EFFECT OF THERMAL RADIATION ON MHD FREE CONVECTIVE OSCILLATORY FLOW BETWEEN TWO VERTICAL PARALLEL PLATES WITH PERIODIC PLATE TEMPERATURE AND DISSIPATIVE HEAT.

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ABSTRACT
This paper focuses on the effect of thermal radiation on unsteady free convection flow of a viscous incompressible electrically conducting fluid with dissipative heat between two long vertical plates where the temperature of one of the plates oscillates about a constant non-zero mean temperature. The governing equations are solved by regular perturbation technique. The expressions for the velocity field, temperature field, the skin friction at the plate and Nusselt number are obtained in non-dimensional forms. Computations are performed for a wide range of the governing flow parameters, viz., the Hartmann number, Grashof Number for heat transfer, Prandtl Number and Radiation parameter. The effects of these flow parameters on the velocity, temperature, skin friction and Nusselt number are presented graphically and results are discussed.

Keywords: MHD, Thermal Radiation, Nusselt number, skin friction, free convection

1. INTRODUCTION:
   The investigation of free convective flows between two long vertical plates is important because of their applications in the field of nuclear reactors, heat exchangers, cooling appliances in electronic instruments and in different engineering fields. Ostrach [1] first presented the fully developed free convection flow between two parallel plates at constant temperature. Aung [2] studied the exact solution for free convection in a vertical plate channel with asymmetric heating for a fluid of constant properties. Natural convection for heated iso-flux boundaries of the channel containing a low Prandtl number fluid was investigated by Compo et al. [3]. The above mentioned works are confined to fully developed steady flows. The flow may be unsteady when the current is periodic due to off control mechanisms or due to partially rectified A.C voltage, there exists periodic heat inputs. Singh et al. [4] considered transient free convection flow between two long vertical plates maintained at constant but unequal temperatures. Jha et al. [5] extended the problem to the case of symmetric heating of the channel walls. Ahmed and Kalita [6] have studied MHD oscillatory free convective past a vertical plate in slip flow-regime with variable suction and periodic plate temperature.
   Recent developments in hypersonic flights, missile re-entry, rocket combustion chamber plants for inter-planetary flight and gas cooled nuclear reactors focused on thermal radiation as a mode of energy transfer and emphasize the need for improved understanding of radiative transfer in these processes.
   The effect of radiation on MHD flow, heat and mass transfer problems has become industrially more important. Many engineering processes occur at high temperatures, the knowledge of radiation heat transfer plays significant role in the design of equipment. Nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering processes. At high operating temperature, the radiation effect can be quite significant.
   Hossain and Takhar [7] studied the radiation effect using Rosseland approximation on mixed convection along a vertical plate with uniform plate temperature. Raptis [8] analyzed both the thermal radiation and free convection flow through a porous medium by using a perturbation technique. Raptis and Perdikis [9] considered the problem of thermal radiation and free convection flow past a moving plate. The interaction of radiation with laminar free convective heat transfer from a vertical plate was investigated by Cess [10]. Recently, Shivaiah and Rao [11], Rajput et al. [12] have studied the effect of thermal radiation on unsteady free convection heat and mass transfer. Very recently M.Mecili and E.Mezache [13] have presented analytical solution for slip flow heat transfer in microtubes including viscous dissipation and axial heat conduction.
   The aim of the present investigation is to analyze the effect of thermal radiation on MHD free convective oscillatory flow between two vertical parallel plates with periodic plate temperature and dissipative heat. This work is an extension of the problem studied by Ahmed et al. [14] to the case when there is an imposed thermal radiation.
2. MATHEMATICAL ANALYSIS:

An unsteady flow of an incompressible viscous electrically conducting fluid between two long vertical parallel plates under the action of a transverse magnetic field is considered. The axis is taken along one of the plates in the vertically upward direction and the normal to the plates. Let \( \vec{u} = u \hat{x} + v \hat{y} \) be the fluid velocity and \( \vec{B} = B_0 \hat{y} \) be the applied magnetic field. Since the plate is infinite in length in the \( x \) direction, therefore all the quantities except possibly the pressure are to be independent of \( x \).

The temperature of the plates \( y = 0 \) and \( y = h \) are assumed to be \( T_0 \) and \( T_0 + \varepsilon (T_0 - T_0) \cos \alpha \). Then under usual Boussinesq’s approximation and closely following Nanahari [15], the flow can be shown to be governed by the system of equations:

\[
\begin{align*}
\frac{\partial \vec{u}}{\partial t} &= -\frac{\sigma \vec{E}_0 \vec{B}_0^2}{\rho} + n - \frac{\nu}{\partial y^2} \\
\rho \Delta T &= \frac{k}{\partial y^2} + \mu \frac{\partial u}{\partial y} + \frac{\partial B_0^2}{\partial y} - \frac{\partial T}{\partial y}
\end{align*}
\]  

(1)

The radiative heat flux \( q_r \) under Rosseland approximation by Brewer [16] has the form:

\[
q_r = \frac{4 \sigma T^4}{3 k_1 \frac{\partial T}{\partial y}}
\]

(3)

Using the equation (3) in equation (2) we get

\[
\rho \frac{\partial \Delta T}{\partial \tau} + \frac{\partial u}{\partial y^2} + \mu \frac{\partial u}{\partial y} + \frac{\partial B_0^2}{\partial y} - \frac{3 \partial T}{\partial y}
\]

(4)

The symbols are defined in the nomenclature.

The relevant boundary conditions are:

At \( y = 0, T = T_0 \)

(5)

At \( y = h, T = T_0 + \varepsilon (T_0 - T_0) \cos \alpha \)

(6)

We introduce the following non-dimensional quantities:

\[
\begin{align*}
\bar{u} &= \frac{u}{h} \\
\bar{T} &= \frac{\Delta T}{T_0} \\
\bar{x} &= \frac{x}{h} \\
\bar{y} &= \frac{y}{h} \\
\bar{v} &= \frac{v}{h} \\
\bar{\rho} &= \frac{\rho}{\rho_0} \\
\bar{\mu} &= \frac{\mu}{\mu_0} \\
\bar{E}_0 &= \frac{E_0}{h^2} \\
Gr &= \frac{g \beta \bar{T} h^2}{\nu^2}
\end{align*}
\]

The non-dimensional forms of the equations (1) and (4) are:

\[
\begin{align*}
\frac{\partial \bar{u}}{\partial \tau} &= Gr \frac{\partial \bar{u}}{\partial \bar{y}} + \frac{N}{\lambda} \\
\frac{\partial \bar{v}}{\partial \tau} &= \frac{\partial \bar{v}}{\partial \bar{y}} \\
\frac{\partial \bar{T}}{\partial \tau} &= \frac{\partial \bar{T}}{\partial \bar{y}} + \frac{N}{\lambda} \\
\frac{\partial \bar{\rho}}{\partial \tau} &= \frac{\partial \bar{\rho}}{\partial \bar{y}} \\
\frac{\partial \bar{\bar{E}_0}}{\partial \bar{y}} &= \frac{\partial \bar{\bar{E}_0}}{\partial \bar{y}} + \frac{N}{\lambda}
\end{align*}
\]

(7)

Using (5) and (6), the corresponding non-dimensional boundary conditions become:

At \( y = 0, \bar{u} = 0 \)

(9)

At \( y = h, \bar{v} = 0 \)

(10)

For ease of calculations, boundary condition (10) is modified as under:

At \( y = h, \bar{v} = 1 \)

(11)

3. METHOD OF SOLUTION:

We assume the solutions of the equations (7) and (8) to be of the following forms:

\[
\bar{u} = \bar{u}_0(\bar{y}) + e^{i\alpha \bar{y}} \bar{u}_1(\bar{y}) + \ldots
\]

(12)

Where \( \varepsilon \) is a small positive quantity and \( \alpha < 1 \).

Where \( \varepsilon \) is a small positive quantity and \( \alpha < 1 \).

Substituting these in equations (7) and (8) and by equating coefficients of the similar terms and neglecting \( \varepsilon^2 \), the following differential equations are obtained:

\[
\begin{align*}
\bar{u}_0' &= -Gr \bar{T} \\
\bar{u}_1' - (M + i \alpha) \bar{u}_0 &= -Gr \bar{u}_1 \\
\lambda \bar{u}_1 + e^{i\alpha \bar{y}} \bar{u}_1 &= -2p_1 E_0 \left[ \bar{u}_0' + M \bar{u}_0 \right]
\end{align*}
\]

(13)

with boundary conditions:

At \( y \to 0, \bar{u}_0 = 0, \bar{u}_1 = 1, \bar{T}_0 = 0 \)

(14)

At \( y \to h, \bar{u}_0 = 0, \bar{u}_1 = 1, \bar{T}_1 = 0 \)

(15)

Now using multiparameter perturbation technique, we make the following substitutions using \( E_0 \) as the perturbation parameter:

\[
\begin{align*}
\bar{u}_0 &= \bar{u}_0 + \varepsilon \bar{u}_1 + \varepsilon^2 \bar{u}_2 + \ldots \\
\bar{u}_1 &= \bar{u}_1 + \varepsilon \bar{u}_2 + \varepsilon^2 \bar{u}_3 + \ldots
\end{align*}
\]

(20)

Using (20) in (14) to (17) and equating the coefficients of \( \varepsilon^0 \), \( \varepsilon^1 \) and neglecting the higher powers of \( \varepsilon \), we obtain the following set of equations:

\[
\begin{align*}
f_0' &= -M \bar{u}_0 \\
f_1' &= -M \bar{u}_1 \\
g_0' &= -Gr \bar{u}_0 \\
g_1' &= -Gr \bar{u}_1 \\
\lambda \bar{u}_1 &= -2p_1 E_0 \bar{u}_1
\end{align*}
\]

(16)

The symbols are defined in the nomenclature.

The relevant boundary conditions are:

At \( y = 0, \bar{u}_0 = 0, \bar{u}_1 = 0, \bar{T}_0 = 0, \bar{T}_1 = 0, \bar{u}_0 = 0, \bar{u}_1 = 0 \)

(29)

At \( y = h, \bar{u}_0 = 0, \bar{u}_1 = 0, \bar{T}_0 = 0, \bar{T}_1 = 0, \bar{u}_0 = 0, \bar{u}_1 = 0 \)

(30)

The solutions of equations (21) to (28) under the boundary Conditions (29) and (30) as well as the results for \( u \) and \( \bar{T} \) are obtained but not presented here for the sake of brevity.

4. SKIN FRICTION:

The non-dimensional shear stress \( \tau \) at the plate \( y = 0 \) is given by:

\[
\tau = \frac{\mu}{\nu} \frac{\partial \bar{u}}{\partial \bar{y}} |_{y=0} = \frac{\partial \bar{u}}{\partial \bar{y}} |_{y=0}
\]

(28)

where \( \tau, \mu \) are respectively the real and imaginary parts of \( \tau \) at \( y = 0 \). Here \( \tau = \bar{u}_1 \bar{u}_1' \) is the dimensional shear stress at the plate \( y = 0 \).
5. COEFFICIENT OF HEAT TRANSFER:

The heat flux from the plate to the fluid in terms of Nusselt number $N_u$ is given by:

$$N_u = \frac{\left( \frac{\partial 
abla}{\partial y} \right)_{y=0}}{\left( \frac{\partial T}{\partial y} \right)_{y=0}} = \frac{T_0 - T_S}{T_0 - T_S}$$

$$= \Theta'(0) + i e^{i \alpha} \Theta'(0) = N_{u_r} + i N_{u_i}$$

where $N_{u_r}$ and $N_{u_i}$ are respectively the real and imaginary parts of $N_u$ at $y=0$. Here $N_u = \left( \frac{\partial T}{\partial y} \right)_{y=0}$ is the dimensional heat flux at the plate $y=0$.

6. RESULTS AND DISCUSSION:

In order to get physical insight into the problem, the numerical calculations are carried out from the solutions for the velocity field, temperature field, rate of heat transfer and shear stress by assigning some solicited values to the parameters involved in the problem. These numerical results have been displayed in figures.

Fig1: velocity profiles for different Gr when $M=10, Ec=0.1, N=-1, t=0.1, Pr=1, \omega=0.5, \varepsilon=0.01$

Fig2: velocity profiles for different N when $M=10, Ec=0.1, Gr=10, t=0.1, Pr=1, \omega=0.5, \varepsilon=0.01$

Fig3: velocity profiles for different M when $Gr=5, Ec=0.1, N=-1, t=0.1, Pr=7, \omega=0.2, \varepsilon=0.01$

Fig4: velocity profiles for different Pr when $M=5, Ec=0.1, N=-1, t=1, Gr=10, \omega=0.2, \varepsilon=0.01$

Fig5: Temperature $\Theta$ against $y$ for different Gr when $M=10, Ec=0.1, N=-1, t=0.1, Pr=1, \omega=0.5, \varepsilon=0.01$
Fig 6: Temperature $\theta$ against $y$ for different $N$ when $M=10$, $Ec=0.1$, $t=0.1$, $Gr=10$, $Pr=1$, $\omega=0.5$, $\epsilon=0.01$

Fig 7: Temperature $\theta$ against $y$ for different $M$ when $N=1$, $Ec=0.1$, $t=1$, $Gr=5$, $Pr=7$, $\omega=0.2$, $\epsilon=0.001$

Fig 8: Temperature $\theta$ against $y$ for different $Pr$ when $N=1$, $Ec=0.1$, $t=1$, $Gr=10$, $M=5$, $\omega=0.2$, $\epsilon=0.001$

Fig 9: Nusselt number $Nu$ against time $t$ for different $M$ when $Ec=0.1$, $N=1$, $Gr=10$, $Pr=7$, $\omega=0.2$, $\epsilon=0.001$

Fig 10: Nusselt number $Nu$ against time $t$ for different $Pr$ when $M=5$, $Ec=0.1$, $N=1$, $Gr=10$, $\omega=0.2$, $\epsilon=0.001$

Fig 11: Nusselt number $Nu$ against time $t$ for different $Gr$ when $M=5$, $Ec=0.1$, $N=1$, $Pr=7$, $\omega=0.2$, $\epsilon=0.001$
Fig 12: Nusselt number $N_u$ against time $t$ for different $N$ when $M=10$, $Ec=0.1$, $Gr=10$, $Pr=1$, $\omega=0.5$, $\varepsilon=0.01$

Fig 13: Shear stress $\tau$ against time $t$ for different $M$ when $Ec=0.1$, $N=1$, $Gr=10$, $Pr=7$, $\omega=0.2$, $\varepsilon=0.001$

Fig 14: Shear stress $\tau$ against time $t$ for different $Pr$ when $Ec=0.1$, $N=1$, $Gr=10$, $M=5$, $\omega=0.2$, $\varepsilon=0.001$

Fig 15: Shear stress $\tau$ against time $t$ for different $Gr$ when $Ec=0.1$, $N=1$, $M=5$, $Pr=7$, $\omega=0.2$, $\varepsilon=0.001$

Fig 16: Shear stress $\tau$ against time $t$ for different $N$ when $Ec=0.1$, $Gr=10$, $M=10$, $Pr=1$, $\omega=0.5$, $\varepsilon=0.01$

Figures 1, 2, 3 and 4 display velocity profile versus $y$ for different values of thermal Grashof number $Gr$, radiation parameter $N$, Hartmann number $M$ and Prandtl number $Pr$. It is observed that velocity increases from zero to its maximum value and then leads to zero as $y$ increases. Figure 1 depicts that velocity increases owing to an increase in the value of $Gr$. The thermal Grashof number $Gr$ signifies the relative effects of the thermal buoyancy force to viscous force. Here, the positive values of $Gr$ correspond to cooling of the plate. Figure 2 shows that velocity decreases with the increasing values of $N$. In figure 3, we notice that an increase in $M$ causes the velocity to fall. It is because that the application of transverse magnetic field will result a resistive type force (Lorenz's force), similar to drag force. Physically, fluids with higher Prandtl number have high viscosity and hence moves slowly. This behaviour is evident from figure 4.

Figures 5, 6, 7 and 8 represent the temperature profiles against $y$ for different values of $Gr$, $N$, $M$ and $Pr$. It is marked in figures 5, 6 and 8 that the temperature profile decreases with the increasing values of $Gr$, $N$ and $Pr$. That is the
thickness of the thermal boundary layer decreases with increasing G, N and Pr. The temperature profile under the effect of M is given in figure 7 which shows that, increasing values of M increase temperature profile.

Figures 9,10, 11 and 12 deal with the effects of M, Pr, Gr, and N over the rate of heat transfer in terms of Nusselt number Nu. We observe from figures 9 and 12 that magnitude of the Nusselt number decreases with the increasing M and N. Figures 10 and 11 show that Nu increases with the increasing values of Pr and Gr while figure 12 indicates that Nusselt number increases with the increasing values of N.

Variations of shear stress $\tau$ against time t for various values of M, Pr, Gr and N are plotted through figures 13, 14, 15, and 16. The shear stress $\tau$ is increased by Hartmann number M and radiation parameter N as observed from figures 13 and 16. Figure 14 and 15 show that $\tau$ is decreased under the influence of Pr and Gr.

7. CONCLUSIONS:
1. The radiation parameter N decelerates the velocity profile and decreases the temperature distribution in the boundary layer.
2. The increase in radiation parameter N leads to the increase of heat transfer.
3. An increase in the value of radiation parameter N leads to an increase of the shear stress $\tau$.
4. An increase in value of magnetic parameter M leads to fall in the velocity which is consistent with the law of physics.

REFERENCES:
1. S.Ostrach, Laminar natural-convection flow and heat transfer of fluids with or without heat sources in channels with constant wall temperature, Technical Report 2863, (1952), NASA, USA.

NOMENCLATURE:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>SI unit</th>
</tr>
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<tbody>
<tr>
<td>g</td>
<td>Acceleration due to gravity</td>
<td>m/s²</td>
</tr>
<tr>
<td>u</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat at constant pressure</td>
<td>J/kg·K</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity of the fluid</td>
<td>W/m·K</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Coefficient of viscosity</td>
<td>N·m/s²</td>
</tr>
<tr>
<td>M</td>
<td>Hartmann number</td>
<td></td>
</tr>
<tr>
<td>$E_C$</td>
<td>Eckert number</td>
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</tr>
<tr>
<td>$Gr$</td>
<td>Grashof number for heat transfer</td>
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<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
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<tr>
<td>N</td>
<td>Radiation parameter</td>
<td></td>
</tr>
<tr>
<td>$k_1$</td>
<td>Mean absorption coefficient</td>
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<tr>
<td>$\bar{T}$</td>
<td>Temperature at the static condition</td>
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</tr>
<tr>
<td>$\mathcal{B}_0$</td>
<td>Strength of the applied magnetic field</td>
<td>Tesla</td>
</tr>
<tr>
<td>$P$</td>
<td>Fluid pressure</td>
<td>N/m²</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>K</td>
</tr>
</tbody>
</table>

Greek symbols:
-$\theta$ Non-dimensional temperature.
-$\epsilon$ Electrical conductivity (ohm·m)$^{-1}$
-$\rho$ Small reference parameter,
-$\rho$ Density of the fluid | kg/m³ |
-$\nu$ Kinematic viscosity | m²/s |
-$\beta$ Coefficient of volume expansion | K$^{-1}$ |
-$\sigma_1$ Stefan-Boltzmann constant |         |
-$\omega_0$ Dimensionless frequency | Hz |
-$\omega$ Frequency of oscillation |         |