

$$h_{313} = \text{trace}(A_1 A_2 Q_3 Q_3 A_1 d'_{33}), \quad h_{321} = h_{312},$$

$$h_{331} = h_{313}, \quad h_{322} = \text{trace}(A_1 Q_2 A_2 Q_3 A_1 d'_{33}),$$

$$h_{323} = \text{trace}(A_1 A_2 Q_3 Q_3 I_3 d'_{33}), \quad h_{332} = h_{323},$$

$$h_{333} = \text{trace}(A_1 A_2 Q_3 Q_3 A_1 d'_{33}),$$

$$H_3 = h_{311} \dot{\theta}_1 \dot{\theta}_1 + h_{312} \dot{\theta}_1 \dot{\theta}_2 + h_{313} \dot{\theta}_1 \dot{\theta}_3 + h_{321} \dot{\theta}_2 \dot{\theta}_1 + h_{322} \dot{\theta}_2 \dot{\theta}_2 + h_{323} \dot{\theta}_2 \dot{\theta}_3 + h_{331} \dot{\theta}_3 \dot{\theta}_1 + h_{332} \dot{\theta}_3 \dot{\theta}_2 + h_{333} \dot{\theta}_3 \dot{\theta}_3$$

$$H = [H_1 \quad H_2 \quad H_3],$$

$$G_1 = -(m_1 g d_{11} r_1 + m_2 g d_{21} r_2 + m_3 g d_{31} r_3),$$

$$G_2 = -(m_2 g d_{22} r_2 + m_3 g d_{32} r_3), \quad G_3 = -(m_3 g d_{33} r_3),$$

$$G = [G_1 \quad G_2 \quad G_3]$$