DESIGN OPTIMIZATION OF THERMAL HEAT ENGINES
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Abstract – Looking for an engine cycle with height output, multi-source of energy and less polluting pushes to reconsider the Stirling cycle. Several prototypes of engine were produced (Ford-Philips 4-215, Ross yoke, GPU-3.. etc), but their performances remain weak compared with other types of internal combustion engine. In order to increase their performances and to analyze their operations, a numerical program of simulation taking into account thermal and mechanical losses was developed and a study of optimization of the design parameters was elaborated. The program which was applied to GPU-3 prototype of the General Motor gave results very close to the experimental results and leads to the optimization of the operating conditions. It also leads to the determination of the optimal values of the geometrical and physical design parameters of the prototype and to the increase of its performances as long as the working liquid pressure is maintained acceptable of the working liquid in the engine.

Keywords – Five (5), Design, Heat losses, Optimization, Performance, Thermal heat engine.

1. INTRODUCTION

The urgent need to preserve fossil fuels and to use renewable energies leads to using of the Stirling engines, these engines present an excellent theoretical output. Several prototypes were produced [2-6] but their outputs remain very weak compared with the excellent theoretical yield. In fact, these engines have an extremely complex phenomena related to the compressible fluid mechanics, the thermodynamics and the heat transfer. The precise description and the comprehension of these highly non stationary phenomena are necessary in order to determine the different engine losses and to find the optimal values of the design variables. G. Popescu [7] studied the influence of the heat losses and irreversibilities on the indices of the engine performance by a thermodynamic optimization which is endo- and exo- irreversible in finished time of the Stirling engine, showing that the most significant reduction in the performances is due to the no adiabatic regenerator.

M. M. Salah El-Din [8] studied the Stirling engines with solar energy; he concluded that the operating temperature and the engine performance depend on the internal irreversibility. I. Kolin [9] showed that the discontinuous movement pushes the cycle to the constant volume lines. He studied several mechanisms of movement transformation and showed that more the movement is discontinuous more the real cycle approaches the ideal cycle. R. Gheith et al [10] developed a dynamic model, taking into account the energy losses in the Stirling engine, the obtained results show that the losses by internal, external conduction and by pressure loss are most significant in the regenerator.

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whereas the losses by Schuttle effect are more significant in the exchanger piston. The other losses are negligible. The reduction of the losses increases the engine performance; it is necessarily based on the determination of the optimal geometrical and physical characteristics of the regenerator and the exchanger piston. A study of optimization based on the dynamic model [11] is developed in this article. It allows to studying the influence of the geometrical and physical parameters on the prototype performance of Stirling engine and so to determine their optimal values.

2. OUTLINE OF THE THEORETICAL MODEL

The dynamic model of the developed Stirling engine is based on the following assumptions:
- The temperatures of the cooler walls and the heater are taken constant.
- The gas temperature in the different compartments is calculated referring to the law of perfect gases.
- The energy losses considered in this model are:
  - Energy Lost by internal conduction, through the solid matrix of the regenerator between the hot and cold parts which are calculated by the expression:

\[
\delta Q_{Pcdr} = k_{cdr} \frac{A_r}{L_r} (T_{r-h} - T_{r-r})
\]

- Losses of energy by external conduction through the regenerator which is not adiabatic:

\[
\delta Q_{Pext} = (1 - e) (\delta Q_{r1} + \delta Q_{r2})
\]

- The pressure losses by friction in the regenerator which leads to power losses:

\[
\delta Q_{Pch} = -\frac{\Delta p m}{\rho}
\]

- The losses of energy by Shuttle effect which comes from the shuttle movement of the exchanger piston between the heater and the cooler:

\[
\delta Q_{Pshl1} = \frac{0.4 Z^2 k_{pis} D_d}{J L_d} (T_d - T_c)
\]
- Losses by irreversibility effect of the compression and the relaxation:
\[
\delta W_{\text{pinc}} = \frac{1}{32} \varepsilon \gamma (\gamma - 1) \frac{T_p}{V_{\text{moy}}} \beta \frac{\Delta P}{V_{\text{moy}}} \Lambda \frac{\Delta T}{V_{\text{moy}}} \Delta T
\]

The dynamic model variables are given on the basis of the energy and mass conservation balance [11]:
- The balance of energy conservation is written:
\[
\delta Q_{\text{r}} + C_p T_p \delta T_p = P \frac{dT}{dt} + C_v \frac{d(m_T)}{dt} + \sum \Delta Q \text{ Diss} (1)
\]
- The balance of the mass conservation in the engine gives:
\[
M = m_d + m_c + m_g + m_r + m_h (2)
\]
- By taking into account the losses by internal and external conduction in the regenerator, the quantities of heat exchanged in the regenerator are written:
\[
\delta Q_{\text{r}} = \varepsilon h_r A_{\text{par}} (T_{\text{par}} - T_r) - \frac{\delta Q_{\text{pedr}}}{2} (3)
\]
- The work given by the cycle is:
\[
\delta W = P_c \frac{dV_c}{dt} + P_d \frac{dV_d}{dt} - \delta W_{\text{irr}} - \delta W_{\text{irr}} (4)
\]
- The thermal effectiveness given by the cycle is:
\[
\eta = \frac{W}{Q_h} (5)
\]

3. RESULTS OF THE MODEL:

The developed model will be applied to a GPU-3 engine of the rhombic type, carried out by General Motor [12]. By introducing the geometrical and physical parameters into the models, the results are given in the following table.1.

3.1 Energy losses in the engine

The energy losses in the engine which are calculated by the model are shown in figures (1) (2) and (3). The energy lost by dissipation is mainly observed in the regenerator, it reaches a maximum equal to 3.94 kW, its average value is 934.9 W. It is 123 W in the heater and 26.57 W in the cooler.

![Figure 1: Lost heat flow.](image)

The energy lost by internal conduction in the different exchangers is given by figure (3). It is negligible in the heater and the cooler and is about 8.48 kW in the regenerator which represents 35% of total lost energy. That is due to the temperature variation which is very significant in the regenerator. The heat flow lost by the Shuttle effect is represented by figure (3). Its average value is 3.119 kW. It represents 13% of total lost energy. The energy lost by external conduction in the regenerator is very significant and depends mainly on the regenerator effectiveness.

4. OPTIMIZATION OF THE STIRLING ENGINE PERFORMANCES

The energy losses are mainly observed in the regenerator, it represents 86% of the total losses in the engine. They are primarily due to the losses by external and internal conduction and pressure losses. The energy lost by Schutte effect in the exchanger piston is also significant, it represents 13% of the other losses are very weak [10]. The reduction of these losses generates an improvement of the engine performances. It is based on the determination of the optimal geometrical and physical parameters. These losses depend mainly on the matrix conductivity of the regenerator, its porosity, the in temperature variation, the working liquid mass, the regenerator volume and the geometrical characteristics of the displacer. To study the influence of these parameters on the prototype performances, we change each time the studied parameter in the model and we maintain the other ones fixed, equal to the prototype parameters.

4.1. Influence of parameters on the engine performances

4.1.1. Effect of the conductivity and the heat capacity of the regenerator matrix

The performances of the engine vary according to the conductivity and to the heat capacity of material constituting the regenerator matrix. Figure (2) shows that increase of matrix regenerator thermal conductivity leads to a reduction of the performances, which is explained by the increase of internal conduction losses in the regenerator according to conductivity [13].

![Figure 2: Effect of regenerator thermal conductivity on the performances.](image)
Figure (3) shows that the engine performances are improved when the heat capacity of the regenerator matrix increases.

![Figure 3: Effect of the regenerator heat capacity on the performances.](image)

The matrix of the regenerator can be made out of different materials. The performances of the engine are given according to the material nature in table 2. To increase heat exchange of the regenerator and to reduce the internal losses by conductivity, we should choose a material with high heat capacity and low conductivity.

Steel, stainless steel, brass and granite are the best materials which can constitute the regenerator matrix.

4.1.2. Effect of the porosity of the regenerator

The engine performances decrease when porosity increases and that is due to the increase in the external conduction losses [13] and to the reduction of the exchanged energy between the fluid and the regenerator figure (4). A porosity of 65.5 % will give a better result.

![Figure 4: Effect of the regenerator porosity on the performances.](image)

4.1.3. Effect of the temperature gradient of the regenerator ($T_{h} - T_{c}$)

Although the engine losses increase when the temperature gradient of the regenerator rises [13]. The performances of the engine increase, figure (5);

![Figure 5: Temp. gradient Effect on performance.](image)

Effect of the fluid mass

The increase of the fluid mass in the engine leads to a rise of the energy lost by pressure loss [13-15], however the engine output increases and the effectiveness reaches a maximum which is about 40 % when the mass is equal to 0.8 g figure (6).

![Figure 6: Effect of the fluid mass on the performances.](image)

4.1.5. Effect of the regenerator volume on the performances

To vary the regenerator volume we can maintain the diameter fixed and vary the length or conversely. When the regenerator diameter is maintained constant equal to 0.0226 m, the length variation influences on the performances. The engine power and effectiveness reach a maximum which corresponds to a length equal to 0.01 m then decrease quickly as shown in figure (7).
If the exchanger piston area is equal to 0.0045 m², we can reach a power more than 5 kW and an output slightly lower than that of the prototype. When the exchanger piston area is constant at 0.0038 m², the effect of the stroke variation on the performances is given by figure (11).

![Figure 11: Effect of the exchanger piston stroke on the engine performances.](image)

When the stroke increases, the engine power decreases but the effectiveness reaches a maximum. The optimal performances are better than the prototype. They are obtained when the area and the stroke are respectively equal to 3.8 \(10^{3}\) m² and 0.042 m, which corresponds to a power of 4500 W and an effectiveness of 41 %. The thermal conductivity of the exchanging piston influences considerably the engine performance figure (12). A weak conductivity reduces the losses by Schuttle effect and increases consequently the engine power and the effectiveness.

![Figure 12: Effect of the displacer thermal conductivity.](image)

4.2. Optimization of the model parameters

The Stirling engine performance depends on the choice of the geometrical and physical parameters [14]. To study their influence on the engine performance, we gradually replaced these parameters by their optimal values. The obtained results are gathered in Table 3.

At the beginning of table 3, we only replaced in the model, the prototype porosity (0.697) by the optimal porosity (0.655). The power and the effectiveness are improved but the average pressure increases slightly.
Therefore, we kept in the model the optimal porosity and we looked for the regenerator optimal length (0.021 m). By introducing it into the model, the power is improved and the effectiveness remains acceptable. Then the optimal diameter (0.024 m) is used; the power and the exchanged energy in the regenerator increase but the effectiveness decreases slightly. By replacing in the model the working fluid mass by the calculated optimal value (1.15g), the power and the effectiveness increase but the average pressure remains acceptable. The exchanger piston conductivity influences the loss by Shuttle effect, its reduction leads to a remarkable increase in the performances. By introducing in the model the optimal values of the area and the stroke of the exchanger piston, the prototype performances are clearly improved.

5. CONCLUSION

The theoretical Stirling cycle presents an excellent theoretical output, but the realized prototypes have lower outputs because of the considerable losses in the regenerator and the exchanger piston, which are primarily due to the losses by external and internal conduction, the pressure losses in the regenerator and by Shuttle effect in the exchanger piston. These losses depend on the geometrical and physical parameters of the prototype design. An optimization study of these parameters that was applied to GPU-3 engine of General Motor had a relatively significant real output (39%) and led to a reduction of these losses and to a notable improvement of the engine performance. To carry out this study, we introduced firstly the parameters of this prototype into a dynamic model; the obtained results were very close to the experimental results, Table 1. Then, we studied the influence of each geometrical and physical parameter on the engine performance and the exchanged energy of the regenerator. The reduction of the matrix porosity and conductivity of the regenerator increases the performance. The increase of the working fluid mass lead to an increase in the engine power and pressure but the effectiveness reaches the maximum. When the exchanger piston section increased and its stroke decreased, the engine output increases and the efficacy reaches the maximum. A low conductivity of the exchanger piston reduces the losses by Shuttle effect and increases consequently the engine power and efficacy. Finally, we optimized these parameters gradually by introducing them into the model and seeking at each time the optimum value. Although the real performance of the used prototype is relatively significant, the obtained results are considerably improved. The output passes from 39 % to 51 %, the power is improved of approximately 20% and the average pressure is slightly increased as shown in Table 2. Applying this study to prototypes with weak performance or to new ones will lead to the determination of their design optimal parameters and consequently to interesting performance.

Nomenclature

- **Cpr**: Heat capacity of each cell matrix
- **CV**: Specific heat at constant volume
- **d**: Hydraulic Diameter
- **D**: Diameter
- **e**: Regenerator efficiency
- **f**: Friction factor
- **J**: Annular gap between displacer and cylinder
- **G**: Working gas mass flow
- **k**: Thermal conductivity
- **L**: Length
- **M**: Mass of working gas in the engine
- **m**: Mass flow rate
- **m**: Mass of working gas in the engine
- **P**: Pressure
- **Q**: Heat
- **Q**: Power
- **R**: Gas constant
- **T**: Temperature
- **U**: Convection heat transfer coefficient
- **V**: Volume
- **W**: Work
- **Z**: Displacer stroke
- **c**: compression space
- **ch**: load
- **cd**: conduction
- **d**: expansion space
- **E**: entered
- **ext**: outside
- **f**: cooler
- **h**: heater
- **irr**: irreversible
- **moy**: average
- **p**: loss
- **Pa**: Wall
- **pis**: piston
- **r**: regenerator
- **r1**: regenerator cell 1
- **r2**: regenerator cell 1
- **S**: left
- **shl**: Shuttle

GREEK LETTERS

- **θ**: Crank angle
- **μ**: Working gas dynamic viscosity
- **ρ**: Density
- **ω**: Angular frequency
- **γ**: Cp Cv⁻¹⁻⁻⁻¹

6. REFERENCES


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**Table 1: Parameters and results of the prototype.**

<table>
<thead>
<tr>
<th>Prototype Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regenerator thermal conductivity</td>
<td>15 W/mK</td>
</tr>
<tr>
<td>Regenerator porosity</td>
<td>0.697</td>
</tr>
<tr>
<td>Regenerator length</td>
<td>0.0226 m</td>
</tr>
<tr>
<td>Regenerator diameter</td>
<td>0.0226 m</td>
</tr>
<tr>
<td>Working fluid mass</td>
<td>1.1362 g</td>
</tr>
<tr>
<td>Displacer piston conductivity</td>
<td>15 W/mK</td>
</tr>
<tr>
<td>Displacer piston surface</td>
<td>3.8 × 10^{-3} m²</td>
</tr>
<tr>
<td>Displacer piston stroke</td>
<td>0.046 m</td>
</tr>
<tr>
<td>Calculated average pressure</td>
<td>46.74 bar</td>
</tr>
<tr>
<td>Calculated exchanged energy</td>
<td>448.7206 J</td>
</tr>
<tr>
<td>Calculated power</td>
<td>4273</td>
</tr>
<tr>
<td>Calculated effectiveness</td>
<td>39 %</td>
</tr>
<tr>
<td>Experimental power</td>
<td>3500</td>
</tr>
<tr>
<td>Experimental effectiveness</td>
<td>35 %</td>
</tr>
</tbody>
</table>

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**Table 2: Regenerator material effect on the engine performances.**

<table>
<thead>
<tr>
<th>Exchanged energy in the regenerator (J)</th>
<th>Engine efficiency (%)</th>
<th>Engine power (W)</th>
<th>Conductivity (W/m K)</th>
<th>Volumetric heat Capacity (10^8 J/m³ K)</th>
<th>Regenerator material</th>
</tr>
</thead>
<tbody>
<tr>
<td>441.25</td>
<td>38.84</td>
<td>4258</td>
<td>46</td>
<td>3.8465</td>
<td>Steel</td>
</tr>
<tr>
<td>448.72</td>
<td>39.29</td>
<td>4273</td>
<td>15</td>
<td>3.545</td>
<td>Stainless steel</td>
</tr>
<tr>
<td></td>
<td>389</td>
<td>3.3972</td>
<td></td>
<td></td>
<td>Copper</td>
</tr>
<tr>
<td>415.67</td>
<td>34.6</td>
<td>4080</td>
<td>100</td>
<td>3.145</td>
<td>Brass</td>
</tr>
<tr>
<td>378.03</td>
<td>29.16</td>
<td>3812</td>
<td>200</td>
<td>2.322</td>
<td>Aluminum</td>
</tr>
<tr>
<td>430.75</td>
<td>34.51</td>
<td>4091</td>
<td>2.5</td>
<td>2.262</td>
<td>Granite</td>
</tr>
<tr>
<td>429.9</td>
<td>34.28</td>
<td>4081</td>
<td>0.92</td>
<td>2.208</td>
<td>Concrete</td>
</tr>
<tr>
<td>427.8</td>
<td>33.85</td>
<td>4062</td>
<td>1.2</td>
<td>2.125</td>
<td>Glass</td>
</tr>
<tr>
<td>407.75</td>
<td>30.04</td>
<td>3863</td>
<td>0.16</td>
<td>1.45</td>
<td>PVC</td>
</tr>
<tr>
<td>367.4</td>
<td>23.75</td>
<td>3207</td>
<td>0.23</td>
<td>0.512</td>
<td>Wood</td>
</tr>
</tbody>
</table>

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**Table 3: Effect of the optimal values on the engine performances.**

<table>
<thead>
<tr>
<th>Regenerator porosity</th>
<th>Optimal Value</th>
<th>Power (W)</th>
<th>Efficiency (%)</th>
<th>Exchanged energy (J)</th>
<th>Average Pressure (bar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>65.5 %</td>
<td>4554.2</td>
<td>43.3</td>
<td>474.31</td>
<td>47.1</td>
<td></td>
</tr>
<tr>
<td>Regenerator length</td>
<td>0.021 m</td>
<td>4675.2</td>
<td>41.04</td>
<td>472.36</td>
<td>49.36</td>
</tr>
<tr>
<td>0.024 m</td>
<td>4765.1</td>
<td>39.91</td>
<td>475.7670</td>
<td>48.457</td>
<td></td>
</tr>
<tr>
<td>Working fluid mass</td>
<td>1.15 g</td>
<td>4849.6</td>
<td>40.14</td>
<td>480.71</td>
<td>49</td>
</tr>
<tr>
<td>Exchanger piston Conductivity</td>
<td>1.2 W/mK</td>
<td>5079.4</td>
<td>50.93</td>
<td>479.87</td>
<td>49.45</td>
</tr>
<tr>
<td>Exchanger piston area</td>
<td>3.86 10^{-3} m²</td>
<td>5183</td>
<td>50.97</td>
<td>483.71</td>
<td>49.225</td>
</tr>
<tr>
<td>Exchanger piston stroke</td>
<td>0.047 m</td>
<td>5106.2</td>
<td>51.05</td>
<td>472.67</td>
<td>49.549</td>
</tr>
</tbody>
</table>