_0^2}{U_0^2} \bar{t}, n = \frac{V_0}{U_0} \bar{n}, \bar{u} = \frac{\bar{u}}{U_0}, h = \frac{V_0 \bar{h}}{U_0},$$

$$C_T = \frac{\bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \theta_r = \frac{\bar{T}_r - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, C = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty},$$

$$G_r = \frac{\nu g \beta_r (\bar{T}_w - \bar{T}_\infty)}{U_0 V_0^2}, U_\infty = \frac{\bar{U}_\infty}{U_0}, G_r = \frac{\nu g \beta_r (\bar{T}_w - \bar{T}_\infty)}{U_0 V_0^2},$$

$$G_m = \frac{\nu g \beta_c (\bar{C}_w - \bar{C}_\infty)}{U_0 V_0^2}, M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, Q_1 = \frac{\nu \bar{Q}_1 (\bar{C}_w - \bar{C}_\infty)}{(\bar{T}_w - \bar{T}_\infty) V_0^2},$$

$$P_r = \frac{\nu}{\alpha}, \gamma = \frac{\nu C_l}{V_0^2}, S_c = \frac{\nu}{D}, R = \frac{Q_0 \nu}{\rho C_p V_0^2}, K = \frac{V_0^2}{U_0^2} \bar{K},$$

$$N = M + \frac{1}{K}$$

Using the equation (7) and Non-dimensional quantities in the equations (2), (3) and (4), we get the non-dimensional governing equations :

$$\frac{\partial}{\partial t} (u - U_\infty) - (1 + \varepsilon A e^{nt}) \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{\theta'}{\theta - \theta_r} \frac{\partial u}{\partial y} - N(u - U_\infty) + G_r \theta + G_m C \dots \dots (8)$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - R \theta + Q_1 C \dots \dots \dots (9)$$

$$\frac{\partial C}{\partial t} - (1 + \varepsilon A e^{nt}) \frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - \gamma C \dots \dots \dots (10)$$

The corresponding boundary conditions are

$$y=0 : u=h \frac{\partial u}{\partial y}, \theta=1+\varepsilon e^{nt}, C=1+\varepsilon e^{nt} \dots \dots \dots (11)$$

$$y \rightarrow \infty : U \rightarrow U_\infty = 1 + \varepsilon e^m, \theta \rightarrow 0, C \rightarrow 0 \dots (12)$$

3. METHOD OF SOLUTIONS:

The equations (8), (9) and (10) are coupled, non linear partial differential equations and these can not be solved in closed form. Moreover, these equations can be solved by reducing to a set of ordinary differential equations in dimensionless form. It is done by representing the velocity, temperature and concentration of the fluid in the neighbourhood of the plat. In order to solve the equations (8),(9) and (10) under the boundary conditions given by (11) and (12), we assume that the non dimensional velocity, temperature and concentration are in the form :

$$u = u_0(y) + \varepsilon e^m u_1(y) + o(\varepsilon^2) \dots (13)$$

$$\theta = \theta_0(y) + \varepsilon e^m \theta_1(y) + o(\varepsilon^2) \dots (14)$$

$$C = C_0(y) + \varepsilon e^m C_1(y) + o(\varepsilon^2) \dots (15)$$

Substituting (13)-(15) in the equations (8)-(10) and equating the harmonic and non-harmonic terms and neglecting the higher order terms of $o(\varepsilon^2)$, we obtain the following differential equations:

Zeroth order:

$$u_0'' + \left[1 - \frac{\theta_0'}{\theta_0 - \theta_r} \right] u_0' - Nu_0 = -Gr\theta_0 - GmC_0 - N \dots (16)$$

$$\theta_0'' + Pr\theta_0' - RP_r\theta_0 = -Q_1P_rC_0 \dots (17)$$

$$C_0'' + ScC_0' - \gamma ScC_0 = 0 \dots (18)$$

The relevant boundary conditions are

$$\text{At } y=0 : u_0 = h \frac{\partial u_0}{\partial y}, \theta_0 = 1, C_0 = 1 \dots (19)$$

$$y \rightarrow \infty : u_0 \rightarrow 1, \theta_0 \rightarrow 0, C_0 \rightarrow 0 \dots (20)$$

First order:

$$u_1'' + \left[1 - \frac{\theta_0'}{\theta_0 - \theta_r} \right] u_1' - (N+n)u_1 = \frac{u_0'}{\theta_0 - \theta_r} \left[\theta_1' - \frac{\theta_1\theta_0'}{\theta_0 - \theta_r} \right] - Au_0' - N - n - Gr\theta_1 - GmC_1 \dots (21)$$

$$\theta_1'' + Pr\theta_1' - P_r(R+n)\theta_1 = -P_rA\theta_0' - QC_1P_r \dots (22)$$

$$C_1'' + ScC_1' - S_c(\gamma+n)C_1 = -S_cAC_0' \dots (23)$$

The relevant boundary conditions are

$$\text{At } y=0 : u_1 = h \frac{\partial u_1}{\partial y}, \theta_1 = 1, C_1 = 1 \dots (24)$$

$$y \rightarrow \infty : u_1 \rightarrow 1, \theta_1 \rightarrow 0, C_1 \rightarrow 0 \dots (25)$$

The solutions of the equations (16)-(18) subject to the boundary conditions (19) and (20) are:

$$u_0 = A_{10}e^{-A_0y} + \frac{Gr + Gm + N}{N} \dots (26)$$

$$\theta_0 = e^{-A_2y} + A_3(e^{-A_1y} - e^{-A_2y}) \dots (27)$$

$$C_0 = e^{-A_3y} \dots (28)$$

The solutions of the equations (21)-(23) subject to the boundary conditions (24) and (25) are

$$u_1 = \frac{A_{11}}{N+n} \left[\frac{e^{-A_0y}}{1+hA_0} - 1 \right] \dots (29)$$

$$\theta_1 = A_{16}e^{-A_8y} + A_{13}e^{-A_1y} + A_{14}e^{-A_2y} + A_{15}e^{-A_7y} \dots (30)$$

$$C_1 = e^{-A_7y} + A_{12}(e^{-A_1y} - e^{-A_7y}) \dots (31)$$

where A's, B's and m's are given in Appendix.

The complete solution of equations (8)-(10) with the boundary conditions (11) and (12) and using equations (13)-(15), we get the general solution as follows:

$$u(y,t) = A_{10}e^{-A_0y} + \frac{Gr + Gm + N}{N} + \varepsilon e^m \left[\frac{A_{11}}{N+n} \left(\frac{e^{-A_0y}}{1+hA_0} - 1 \right) \right] \dots (29)$$

$$\theta(y,t) = e^{-A_2y} + A_3(e^{-A_1y} - e^{-A_2y}) + \varepsilon e^m (A_{13}e^{-A_1y} + A_{14}e^{-A_2y} + A_{15}e^{-A_7y} + A_{16}e^{-A_8y}) \dots (30)$$

$$C(y,t) = e^{-A_3y} + \varepsilon e^m [e^{-A_7y} + A_{12}(e^{-A_1y} - e^{-A_7y})] \dots (31)$$

COEFFICIENT OF SKIN FRICTION:

The coefficient of skin friction in the non-dimensional form at the plate $y=0$ is given by

$$\tau = \left(\frac{\partial u}{\partial y} \right)_{y=0} = -A_0A_{10} - \frac{\varepsilon A_0A_{11}}{(N+n)(1+hA_0)} e^m$$

4. RESULTS AND DISCUSSION

Some numerical calculations have been carried out for the non-dimensional velocity u and skin friction coefficient τ for the flow under consideration and their behaviour have been discussed for variations in the governing parameters. We have restricted our investigation to Prandtl number Pr equal to 0.7, 1 and 7 which corresponds to air, electrolyte and water respectively. Here we fixed Eckert number $\varepsilon = 0.2$.

Figures 1 to 12 show the effects of h , Pr , Gm , M , γ , θ_r , t , Q_1 , Gr , K , Sc and R respectively over dimensionless velocity profile u versus y . The result obtained show that velocity is affected by these physical parameters. Figures 1, 2, 3 and 4 depict the effects of rarefaction parameter h , Prandtl number Pr , Solutal Grashof number Gm and Hartmann number M respectively over the velocity u . The effect of these parameters is to decelerate the magnitude of the velocity. Prandtl number is the ratio of viscous force to the thermal force. Pr increases means momentum diffusivity increases or thermal diffusivity decreases. Therefore magnitude of velocity decreases with the increase of momentum diffusivity (In figure 2). Solutal Grashof number is the ratio of buoyancy force due to concentration gradient to viscous force. Buoyancy force increases Solutal Grashof number Gm . As seen from the fig. 3, Buoyancy force decreases the magnitude of the velocity. In figure 4, it is seen that magnitude of u decreases with the increasing value of M . This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the convection flow. Figure 5 depicts the influence of the chemical reaction parameter γ . It is observed u increases with increasing γ .

Figures 6, 7, 8, 9 and 10 display the influence of variable viscosity θ_r , time t , radiation parameter Q_1 , Grashof number for heat transfer Gr and permeability parameter K over the dimensionless velocity. The effect of these parameters are to accelerate the magnitude of the velocity. It is concluded (from figure 9) that by increasing Grashof number Gr , $|u|$ increases at the wall, which shows that Gr causes to strengthen the fluid slip at the wall. Since the effects of

Grashof number tends to increase of buoyancy force which results in promotion of velocity profile. In figures 11 and 12, we observed that Schmidt number Sc and heat source parameter R are to accelerate the velocity fields.

From these twelve figures we see that the velocity increases or decreases rapidly near the wall but it remains uniform as we move from the wall. Also, the parameters h , Pr , M , θ_r , t , Q_1 , Gr , K have more prominent effect than γ , Gm , Sc and R .

Variation of skin- friction coefficient τ against rarefaction parameter h various values of Gr , θ_r , K , t , Q_1 , γ , Sc , R , M and Pr are plotted through figures 13 to 22. The thermal Grashof number Gm increases the magnitude of the skin friction coefficient τ which is seen in figure 13. The increase in variable viscosity θ_r results in increase of the magnitude of τ as observed from figure 14.

Figure 15 displays the effects of the permeability parameter K on the skin friction coefficient. As shown, $|\tau|$ is increasing with the increasing dimensionless porous medium parameter K . The variation of τ for different values of t is elucidated in figure 16. The effect of time t is to increase the magnitude of the skin friction coefficient. Figure 17 brings out the significance of radiation parameter Q_1 over τ . The radiation parameter increases the magnitude of the skin friction coefficient.

Figures 18, 19 and 20 illustrate the effects of chemical reaction parameter γ , Schmidt number Sc and heat source parameter R over τ . These figures show that τ increase with the increase of γ , Sc and R .

The effects of Hartmann number M and Prandtl number Pr on τ are depicted in figures 21 and 22. It is interesting to note that the effect of magnetic field is to decrease the magnitude of τ which is seen from figure 21. From figure 22, it can be seen that the increase in the value of Prandtl number Pr leads to decrease $|\tau|$.

From the figures 13 to 22, we observed that τ or $|\tau|$ decrease as increasing h and it increases or decreases rapidly near the wall/plate but it remains uniform as we move from the plate. Also variation of τ is more prominent for small values of h .

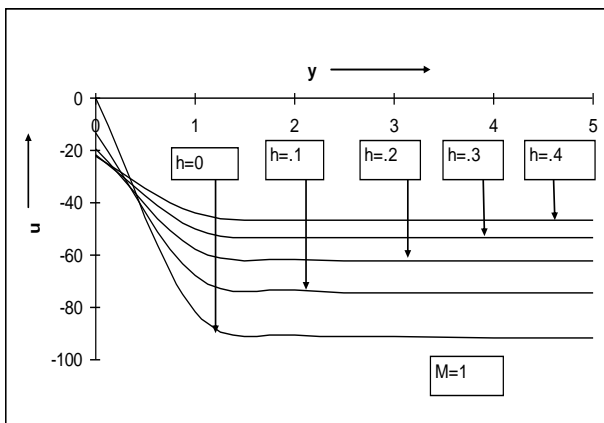


Figure1-Velocity profile u versus rarefaction parameter h against y for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, \epsilon=.2, t=3, R=2$.

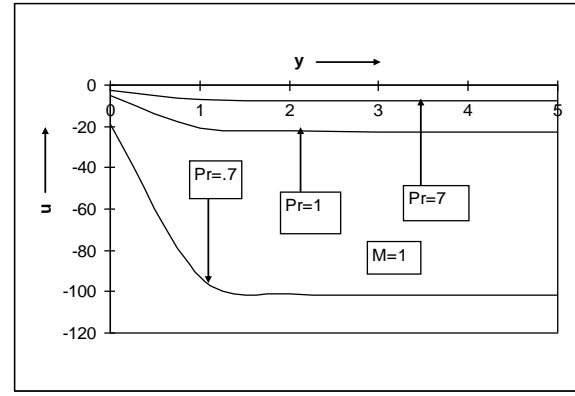


Figure 2- Velocity profile u versus Prandtl number Pr against y for $Q_1=2, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, h=.1, \epsilon=.2, t=3, R=2$.

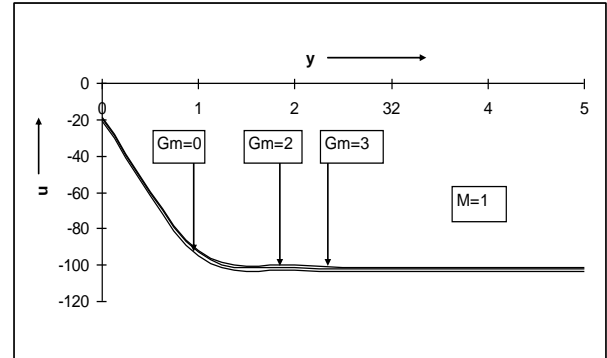


Figure 3-Velocity profile u versus Grashof number Gm against y for $Pr=.7, Sc=.6, Q_1=2, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, h=.1, \epsilon=.2, t=3, R=2$.

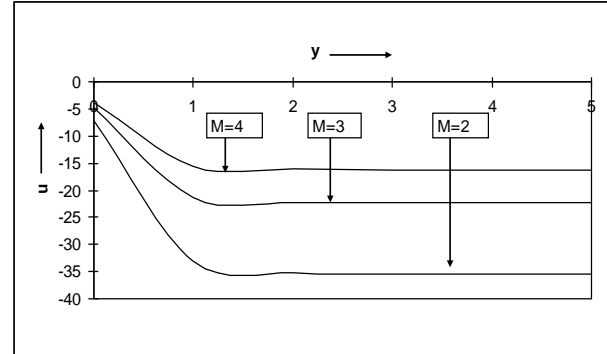


Figure 4-Velocity profile u versus magnetic field M against y for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, \epsilon=.2, t=3, R=2$.

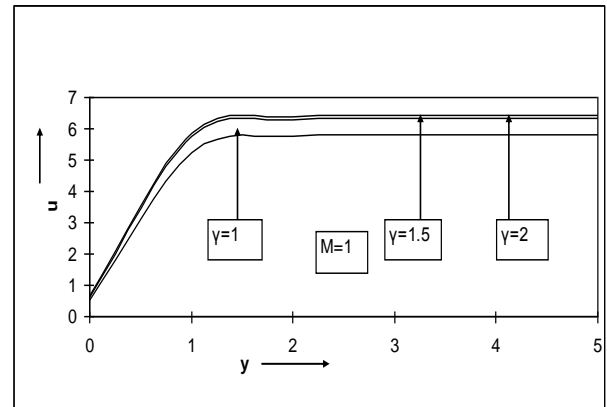


Figure 5-Velocity profile u versus Chemical reaction γ against y for $Pr=.7, Sc=.6, A=.5, Gr=3, Gm=2, K=2, \theta_r=-.6, n=.1, h=.1, \epsilon=.2, t=3, R=2$.

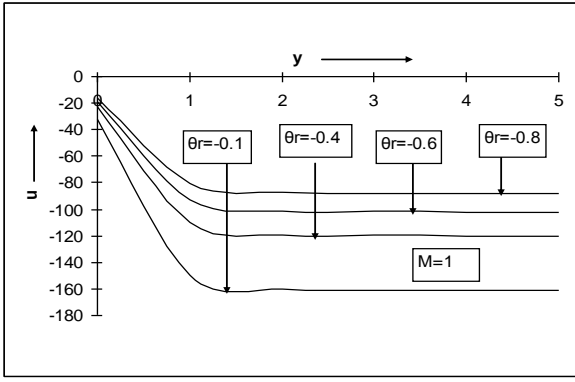


Figure 6-Velocity profile u versus Variable viscosity θ_r against y for $Pr=.7, Sc=.6, Q1=2, \gamma=.5, A=.5, Gr=3, Gm=2, K=2, n=.1, h=.1, \epsilon=.2, t=3, R=2$.

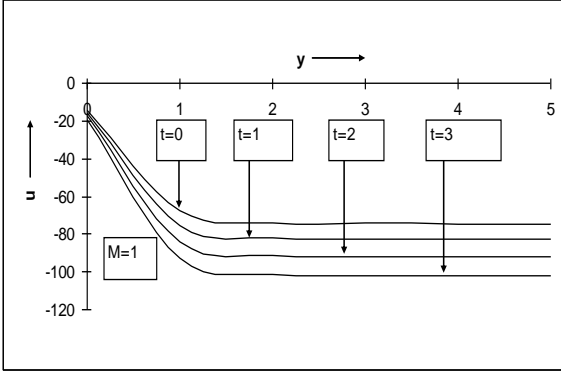


Figure 7-Velocity profile u versus time t against y for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, \epsilon=.2, t=3, R=2$.

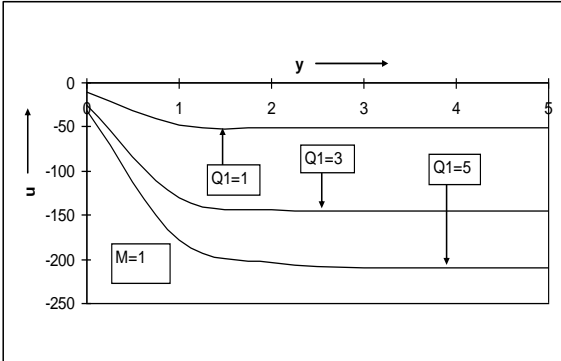


Figure 8-Velocity profile u versus radiation parameter $Q1$ against y for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, h=.1, \epsilon=.2, t=3, R=2$.

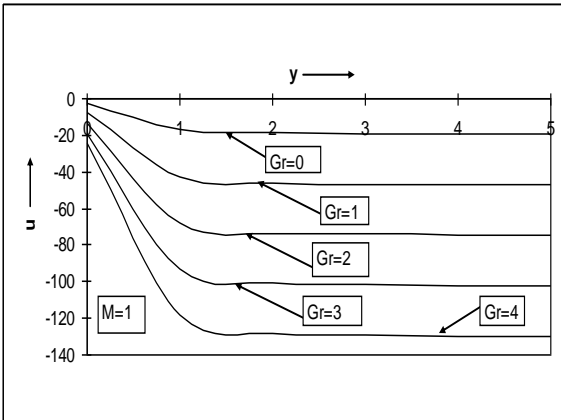


Figure 9-Velocity profile u versus Gr against y for $Pr=.7, Sc=.6, \gamma=.5, A=.5, h=.1, Gm=2, K=2, \theta_r=-.6, n=.1, \epsilon=.2, t=3, R=2$.

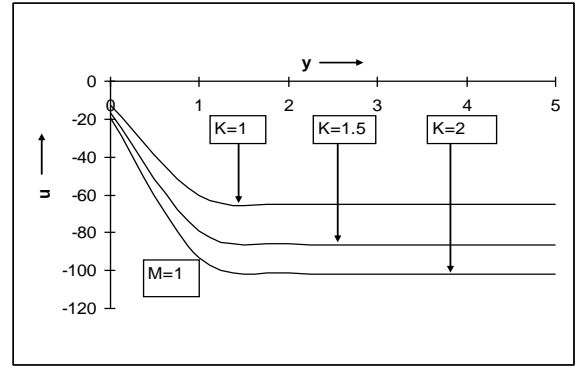


Figure 10-Velocity profile u versus rarefaction K against y for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, h=.1, \theta_r=-.6, n=.1, \epsilon=.2, t=3, R=2$.

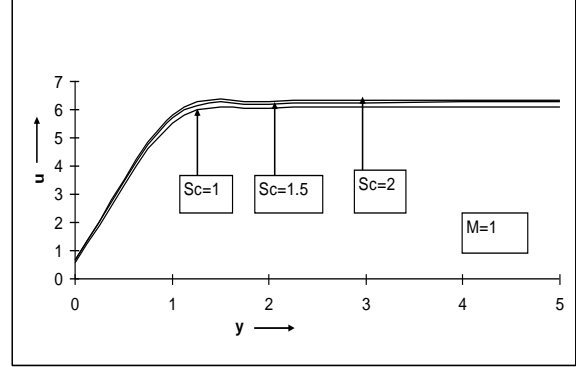


Figure 11-Velocity profile u versus Schmidt number Sc against y for $Pr=.7, Q1=2, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, h=.1, \epsilon=.2, t=3, R=2$.

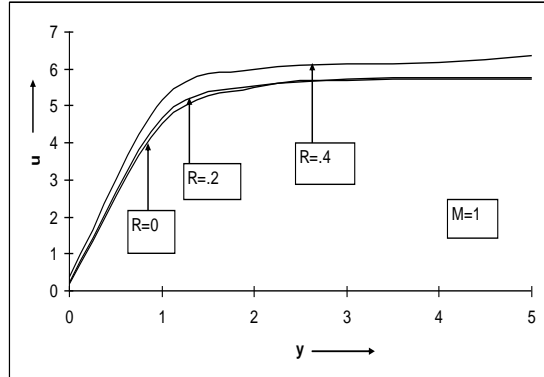


Figure 12-Velocity profile u versus rarefaction R against y for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, \epsilon=.2, t=3, h=.1$.

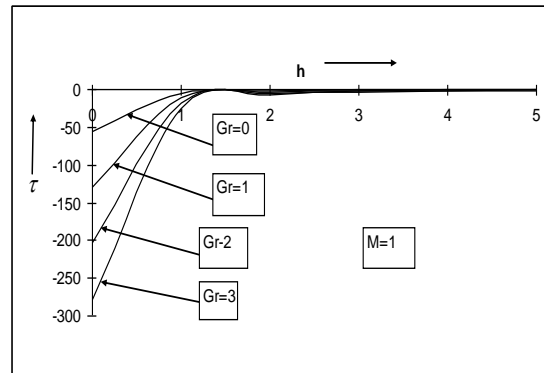


Figure 13-Co-efficient of skin friction τ versus Gr against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-.6, n=.1, \epsilon=.2, t=3, R=2, y=0$.

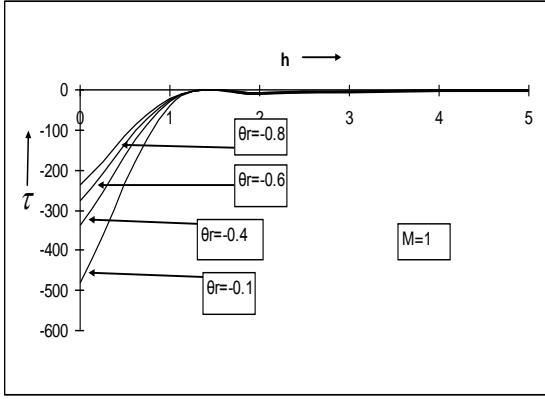


Figure 14-Co-efficient of skin friction τ versus θ_r against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-0.6, n=.1, \epsilon=.2, t=3, R=2, y=0$.

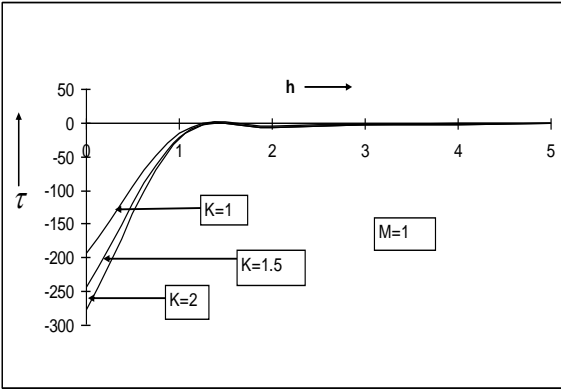


Figure 15-Co-efficient of skin friction τ versus K against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-0.6, n=.1, \epsilon=.2, t=3, R=2, y=0$.

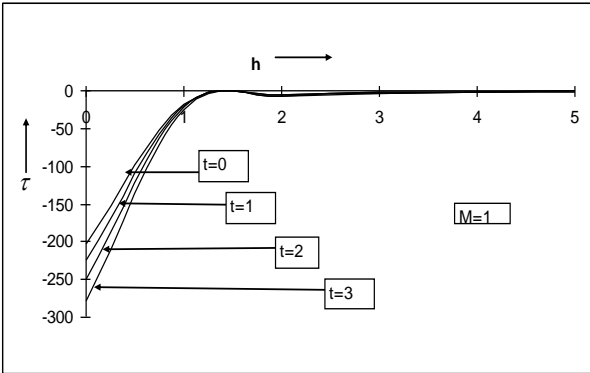


Figure 16-Co-efficient of skin friction τ versus t against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=0.6, n=.1, \epsilon=.2, t=3, R=2, y=0$.

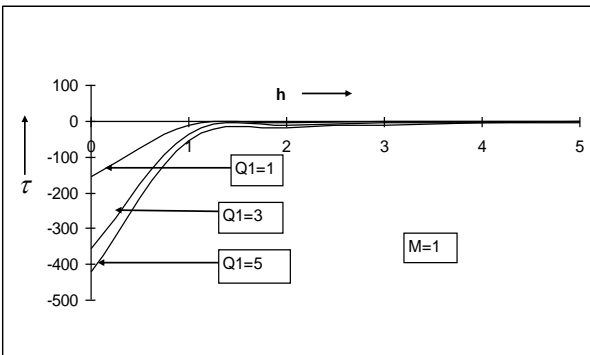


Figure 17-Co-efficient of skin friction τ versus $Q1$ against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-0.6, n=.1, \epsilon=.2, t=3, R=2, y=0$.

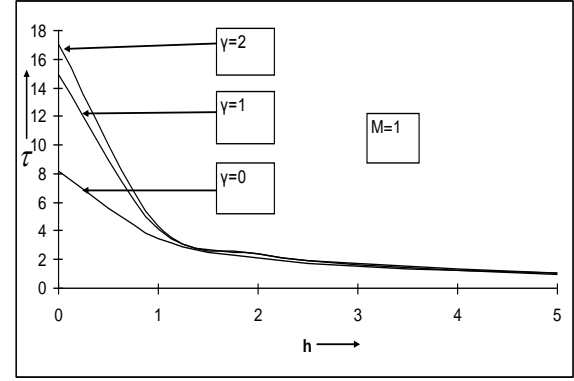


Figure 18-Co-efficient of skin friction τ versus γ against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-0.6, n=.1, \epsilon=.2, t=3, R=2, y=0$.

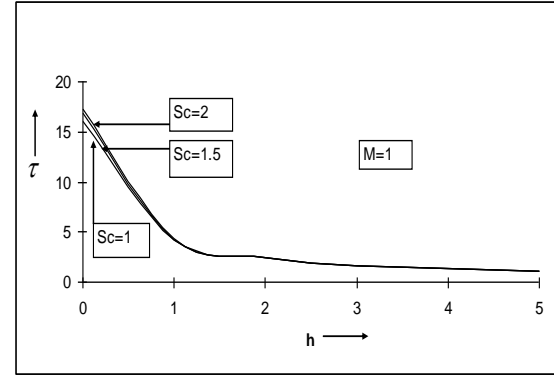


Figure 19-Co-efficient of skin friction τ versus Sc against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-0.6, n=.1, \epsilon=.2, t=3, R=2, y=0$.

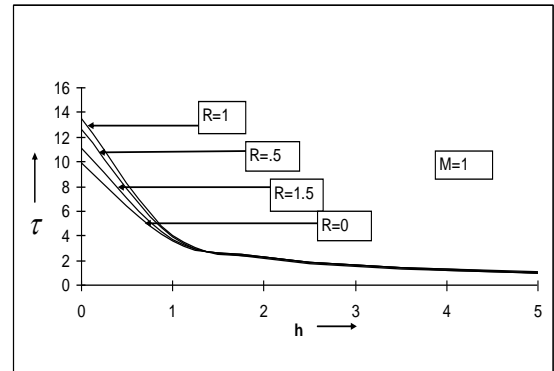


Figure 20-Co-efficient of skin friction τ versus R against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-0.6, n=.1, \epsilon=.2, t=3, y=0$.

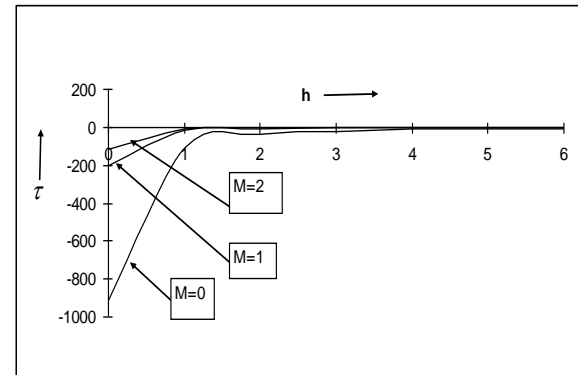


Figure 21-Co-efficient of skin friction τ versus M against h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gr=2, Gm=2, K=2, \theta_r=-0.6, n=.1, \epsilon=.2, t=3, R=2, y=0$.

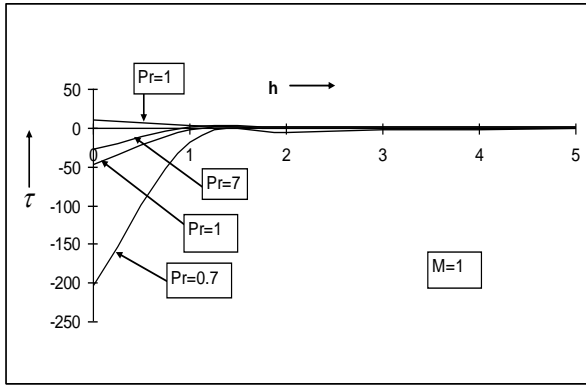


Figure 22-Co-efficient of skin friction τ versus h for $Pr=.7, Sc=.6, \gamma=.5, A=.5, Gm=2, K=2, \theta_r=-0.6, n=.1, \varepsilon=.2, t=3, R=2, y=0$.

5. CONCLUSIONS

1. The magnitude of the velocity decreases with the increasing value of the rarefaction parameter h .
2. The chemical reaction parameter γ accelerates the velocity.
3. The velocity profile increases or decreases rapidly near the wall for different values of the parameters involved.
4. The thermal Grashof number Gr increases the magnitude of the skin friction coefficient.
5. The skin friction coefficient decreases with the increasing value of Hartmann number M .
6. The magnitude of the skin friction decreases for increasing rarefaction parameter h and it decreases rapidly near the wall.

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NOMENCLATURE

- A : Suction parameter
 B_0 : Magnetic induction (T)
 \bar{C} :Species concentration of the fluid in the boundary layer
 C_l :Chemical reaction parameter
 \bar{C}_w : Concentration of the wall
 \bar{C}_∞ :Species concentration of the fluid far away from the plate
 D :Coefficient of chemical molecular diffusivity
 g : Acceleration due to gravity [m/s^2]
 Gm :Grashof number for mass transfer
 Gr : Grashof number for heat transfer
 h : Rarefaction parameter
 \bar{K} :Permeability of the porous medium(H/m)
 M : Magnetic field($m^{-1}A$)
 n :dimensionless exponential index
 Pr : Prandtl number
 \bar{Q}_1 : Dimensional absorption of radiation parameter
 R : Heat source parameter
 Sc : Schmidt number
 \bar{t} :Dimensional time(s)
 \bar{T} :Temperature of the fluid in the boundary layer(K)
 \bar{T}_w :Temperature of the wall (K)
 \bar{T}_∞ :Temperature of the fluid far away from the plate (K)
 (\bar{u}, \bar{v}) : Components of the fluid velocity (m/s)
 U_∞ : Free stream velocity(m/s)
 V_0 : Scale of suction velocity(m/s)

Greek symbols

- β_T :Thermal expansion coefficient(K^{-1})
 β_C :Concentration expansion coefficient
 γ : Chemical reaction parameter

ν : Kinematic viscosity(m^2 / s)
 ρ : Fluid density(kg/m^3)
 θ : Non-dimensional temperature
 θ_r : Variable viscosity(Ns/m^2)
 μ : Fluid viscosity(Pa.s)
 σ : Electrical conductivity of the fluid(s.m^{-1})

APPENDIX

$$\begin{aligned}
 A_1 &= \frac{Sc + \sqrt{Sc^2 + 4\gamma Sc}}{2}, A_2 = \frac{Pr + \sqrt{Pr^2 + 4RPr}}{2}, \\
 A_3 &= \frac{-Pr Q_1}{A_1^2 - A_1 Pr - RPr}, A_4 = (A_3 - 1)A_2 - A_3 A_1, \\
 A_5 &= 1 - \frac{A_4}{1 - \theta_r}, \quad N = M + \frac{1}{K}, \\
 A_6 &= \frac{A_5 + \sqrt{A_5^2 + 4N}}{2}, A_7 = \frac{Sc + \sqrt{Sc^2 + 4Sc(\gamma + n)}}{2},
 \end{aligned}$$

$$\begin{aligned}
 A_8 &= \frac{Pr + \sqrt{Pr^2 + 4Pr(R + n)}}{2}, A_9 = \frac{A_5 + \sqrt{A_5^2 + 4(N + n)}}{2} \\
 A_{10} &= -\frac{Gm + Gr + N}{N(1 + hA_6)}, \\
 A_{11} &= AA_{10}A_6 - \frac{A_{10}A_6}{1 - \theta_r} [A_{17} + A_5 - 1] - Gr - Gm + N - n \\
 A_{12} &= \frac{AScA_1}{A_1^2 - ScA_1 - Sc(\gamma + n)}, A_{13} = \frac{Pr(A_1AA_3 - A_{12}Q_1)}{A_1^2 + PrA_1 - Pr(R + n)}, \\
 A_{14} &= \frac{PrAA_2(1 - A_3)}{A_2^2 - PrA_2 - Pr(R + n)}, A_{15} = \frac{PrQ_1(A_{12} - 1)}{A_7^2 - PrA_7 - Pr(R + n)}, \\
 A_{16} &= 1 - A_{13} - A_{14} - A_{15}, \\
 A_{17} &= -(A_{16}A_8 + A_{15}A_7 + A_{14}A_2 + A_{13}A_1).
 \end{aligned}$$