MATHEMATICAL MODEL OF DIRECT EVAPORATIVE COOLING AIR CONDITIONER

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ABSTRACT

A mathematical model for the direct evaporative cooling air conditioning is presented. Three dimensions solid evaporative cooling pad is regard as a big two dimensions plane. In order to avoid determining length and width of the plane, the set of partial differential equation is transformed to nondimensional form. Then directly calculate the product \((\alpha, A)\) so as to avoid measuring \(\alpha_a\) and \(A\). Finally, the set of partial differential equation can be solved.

1. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat transfer total surface area of pad</td>
<td>(m^2)</td>
<td></td>
</tr>
<tr>
<td>Pad area coefficient</td>
<td>(\text{m}^2/\text{m}^3)</td>
<td></td>
</tr>
<tr>
<td>Air pressure</td>
<td>(\text{Pa})</td>
<td></td>
</tr>
<tr>
<td>Specific heat capacity at constant pressure</td>
<td>(\text{J.kg}^{-1}.\text{K}^{-1})</td>
<td></td>
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<tr>
<td>Specific heat capacity of water</td>
<td>(\text{J.kg}^{-1}.\text{K}^{-1})</td>
<td></td>
</tr>
<tr>
<td>Length of the surface in the air flow direction</td>
<td>(\text{m})</td>
<td></td>
</tr>
<tr>
<td>Length of the surface in the water flow direction</td>
<td>(\text{m})</td>
<td></td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>(\text{kg.s}^{-1})</td>
<td></td>
</tr>
<tr>
<td>Heat flow</td>
<td>(\text{W})</td>
<td></td>
</tr>
<tr>
<td>Heat of vaporization of water</td>
<td>(\text{J.kg}^{-1})</td>
<td></td>
</tr>
<tr>
<td>Heat of vaporization of water at (0^\circ\text{C})</td>
<td>(\text{J.kg}^{-1})</td>
<td></td>
</tr>
<tr>
<td>Nondimensional temperature</td>
<td>(^\circ\text{C})</td>
<td></td>
</tr>
<tr>
<td>Space coordinate in the air flow direction</td>
<td>(\text{m})</td>
<td></td>
</tr>
<tr>
<td>Space coordinate in the water flow direction</td>
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</tr>
<tr>
<td>Nondimensional space coordinate in the air flow direction</td>
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</tr>
<tr>
<td>Nondimensional space coordinate in the water flow direction; Volume ((\text{m}^3))</td>
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<td></td>
</tr>
<tr>
<td>Moisture content</td>
<td>(\text{kg.kgda}^{-1})</td>
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<tr>
<td>Nondimensional moisture content</td>
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<td></td>
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<tr>
<td>Heat transfer coefficient</td>
<td>(\text{W.m}^{-2}.\text{K}^{-1})</td>
<td></td>
</tr>
<tr>
<td>Pad thickness</td>
<td>(\text{m})</td>
<td></td>
</tr>
<tr>
<td>Pad evaporative cooling efficiency</td>
<td>(%)</td>
<td></td>
</tr>
</tbody>
</table>

\(\rho\) Air density \(\text{kg/m}^3\)
\(\sigma\) Mass heat transfer coefficient \(\text{kgda.m}^{-2}.\text{s}^{-1}\)

Subscripts

\(0\) Initial value, At \(0^\circ\text{C}\)
\(a\) Moist air
\(c\) Coefficient
\(da\) Dry air
\(i\) Input
\(L\) Latent
\(o\) Output
\(pad\) Evaporative cooling pad
\(s\) Sensible; On the water-air interface surface
\(v\) Vaporization
\(w\) Water; At water temperature
\(wb\) Wet bulb

Superscripts

\('\) Saturated air
\(\bar{}\) Mean value

2. INTRODUCTION

It’s well known that refrigerant is water in evaporative cooling air conditioner, not CFCs. It has no pollution to the atmosphere. Its COP is higher than that of conventional mechanical refrigerator, and its energy consumption and initial investment are both lower than that of conventional air conditioning equipment. So this environmental protection, energy saving and economical technology is widely used in southwest America, most areas in Australia and a lot of arid and half-arid areas in the world. This technology was introduced into China in 1980s. Now, evaporative cooling air conditioner is applied widely in China [1].

Simultaneous heat and mass transfer phenomena in evaporative cooling process usually can be described by a set of partial differential equations. This process is very complex, owing to nonlinear relation of various parameters. Although those differential equations are well known, the evaporative
cooling pad is a cross-flow cooling device, it’s difficult to obtain their solution\(^{[2,3]}\). Generally in order to simplify the model, some assumptions are introduced, that made it easy to solve, but the accuracy was sometimes insufficient. However, with computer, complex differential equations can be solved quickly.

In this paper, first, a set of partial differential equations is created. Then, it is transformed to nondimensional form. Finally, it is solved by the finite difference method with iterations.

3. MATHEMATICAL MODEL

In a cross-flow evaporative cooling device air streams horizontally, while water drops downwards through the packing. Recirculating water is poured the packing (pad) at the water temperature, evaporated water is replenished in the tank for recirculating.

Fig. 1 shows a solid evaporative cooling pad. Fig. 2 shows two dimensions plane evaporative cooling pad. Fig. 3 shows infinitesimal area of water-air interface.

We have some basic assumptions. The housing of the device is insulated, no hear transfer to the surroundings. The process is steady-state, pressures and mass flow rates are constant and uniform for both streams. The Lewis relation is satisfied. Longitudinal heat transfer and diffusion are neglected along the evaporative cooling pad.

3.1 Mass balance equation of air and water

The mass balance of moisture in a moist air stream:

\[
\frac{q_{ma}}{L_w} \frac{\partial x_a}{\partial u} \, du = \sigma(x_s - x_u) \, dudv
\]  

(1)

The mass balance of water:

\[
\frac{\partial}{\partial v} \left( \frac{q_{ma}}{L_a} \frac{du}{dv} \right) \, dv = -\sigma(x_s - x_u) \, dudv
\]  

(2)

In the water-air interface:

\[
t_{ai}' = t_{ao}' = t_{vo}' \quad x_{ai}' = x_{ao}' = x_{vo}' \quad h_{x} = h_{w}
\]  

(3)

In the water-air interface, the enthalpy of vapor:

\[
h_{xt} = c_{pt} f_{t_{xt}} + r_0
\]  

(4)

The enthalpy of vapor at the water temperature:

\[
h_{wv} = c_{pt} f_{t_{wv}} + r_0 \approx c_{pt} f_{t_{w}} + r_0
\]  

(5)

From equations (1)(2)(3), the following equation is obtained:

\[
\frac{\partial q_{ma}}{\partial v} \left( \frac{1}{L_w} \frac{du}{dv} \right) \, dv = -\frac{q_{ma}}{L_w} \left( \frac{\partial x_a}{\partial u} \right) \, dudv
\]  

\[
= -\sigma(x_s' - x_u') \, dudv
\]  

(6)

Simplify equation (6):

\[
\frac{\partial x_a}{\partial u} = \frac{c_{pt}}{q_{ma}} (x_s' - x_u')
\]  

(7)
3.2 Energy balance equation of air

The total heat flow consists of sensible heat $dQ_s$ and latent heat $dQ_a$:

$$dQ_s = \alpha_a(t_a - t_s)du\,dv$$  \hspace{1cm} (8)

$$dQ_L = -\frac{\partial}{\partial v}\left(\frac{q_{ma}}{L_a}du\right)\,dy\,h_{vs} = \frac{q_{ma}}{L_a}du\left(\frac{\partial x_c}{\partial u}\right)\,dy\,h_{vs}$$  \hspace{1cm} (9)

$$= \sigma h_{vs}(x'_s - x_s)du\,dv$$

The change of air enthalpy is caused by the sensible and the latent heat:

$$[h_a + \left(\frac{\partial h_a}{\partial u}\right)du]\frac{q_{ma}}{L_a}dv - h_a\frac{q_{ma}}{L_a}dv = -dQ_s - dQ_L$$  \hspace{1cm} (10)

Substituting equations (8)(9) into equation (10):

$$\left(\frac{\partial h_a}{\partial u}\right)\frac{q_{ma}}{L_a}duv = -\alpha_a(t_a - t_s)du\,dv - \frac{\partial}{\partial v}\left(\frac{q_{ma}}{L_a}du\right)\,dy\,h_{vs}$$

$$= -\left[\alpha_a(t_a - t_s) + \left(\frac{\partial q_{ma}}{\partial v}\right)\frac{h_{vs}}{L_a}\right]du\,dv$$

$$= -[\alpha_a(t_a - t_s) - \sigma(x'_s - x_s)h_{vs}]du\,dv$$

With the equation (3), the above equation can be simplified:

$$\left(\frac{\partial h_a}{\partial u}\right)\frac{q_{ma}}{L_a} = -\alpha_a(t_a - t_s) + \sigma(x'_s - x_s)h_{vs}$$  \hspace{1cm} (11)

Air enthalpy:

$$h_a = c_{pad}f_a + x_a(r_0 + c_{pv}t_a)$$  \hspace{1cm} (12)

Specific heat capacity at constant pressure of air:

$$c_{pa} = c_{pad} + x_a c_{pv}$$  \hspace{1cm} (13)

According to (12)(13):

$$\frac{\partial h_a}{\partial u} = \left(c_{pad} + x_a c_{pv}\right)\frac{\partial t_a}{\partial u} + \left(c_{pv}f_a + r_0\right)\frac{\partial x_a}{\partial u}$$  \hspace{1cm} (14)

$$= c_{pa}\left(\frac{\partial t_a}{\partial u} + \left(c_{pv}f_a + r_0\right)\frac{\partial x_a}{\partial u}\right)$$

Substituting equations (5)(6)(14) into equation (11):

$$\left[c_{pad} + x_a c_{pv}\left(\frac{\partial t_a}{\partial u} + \left(c_{pv}f_a + r_0\right)\frac{\partial x_a}{\partial u}\right)\right]\frac{q_{ma}}{L_a}$$

$$= -\left[\alpha_a(t_a - t_s) + \left(\frac{\partial q_{ma}}{\partial v}\right)\frac{h_{vs}}{L_a}\right]$$

$$= -\left[\alpha_a(t_a - t_s) - \frac{q_{ma}}{L_a}\left(\frac{\partial x_a}{\partial u}\right)\left(c_{pv}f_{tw} + r_0\right)\right]$$

Simplify equation (15):

$$c_{pa}\left(\frac{\partial t_a}{\partial u}\right)\frac{q_{ma}}{L_a} = -\alpha_a(t_a - t_s) + \left(\frac{\partial x_a}{\partial u}\right)\left[c_{pv}f_{tw} + r_0\right] - \left(c_{pv}f_{tw} + r_0\right)$$

$$\frac{\partial t_a}{\partial u} = -(t_a - t_w)\frac{\alpha_a}{c_{pa}} \left(\frac{L_a}{q_{ma}}\right) \left[1 + \sigma c_{pv}\left(x'_w - x_a\right)\right]$$  \hspace{1cm} (16)

3.3 Energy balance equation of water

The energy change of water is also caused by the sensible and the latent heat:

From (8)(9)(11)(15), the equation of energy change of water is obtained:

$$dQ_w = dQ_s - dQ_L$$

$$= \left[\alpha_a(t_a - t_w) - q_{ma}\left(\frac{\partial x_a}{\partial u}\right)\left(c_{pv}f_{tw} + r_0\right)\right]du\,dv$$  \hspace{1cm} (17)

From equation (3), we obtain: $t_{vw} = t_w$

Substituting equations (7)(13), $t_{vw} = t_w$, and $L_w = \alpha c_{pv} / \sigma_a = 1$ into equation (17), the follow equation is obtained:

$$dQ_w = \left[\alpha_c(t_a - t_w) - q_{ma}\left(c_{pv}f_{tw} + r_0\right)\right]du\,dv$$

$$= \sigma \left[c_{pa} \left(t_a - t_w\right) - (x'_w - x_a)\left(c_{pv}f_{tw} + r_0\right)\right]du\,dv$$

$$= \sigma \left[c_{pad} + x_a c_{pv} c_{pv} - c_{pad} c_{pw} - x_a t_w c_{pv} - x'_w t_w c_{pv} + x_a r_0\right]du\,dv$$

$$= \sigma(h_a - h_{vw})du\,dv$$  \hspace{1cm} (18)

The energy change of water can be expressed with the following equation:

$$dQ_w = \sigma \left[\frac{q_{ma}}{L_a}du + \left(\frac{\partial x_a}{\partial u}\right)\left(c_{pv}f_{tw} + r_0\right)\right]$$

From equation (18)(19), we obtain the following equation:

$$c_{pa}\left[\frac{q_{ma}}{L_a}du + \left(\frac{\partial x_a}{\partial u}\right)\left(c_{pv}f_{tw} + r_0\right)\right]$$

$$= \sigma(h_a - h_{vw})du\,dv$$

Expand the above equation:
\[
\frac{q_{m}}{L_{a}} \left[ \frac{\partial t_{w}}{\partial v} + \left( \frac{\partial q_{m}}{\partial v} \right) \frac{t_{w}}{L_{a}} \right] dudv
+ \frac{c_{w}}{L_{a}} \left[ \frac{\partial q_{m}}{\partial v} \left( \frac{\partial t_{w}}{\partial v} \right) dudv \right]
= \sigma \left(h_{a} - h_{w} \right) dudv
\]

Neglecting the third member in the brackets, the equation (20) is changed:
\[
c_{w} \left[ \frac{q_{m}}{L_{a}} \left( \frac{\partial t_{w}}{\partial v} \right) + \left( \frac{\partial q_{m}}{\partial v} \right) \frac{t_{w}}{L_{a}} \right] = \sigma \left(h_{a} - h_{w} \right)
\]

Simplify the above equation:
\[
\frac{\partial t_{w}}{\partial v} = \frac{\sigma L_{a}}{c_{w} q_{m}} \left( h_{a} - h_{w} \right) - \frac{t_{w}}{q_{m}} \left( \frac{\partial q_{m}}{\partial v} \right)
\]

From equation (7)(16)(21), we can obtain a set of partial differential equations:
\[
\frac{\partial x_{w}}{\partial t} = \frac{\sigma L_{a}}{q_{ma}} \left(x_{w}' - x_{a} \right)
\]

\[
\frac{\partial t_{a}}{\partial u} = -\left(t_{a} - t_{w} \right) \frac{L_{a}}{c_{ps} q_{ma}} \left[ 1 + \frac{\sigma c_{pv}}{\alpha_{a}} \left(x_{w}' - x_{a} \right) \right]
\]

\[
\frac{\partial t_{w}}{\partial v} = \frac{\sigma L_{a}}{c_{w} q_{m}} \left( h_{a} - h_{w} \right) + \frac{q_{ma} t_{w} L_{a}}{L_{a} q_{m}} \left( \frac{\partial x_{w}}{\partial u} \right)
\]

It is difficult to determine the length and width of the plane, so the set of partial differential equations can be transformed to a nondimensional form.

4. NONDIMENSIONAL MATHEMATICAL MODEL

4.1 Nondimensional space coordinates

First, space coordinates u, v is transformed to U, V which range is from 0–1.

\[
U = \frac{u}{L_{a}}, \quad V = \frac{v}{L_{a}}
\]

From figure 2, we can obtain the total area of the water-air interface surface, A.

Using equation (25)(26), the equation (22)(23)(24) can be transformed to:
\[
\frac{\partial x_{w}}{\partial U} = \frac{c_{w} A}{q_{ma}} \left( x_{w} - x_{a} \right)
\]

\[
\frac{\partial t_{a}}{\partial U} = -\left(t_{a} - t_{w} \right) \frac{L_{a}}{c_{ps} q_{ma}} \left[ 1 + \frac{\sigma c_{pv}}{\alpha_{a}} \left(x_{w}' - x_{a} \right) \right]
\]

\[
\frac{\partial t_{w}}{\partial V} = \frac{c_{w} q_{m}}{A} \left( h_{a} - h_{w} \right) + \frac{q_{ma} t_{w} L_{a}}{q_{m}} \left( \frac{\partial x_{w}}{\partial U} \right)
\]

4.2 Nondimensional temperature and humidity

The difference of inlet air dry bulb temperature \( t_{a} \) and inlet air wet bulb temperature \( t_{wb} \) is chosen as reference values for all temperature.

\[
T_{a} = \frac{t_{a} - t_{wb}}{t_{ai} - t_{wb}} \quad T_{w} = \frac{t_{w} - t_{wb}}{t_{ai} - t_{wb}}
\]

Similarly, air moisture content can be obtained:
\[
X_{a} = \frac{x_{wb} - x_{a}}{x_{wb} - x_{ai}}
\]

In equation (27)(28)(29), \( x_{w}' \) is saturated air moisture content at water temperature and it depends upon \( t_{w} \). It is well known that \( x_{w}' \) be can be calculated by the follow equation:

\[
x_{w}' = 0.62198 \left( \frac{a_{0} + a_{1} t_{w} + a_{2} t_{w}^{2}}{1 + a_{3} t_{w}^{3} + a_{4} t_{w}^{4} + a_{5} t_{w}^{5}} \right) * 1000
\]

Substituting equations (30)(31)(32), and \( L_{a}=\alpha_{a} c_{ps}/\alpha_{a}=1 \), into equation (27)(28)(29), thus, all the equation s of the original system equation (22)(23)(24) have been transformed to a nondimensional form: equation (34)(35)(36).
\[ \frac{\partial T_a}{\partial U} = \frac{1}{(t_{ai} - t_{wb})} \frac{\partial t_a}{\partial U} \]

\[ = -(T_a - T_w) \frac{\alpha A}{c_{pa} q_m} \left[ 1 + \frac{c_w}{c_{pa}} + \frac{X_a}{X_{wb}} \right] \]

\[ \frac{\partial w_t}{\partial V} = \frac{1}{(t_{ai} - t_{wb})} \frac{\partial t_w}{\partial V} \]

\[ = -\frac{\alpha A}{c_{pa} q_m} \left[ \frac{dss(T_a(t_{ai} - t_{wb}) + t_{wb})}{\partial w_t} \right] \]

\[
\frac{1}{(t_{ai} - t_{wb})} + \frac{\alpha A}{c_{pa} q_m} \left[ T_w(t_{ai} - t_{wb}) + t_{wb} \right] + \frac{dss(T_a(t_{ai} - t_{wb}) + t_{wb})}{\partial w_t} \]

\[ = \frac{1}{(t_{ai} - t_{wb})} \]

4.3 Parameter calculate

These equations can be rearranged which contains three unknown parameter \( T_a, T_w, X_a \), and two unknown parameter \( \alpha_a \) and \( A \).

For a evaporative cooling device with known mass flow rates of the two streams (air and water), volume of pad, initial conditions of air and water, and the parameter \( \alpha_a \) (convective heat transfer coefficient). Then, the set of partial differential equations can be solved. It is difficult to measure or calculate \( \alpha_a \) and \( A \), but the product \( \alpha_a A \) can be calculated by the follow method.

The efficiency of evaporative cooling pad can be expressed by the follow equation \[4,5]\).

\[ \eta = 1 - \exp \left( -\frac{\alpha A \delta}{v_a \rho_a c_{pa}} \right) \]

Then,

\[ \alpha_a = -\log(1-\eta) \frac{\rho_a c_{pa}}{V_{pad}} \]

Because, \( A = \alpha_a * V_{pad} \)

So,

\[ \alpha_a A = -\log(1-\eta) \frac{\rho_a c_{pa}}{V_{pad}} \]

The last problem is water temperature. No matter what the water temperature is high or low. For every outdoor air condition, as soon as evaporative cooling device comes into operation, the water temperature varies within the device, and its value gradually tends to the web bulb temperature of input air. Finally, the water temperature doesn’t change, and the device operates steady. When outdoor air condition changes, the water temperature changes, too.

According to the finite difference method with iterations, These equations (equation (34)(35)(36)) can be solved. Solution gives parameters \( (t_a, t_{wb}, X_a) \) for the water and air in every element of the heat transfer area, as well as the mean outlet values of the parameters \( (T_{oa}, T_{ow}, X_{oa}) \).

4.4 Testing the nondimensional mathematical model using the experiment data

In order to test the accuracy of the nondimensional mathematical model, accurate experiment data must be obtained.

<table>
<thead>
<tr>
<th>No.</th>
<th>Input air temp. (℃)</th>
<th>Input air relative humidity (%)</th>
<th>Air web temp. (℃)</th>
<th>Tested output air temp. (℃)</th>
<th>Calculated output air temp. (℃)</th>
<th>Error (℃)</th>
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</thead>
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<td>-0.195</td>
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</tbody>
</table>

As shown in table 1 [9], in 2001, the performance parameter of a direct evaporative cooling (DEC) air conditioner were measured, the evaporative cooling pad of the experimental DEC air conditioner is GLASdek. In this paper, some data of the experimental DEC air conditioner is used to test the accuracy of the nondimensional mathematical model.

5. CONCLUSION

The presented model for direct evaporative cooling (DEC) device makes it possible to estimate the output air temperature of an DEC air conditioner under real conditions according to the finite difference method.

Three dimensions Solid evaporative cooling pad is regard as a big two dimensions plane. Then, directly calculate the product \( \alpha_a A \) so as to avoid measuring \( \alpha_a \) and \( A \). Finally, the set of partial differential equation can be solved.
6. ACKNOWLEDGEMENTS

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7. REFERENCES