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CONSTRUCTAL MODEL OF FITTS'S LAW TO PREDICT SPEED–ACCURACY TRADE-OFF

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ABSTRACT
The Constructal law provides novel modeling tools to better understand the complexity in physical and biological processes. The law begins with the design and optimization of engineered systems and discovers a deterministic principle for the generation of geometric form in natural systems. The intricate design of all human–machine interfaces can benefit from the Constructal law. In this paper, the Constructal law is used to predict minimum time (optimal) of travel using experimental data obtained from a large-scale human–machine interface study along with the use of Fitts's law. The results demonstrate a great potential to utilize Constructal law as complementary to Fitts's law in the design of human–machine systems. Future efforts will include the application of the integrated Constructal–Fitts's law concept to assist in the prediction of effective outcomes for various ergonomic and human factor-related conditions that exist in the work environment.
Keywords: adaptive, Constructal law, Fitts's law, human–machine interface, minimum travel time.

1 INTRODUCTION

It is an observed fact that humans act and react in certain ways to the environment. The gross human movements, such as walking and running, are characterized by human's intention or objective to conserve mass, momentum, and energy. This is restricted to not just gross movements but also finer movements of arms, wrists, and fingers in less physically intense activities such as playing of piano, typing, and human–machine interactions. The economy of movement is the key here. However, why do humans take the path of least resistance such as walking through the lawn, as shown in Fig. 1? Is the landscape architect at fault or is the pedestrian too lazy to take the sidewalks? How do you quantify the effectiveness of a human–machine interface design? The Constructal law answers these questions by quantifying the optimal (or minimal) movement required by the user or the subject to reach a target (A) separated by a distance (D). This is demonstrated with the use of virtual experimental data produced by Goldberg *et al*. [1].

2 THEORY AND BACKGROUND

The design of human–machine interfaces can benefit from intuitive physics-based models and laws. This is true from computers to iPhones to games to assembly lines to websites to automobiles. A fundamental principle in the design of human–machine systems is being able to describe the constituent parts to understand the system as a whole. A relationship known as the Constructal law establishes the fact that optimal distribution of imperfection is the principle that generates form [2,3]. The Constructal law explains and predicts natural self-organization including fine human finger-arm movement, which is characterized by its complexity and rhythmicity. The movements are triggered and coordinated by the central nervous system (CNS) by a group of integrated structures in a manner that is not clearly understood at this time. The Constructal law can provide a physical explanation of these internal events that occur in the mind–brain via geometry of human motion dynamics. Thus,

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Figure 1: The path of least resistance (www.baddesigns.com).

the law could be extended to explain the flow of information in a human mind–brain combination that leads to motor skill acquisition, as demonstrated in Section 5 of this paper. The psychology of human motion is intrinsically linked to the energetic aspects such as metabolism and density of the body, which in turn are governed by the autonomic nervous system under the influence of the CNS.

It can be argued that a somewhat germane relationship exists between the Constructal law and what is known as Fitts's law [1]. The latter predicts that, for single-limb movements, the easy target should be reached first; that is, movement time should be much shorter for the easier to reach target than for the difficult to reach target [4]. In other words, if a large object is close to the starting position of one's hand, the subject should be able to pick it up in a shorter time than a small object that is far away from the individual. But what happens when the subject is expected to do both activities at the same time? The brain has many different muscles to coordinate; how does it accomplish this objective?

Humans act and react! A characteristic feature of human beings is that they respond to their environment. For the purpose of quantitative analysis, consider the following axiom: 'For every response (action time), there is both a reaction (reaction time) and a movement (movement time).' Mathematically, this may be expressed as follows:

$$\mathrm{AT} = \mathrm{RT} + \mathrm{MT}, \tag{1}$$

where AT is the action time (s), RT is the reaction time (seconds), and MT is the movement time (s).

The RT may be mathematically related to the information content of the stimulus event ($H_S = \log_2 N$) that is present in a set of equally probable N alternatives as

$$\mathrm{RT} = a + bH_S, \tag{2}$$

where *a* is the simple reaction time ($H_S = 0$) and b is a proportionality constant, which is equal to 1/baud rate (bits per second).

Let us now consider MT. A simple form of movement is that of directly moving from point A (the starting point) to point B (the end point). This is referred to as step tracking. There are various examples in the real world of step tracking. Examples include the aiming of a camera upon a stationary object, changing lanes while driving on a multilane highway, and reaching over a distance to place the finger on a flip switch. In the performance of such tasks, the movement time would be affected by the distance of the movement as well as the accuracy required by the size of the object towards which one is moving. The relationship between the distance moved and the object size represents the difficulty of the movement response. The details of Fitts's law derivation are provided in Ref. [4].

Fitts's law defines an index of difficulty (ID) with an analogy to information theory:

$$\text{ID (bits)} = \log_2(2A/W), \tag{3}$$

where A is the distance of movement from start to target centre and W is the width of the target.

Equation (3) can be rewritten as

$$\text{ID (bits)} = \log_2[A/(W/2)]. \tag{4}$$

Since A is the distance of movement from the start to the target center, the target can be acquired by overshooting the center, but within the limit of plus W/2. However, it is equally possible to acquire the target by undershooting the center, but within the limit of minus W/2. In the original experiments that led to Fitts's law, it was demonstrated that for movement between 2 and 16 in., a longer time is required when that movement must end within a small target area compared with when the target area is larger. In these original experiments, the two targets were placed at fixed distance apart and the subject had to tap them alternately and as quickly as possible.

The MT may be defined as the time required to physically make a response that begins when the movement is initiated and ends when the target is acquired. The basis for Fitts's law is that the movement time can be predicted from the index of movement difficulty:

$$\text{MT} = a + b\ (\text{ID}). \tag{5}$$

The empirical constants a and b in eqn (5) depend upon the type and nature of movement involved, where a is the delay constant that depends upon the body member being used and b (s/bit) is the measure of the information handling capacity.

The generally used form of Fitts's law is as follows:

$$\text{MT} = a + b \log_2 [2A/W]. \tag{6}$$

What is the optimal $(\text{MT})_{\text{optimal}}$? Can we relate this to constructal optimal time T_{optimal}? Are they one and the same? Is MT just a special case of more general constructal time T? There are various types of movements for which Fitts's law does not hold. However, it has been demonstrated that the rule does apply for single prepared movements, provided that they are reasonably large. As Fitts's law is based on information theory, the equations are descriptive rather than explanatory so that, per se, they do not provide information regarding the specific mechanics of the information processing scheme. In contrast, Constructal law overcomes the limitation of Fitts's law in predicting the travel time in all geometries and flow architectures involving complex human motion that includes the specific mechanics of information processing as demonstrated in the eqns (7)–(10) in the following section and in the results.

3 GENERAL FORMULATION OF CONSTRUCTAL MODEL OF FITTS'S LAW

Let us begin with the first constructal element ratio (C_1) defined by the shape A_1/W_1, which varies as the subject moves from one construct to another. Our goal is to design the user interface in such a manner that the time of travel between the source (red rectangle or circle in Fig. 2) to the destination (green rectangle or circle in Fig. 2) is minimized. The Constructal law approach is demonstrated using data from the experimental study conducted in the Berkeley lab [1], and data from an online test using Java applet in which subjects were told to seek and hit the target with width (W) that was placed at distance (A)

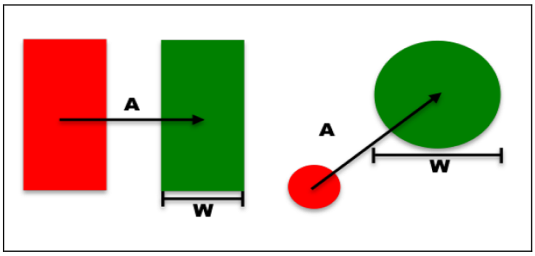


Figure 2: The Java applet presented to human subjects in the Berkeley lab and online experiments [1] with a sequence of rectangular and circular targets, which record timing data.

(see Fig. 2). Note that constructal elements evolve in time, either becoming too difficult to hit the target or easier with practice. The mathematical relationships between constructal element and the target (A) and width are not known at this time.

The starting and target points are located on a constructal ratio defined by the ratio of target distance (A) to the width (W), as in Fitts's law convention.

In the current study, the constructal elements are probabilistic and depend on the level of difficulty experienced during the performance of a task. In other words, the constructal movement is probabilistic and will indicate the individual's ability to develop skills.

C_1 = constructal element ratio = A_1/W_1
C_2 = constructal element ratio= A_2/W_2
C_3 = constructal element ratio= A_3/W_3
C_i = constructal element ratio= A_i/W_i

The observer can move from a smaller construct (C_1) to larger constructs (e.g. C_2, C_3, ...), or vice versa, or between the constructs as and when required by a task.

As developed by Bejan [2], the optimal features that manifest dynamically into a constructal pattern are summarized as follows:

$$(A_i/W_i)_{opt} = V_{i-1}/(f_i*V_i), \tag{7}$$

$$\alpha_{i,opt} = \sin^{-1}(V_{i-1}/2*V_i), \tag{8}$$

$$f_i = (1/\cos\alpha_{i,opt}) - (V_{i-1}/(2*V_i))*\tan\alpha_{i,opt}, \tag{9}$$

$$T_{i,opt} = 2*(f_i*A_i/V_{i-1}*V_i)^{1/2}, \tag{10}$$

where $\alpha_{i,opt}$ is the optimal angle traversed while moving from one construct area to another. The angles subtended in relationship to the vertical or horizontal plane of view that minimizes (or optimizes) the travel time to reach the target accurately. $T_{i,opt}$ is the optimal time to reach the target in a constructal area (C_{Ai}), where i takes on values 1, 2, 3, …, k.

The ability to quantify the movement time from one construct to another is missing in Fitts's law. People always change the direction of flow of their finger, wrists, and arms to optimize travel time as they acquire skills. Even the most difficult tasks become easier if they are well practiced. However, the initial travel time to a website or a device or a search engine determines the success of the human–computer interface, as this creates a first impression in the mind of the user. Perhaps, usability or ease of use is one of the reasons why online education has not become that popular, considering the level of technological development. This is where the Constructal law would play a critical role—as a tool to quantify the effectiveness of human–machine interface design.

The Constructal law provides an expression of evolutionary optimal time as movements continue to explore newer target areas (C_1, C_2, C_3, …, C_i). It is stated by Bejan [2] that $C_2 = n_2*C_1$, where C_1 is within C_2 and so on. The n_2 is the number of elements of C_1 that are within the next construct area, C_2. While this applies to a situation where the user is moving from a smaller construct to a larger construct, it can also be expressed as $C_1 = C_2/n_2$. In dynamic terms, the user will improve their skill while scoping the larger construct (C_2), while potentially returning to a smaller construct (C_1). In dynamic terms, this can happen repeatedly over a period of time, from smaller to larger and back and forth. As the user seeks newer targets to reach, there is skill acquisition, which improves with practice. This is clearly observed in field sports such as football, soccer, and basketball, where athletes cover more area to reach the target with improved accuracy. This also happens in individual court sports such as tennis, racquet ball, and squash. In summary, the Constructal law provides a quantitative measure to predict the optimal time of travel or, in other words, quantifying skill acquisition.

4 EXPERIMENTAL METHODOLOGY

The controlled (in-lab) experimental data [1] were used to demonstrate the utility of the Constructal law to predict optimal time. This study involved two conditions: a ‘homogeneous targets’ condition with rectangles where sequential targets are constant in distance and size, and a ‘heterogeneous targets’ condition where sequential targets are circular and vary in distance and size. The experiments considered targets of different difficulty, as defined by the ratio of target distance over target size, as listed in Table 1. These user studies were conducted under University of California, Berkeley, human subject certification. A Java applet was developed that asks each subject to complete two experiments by using his or her cursor to click on a sequence of rectangular or circular targets as they are presented on the screen. This Java applet is available online at http://automation.berkeley.edu/fitts/ [5]. The applet records the time in milliseconds between when the target appears until when the subject clicks on the target. A subject may click when the cursor is outside the target, but the increments are only timed when the target is successfully clicked.

The study involved 46 subjects, selected to participate via an advertisement posted on the bulletin boards of several buildings on the UC Berkeley campus, as well as on Facebook. As a reward for their participation, an amazon.com gift certificate was offered to all of the study’s participants. The study’s subjects included 17 females (37%) and 29 males (63%), with an average age of 24.7 years old (standard deviation = 4.9). Each participant performed the set of homogeneous target and heterogeneous target experiments in 10 trials. For the controlled experiment, 490 trajectories were collected for each of the 46 subjects, giving a total of 22,540 timing measurements in milliseconds (11,040 for homogeneous targets and 11,500 for heterogeneous targets). For more details about the experiment, readers can refer to Goldberg *et al*. [1] or the website http://automation.berkeley.edu/fitts/ [5].

5 RESULTS AND DISCUSSION

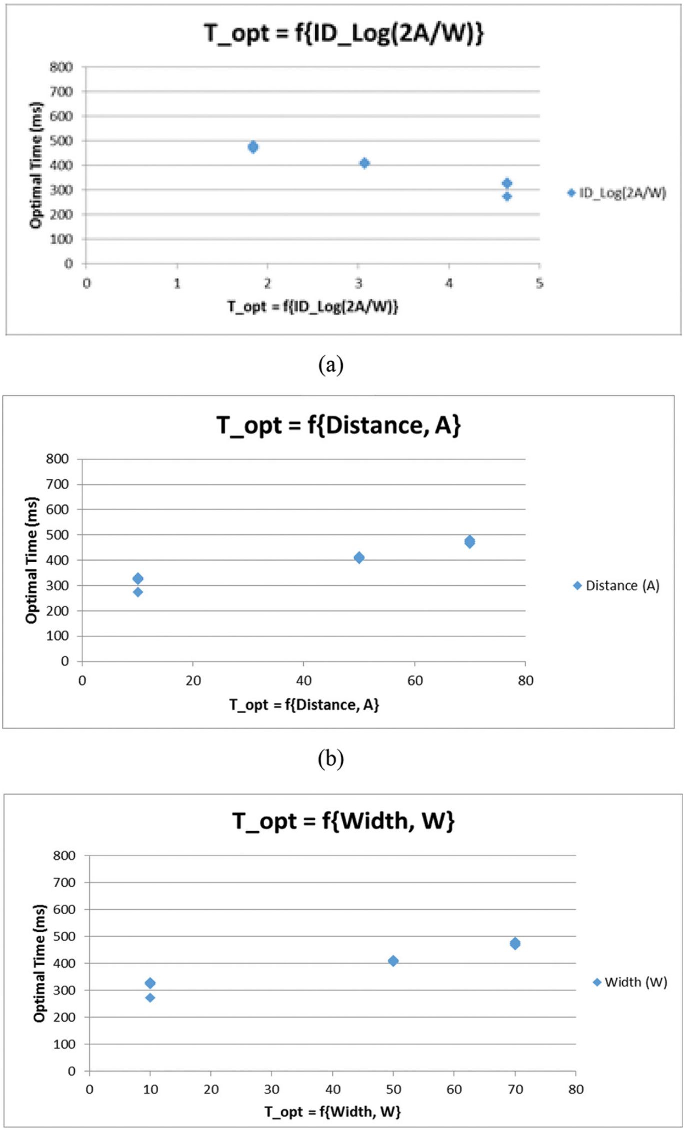
The dynamics of fine human movement have been modeled as a constructal process. As the time of travel depends on the construct area rather than just the ratio of A/W as in Fitts’s law, the Constructal law provides a complementary approach to assess human–machine (or

Table 1: Target distance (A) and width (W), in display pixels, for the 24 recorded fixed rectangles trials and 25 variable circles trials.

Trial	Homogeneous (rectangular) targets			Heterogeneous (circular) targets		
	A	W	A/W	A	W	A/W
1	370	50	7.40	67	20	3.35
2	370	50	7.40	184	38	4.84
3	370	50	7.40	280	14	20.00
4	370	50	7.40	230	29	7.93
5	370	50	7.40	144	55	2.62
6	370	50	7.40	249	29	8.59
7	370	50	7.40	255	14	18.21
8	370	50	7.40	96	50	1.92
9	240	10	24.00	225	19	11.84
10	240	10	24.00	263	12	21.92
11	240	10	24.00	259	25	10.36
12	240	10	24.00	229	20	11.45
13	240	10	24.00	215	31	6.94
14	240	10	24.00	198	83	2.39
15	240	10	24.00	301	16	18.81
16	240	10	24.00	194	66	2.94
17	180	70	2.57	260	12	21.67
18	180	70	2.57	296	14	21.14
19	180	70	2.57	180	44	4.09
20	180	70	2.57	278	11	25.27
21	180	70	2.57	283	37	7.65
22	180	70	2.57	40	32	1.25
23	180	70	2.57	233	10	23.3
24	180	70	2.57	191	50	3.82
25	–	–	–	179	18	9.94

human–computer) interfaces. The results from the two experiments, homogeneous (rectangular targets) and heterogeneous (circular targets), are presented in Figs 3 and 4, respectively. The calculated optimal time value, determined by using eqn (10), is plotted as a function of ID, distance, width, distance/width ratio, and constructal areas for both experimental types.

It is observed in Figs 3a–d and 4a–d that Fitts's law predicts the optimal time required to reach the target in terms of area, width, and distance that do not evolve in time. In contrast, the constructal areas evolve in time, changing geometries, and shapes, resulting in more realistic representations of physical tasks performed by humans in the real world. In Fig. 3e, for the homogeneous rectangular targets, the constructal areas are predictable and the optimum time is more linear. However, in Fig. 4e for heterogeneous circular targets, the constructal areas represent more complex dynamical shapes that evolve in time, challenging the humans to adapt. The optimal time required to achieve perfection is shown in Fig. 4e. As the complexity of constructal circular targets increases, the optimal time increases as well.



(a)

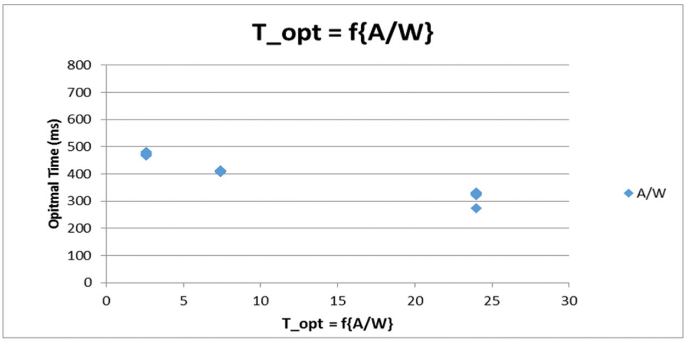
(b)

(c)

Figure 3: Optimal time (T_{opt}, ms) for homogeneous (rectangle) targets.

It is valuable to compare the pattern of optimal travel response between that of homogeneous (rectangular) and heterogeneous (circular), as in Fig. 5. Methods need to be investigated to extract patterns that could be used to enhance subject training to improve motor skills or to improve the human–machine interface design.

The Constructal law-based optimal time in milliseconds for two experiments is shown in Table 2. The difference in mean optimal time between homogeneous (rhythmic) and heterogeneous (chaotic) does not appear to be appreciable, but a variation in the pattern is discernible.



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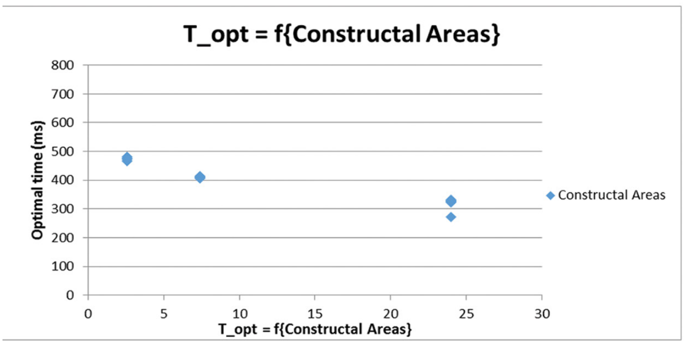


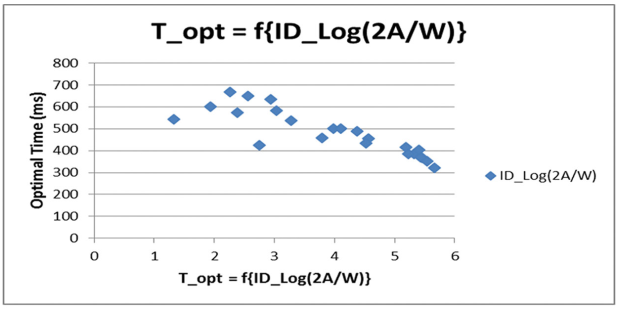
(e)

Figure 3: (continued)

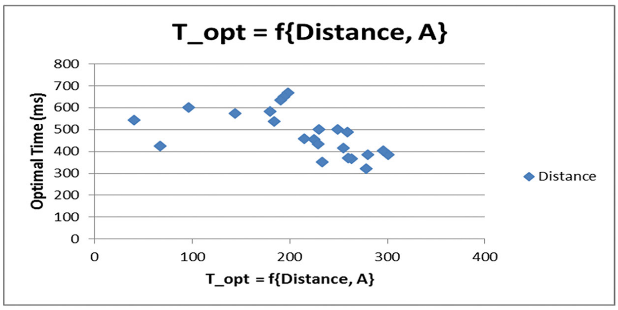
6 CONCLUSION

As nature affords us with rhythmicity and fine-tuned frequencies along with non-linear chaotic behavior, the approach used in this paper is worthy of serious investigation by interested researchers in the fields of motor control, experimental psychology, and human factors engineering. For example, many small component manufacturing companies require a workforce that is very comfortable in interfacing with various types of sophisticated machinery and equipment in order to effectively and efficiently produce a quality product. The application of this construct at work stations where the interaction of workers with their tools, instruments, and machinery is critical to the accuracy and speed of which the product is made should lead to a more productive workforce.

The Constructal law provides a unique methodology to model speed–accuracy trade-off in complex tasks involving shapes that change and/or evolve in time. This is clearly demonstrated in the results shown in Figs 3 and 4, and in Table 2 as well. The optimal time obtained using the Constructal law provides a more realistic dynamic trade-off between speed and accuracy. It also provides specific information about the mechanics of motion in terms of deterministic A/W ratios, velocities, and optimal angles subtended by the complex movements. In summary, the deterministic Constructal law, in conjunction with the probabilistic Fitts's law, could provide an explanatory prediction of human motion in the design of



(a)



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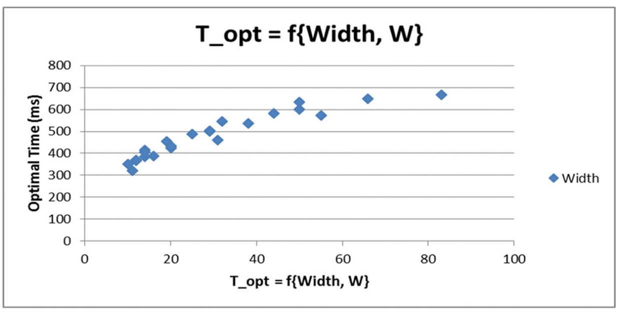

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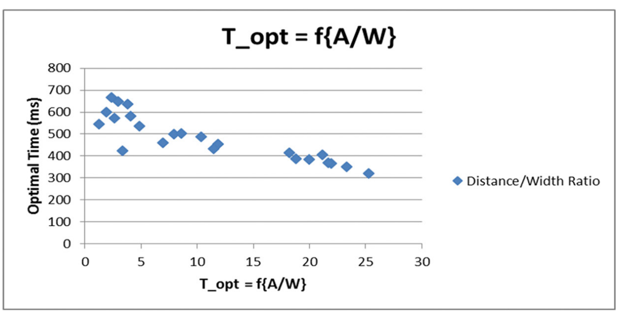
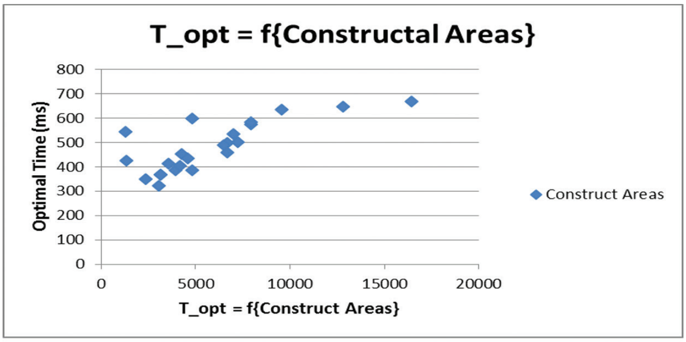


(d)

Figure 4: Optimal time (T_{opt}, ms) for heterogeneous (circular) targets.



(e)

Figure 4: (Continued)

Table 2: Optimal time statistics, in milliseconds, for the 24 recorded fixed rectangles (fixed rectangles) trials and 25 variable circles trials.

Statistics	Homogeneous	Heterogeneous
Mean	397.94	473.33
Median	408.87	456.75
Standard deviation	66.29	98.89
Sample variance	4394.85	9779.90
Range	208.75	345.70
Minimum	272.92	321.91
Maximum	481.68	667.60

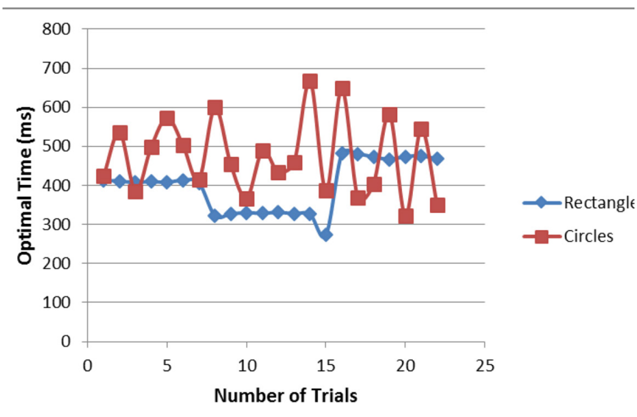


Figure 5: Optimal time (T_{opt}) in milliseconds versus number of trials for homogeneous (rectangular) and heterogeneous (circular) experiments.

complex, adaptive human–machine systems. Future efforts will be aimed at applying this theory to various workplace conditions to test its potential predictive abilities in the field.

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