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## NOMENCLATURE

$x$	longitudinal (axial) coordinate variable
$l$	length of the column
$yz$	plane of the cross-section of the column
$A$	area of the cross-section
$t(x)$	applied torque
$M_{tz} M_{ty}$	moments due to transverse loads
$J$	Saint Venant torsional stiffness of the cross-section
$E$	Young's modulus of elasticity of the column material
$G$	shear modulus or modulus of rigidity
$I_{yy} I_{zz}$	moments of inertia about the $yy$ and $zz$ coordinate axes
$C_w$	warping constant

$n$	buckling mode number
$I_E$	polar moment of inertia about the shear centre
$I_{yz}$	product of inertia
$e_y, e_z$	coordinates of the shear centre
$V_y, V_z$	shear forces
$q_y, q_z$	transverse loads
$N_x$	axial force
$v(x) w(x)$	transverse displacement components in the $y$ and $z$ coordinate directions
$\theta(x)$	twist or rotational displacement
$H_y, H_z, H_w, C_y, C_z$	geometrical properties of the cross-section
$F_1, F_2, F_3$	functionals to be minimized
$\bar{v}(x), \bar{w}(x), \bar{\theta}(x)$	approximations to the unknown displacement functions
$\varphi_n(x)$	displacement functions for the $n$ th buckling mode
$Q_{zz}$	Euler flexural buckling load about the $zz$ axis
$Q_{yy}$	Euler flexural buckling load about the $yy$ axis
$Q_\phi$	torsional buckling load
$r_0$	radius of gyration
$A_n, B_n, \theta_n$	buckling modal amplitudes of the unknown displacement $v(x), w(x)$ and $\theta(x)$ for the $n$ th mode
$\frac{d}{dx}$	first ordinary differential operator
$\frac{d^n}{dx^n}$	$n$ th ordinary differential operator
$L_1 = \frac{d^4}{dx^4}$	notation for the fourth ordinary differential operator
$L_2 = \frac{d^2}{dx^2}$	notation for the second ordinary differential operator
$\iint_A$	double integral over the domain of the cross sectional area, $A$
$\int_0^l$	integral over the length $l$ of the column
$\sum_{n=1}^{\infty}$	summation for $n = 1, 2, 3, \dots, \infty$
$v''(x)$	second derivative of $v(x)$ with respect to $x$
$w''(x)$	second derivative of $w(x)$ with respect to $x$
$\theta''(x)$	second derivative of $\theta(x)$ with respect to $x$