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Statistical Summability for Triple Sequences over Non-Archimedean Fields

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ABSTRACT

This paper aims to explore the concepts of statistical convergence sequence and statistical summability in non-Archimedean fields (NAF). Statistical convergence has been studied in various mathematical fields such as measure theory, probability theory, and number theory, and plays a significant role in summability theory and functional analysis. The goal of this study is to provide characterizations for triple sequences using the $(M_{\lambda_{t,u,v}})$ method of summability over NAF and to prove inclusion relations between statistical convergence and statistical $(M_{\lambda_{t,u,v}})$ summability for triple sequences over such fields. Furthermore, statistics $A_{t,u,v}$-summability for triple sequence has been examined in non-Archimedean fields. The non-trivially valued, complete, non-Archimedean fields are denoted by $\mathcal{K}$ throughout the article.

1. INTRODUCTION

The theory of statistical convergence has been analyzed by numerous researchers since the concept of statistical convergence was first proposed in 1935. The statistical convergence was initiated by Fast [1] and Steinhaus [2] independently in the same year as a generalization, it was reintroduced by Schoenberg [3] in 1959. Furthermore, statistical convergence has become an active research area in various fields of mathematical analysis. Later, the notion was investigated with summability theory by Fridy [4] in 1985. In recent years, quite a few researchers have discussed the concept of statistical convergence in summability theory such as Kolk [5], Connor [6], Edely and Mursaleen [7], Edely [8], Natarajan [9], Mursaleen [10], Suja and srinivasan [11], and so on.

Recently, the concept of statistical convergence has also been extended to double sequences and triple sequences. The notion of the statistical convergence of triple sequences was initially proposed established by Sahiner and Tripathy [12].

A triple sequence $x=\{x_{(g,h,i)}\}$ is defined as statistically convergent to $\mathcal{L}$, for every $\varepsilon>0$: $\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{g\le t; h\le u; i\le v; (t,u,v)\in\mathbb{N}: |x_{g,h,i}-\mathcal{L}|\ge\varepsilon\}|=0$.

We write: $stat-\lim_{g,h,i\to\infty}\{x_{(g,h,i)}\}=\mathcal{L}$ or $x_{(g,h,i)}\xrightarrow{stat}\mathcal{L}$.

2. PRELIMINARIES

This section introduces the definition for statistical $(M_{\lambda_{t,u,v}})$ summability method.

Definition 2.1 [13] A complete, non-trivially valued, NAF $\mathcal{K}$ is said to be an ultrametric valued field such that

1. $|x|\ge 0$ and $|x|=0$ if $x=0$;
2. $|xy|=|x||y|$;
3. $|x+y|\le max\{|x|,|y|\}$ for all $x,y\in\mathcal{K}$. (Ultrametric inequality).

Definition 2.2 [14] Let $\lambda=\{\lambda_{t,u,v}\}$ be a triple sequence such that $\sum_{t,u,v=0}^{\infty}|\lambda_{t,u,v}|<\infty$. Then, $(M, \lambda_{t,u,v})$-method is determined by 6-dimensional infinite matrix $M=(a_{(t,u,v,g,h,i)})$, where, $a_{(t,u,v,g,h,i)}=\begin{cases}\lambda_{g-t,h-u,i-v}; & if\ g\le t, h\le u, i\le v\\ 0; & otherwise.\end{cases}$.

It is well known from Aral et al. [15, 16] that $\lambda_0=\lambda_1=\frac{1}{2}$ and $\lambda_{t,u,v}=0$ if $t,u,v\ge 2$, the $(M, \lambda_{t,u,v})$-method is regular if $\sum_{t,u,v=0}^{\infty}|\lambda_{t,u,v}|=1$.

Thus $\sum_{t,u,v=0}^{\infty}|\lambda_{t,u,v}|=1$.

Remark: The sequence $x=\{x_{g,h,i}\}$ is defined as $M_{\lambda_{t,u,v}}$-summable to $\mathcal{L}$ and it is symbolized by: $x_{g,h,i}\xrightarrow{M_{\lambda_{t,u,v}}}\mathcal{L}$.

Definition 2.3 The sequence $x=\{x_{g,h,i}\}$ is defined as $M_{\lambda_{t,u,v}}$-summable to $\mathcal{L}$ if $|x_{g,h,i}-\mathcal{L}|$ is $M_{\lambda_{t,u,v}}$ summable to $\mathcal{L}$ and it is symbolized by $x_{g,h,i}\xrightarrow{M_{\lambda_{t,u,v}}}\mathcal{L}$. The set of all $M_{\lambda_{t,u,v}}$ summable sequence is defined by:

$$\lim_{t,u,v\to\infty}\sum_{t,u,v=0}^{\infty}\lambda_{g-t,h-u,i-v}|x_{g,h,i}-\mathcal{L}|=0.$$

Definition 2.4 The sequence $x=\{x_{g,h,i}\}$ is defined as $M_{\lambda_{t,u,v}}$-statistically convergent to $\mathcal{L}$ over NAF $\mathcal{K}$, if for all ε>0:

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{g\leq t; h\leq u; i\leq v; \{t,u,v\}\in\mathbb{N}: |\sum_{t,u,v=0}^{\infty}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})|\geq\varepsilon\}|=0.$$

It is symbolized by: $stat_{M_{\lambda_{t,u,v}}}-\lim_{g,h,i\to\infty}x_{g,h,i}=\mathcal{L}$ or $x_{g,h,i}\xrightarrow{M_{\lambda_{t,u,v}}-stat}\mathcal{L}$.

Definition 2.5 A triple sequence $x=\{x_{g,h,i}\}$ is defined as $M_{\lambda_{t,u,v}}$-statistically convergent to $\mathcal{L}$ over NAF $\mathcal{K}$, if for all ε>0:

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{g\leq t; h\leq u; i\leq v; \{t,u,v\}\in\mathbb{N}: |\sum_{t,u,v=0}^{\infty}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})|\geq\varepsilon\}|=0.$$

It is symbolized by: $stat_{M_{\lambda_{t,u,v}}}-\lim_{g,h,i\to\infty}|x_{g,h,i}|=|\mathcal{L}|$ or $|x_{g,h,i}|\xrightarrow{M^{stat}_{\lambda_{t,u,v}}}|\mathcal{L}|$.

Definition 2.6 Let $\lambda=\{\lambda_{t,u,v}\}$, $\eta=\{\eta_{t,u,v}\}$ be two increasing sequences of non-negative integers tending to ∞. Then, we say that $\lambda(x,y,z)=\sum_{t,u,v=0}^{\infty}\lambda_{t,u,v}x^{t,u,v}y^{t,u,v}z^{t,u,v}$ and $\eta(x)=\sum_{t,u,v=0}^{\infty}\eta_{t,u,v}x^{t,u,v}y^{t,u,v}z^{t,u,v}$, where,

$$\frac{\eta(x,y,z)}{\lambda(x,y,z)}=r(x,y,z)=\sum_{t,u,v=0}^{\infty}r_{t,u,v}x^{t,u,v}y^{t,u,v}z^{t,u,v},$$
$$\frac{\lambda(x,y,z)}{\eta(x,y,z)}=s(x,y,z)=\sum_{t,u,v=0}^{\infty}s_{t,u,v}x^{t,u,v}y^{t,u,v}z^{t,u,v}.$$

If $\lambda(x, y, z)$, $\eta(x, y, z)$ are convergent for all $|x|<1$, $|y|<1$, $|z|<1$ because λ and η are bounded sequences.

$$\varLambda=\{\lambda=(\lambda_{t,u,v}):\sum_{t,u,v=0}^{\infty}|\lambda_{t,u,v}|<\infty\},$$
$$\varLambda_0=\{\lambda=(\varLambda):\sum_{t,u,v=0}^{\infty}\lambda_{t,u,v}=1\}.$$

Definition 2.7 Let $\lambda=\{\lambda_{t,u,v}\}$ and $\eta=\{\eta_{t,u,v}\}$ be two increasing triple sequence of positive integers tending to ∞, from $\varLambda$ are said to be equivalent if: $\lim_{t,u,v\to\infty}\frac{\lambda_{t,u,v}}{\eta_{t,u,v}}=1$, and it is denoted by $\lambda\equiv\eta$.

3. RESULTS ON STATISTICAL ($M_{\lambda_{t,u,v}}$) SUMMABILITY FOR TRIPLE SEQUENCES

Theorem 3.1: If for any $\lambda,\eta\in\varLambda$, there exists a triple sequences $\xi=\{\xi_{t,u,v}\}\in\Lambda$, where, $\varLambda=\{\lambda=\{\lambda_{t,u,v}\}:\sum_{t,u,v=0}^{\infty}|\lambda_{t,u,v}|<\infty\}$. Then, $M^{stat}_{\lambda_{t,u,v}}\subset M^{stat}_{\xi_{t,u,v}}$ and $M^{stat}_{\eta_{t,u,v}}\subset M^{stat}_{\xi_{t,u,v}}$.

Proof. Let $\lambda=\{\lambda_{t,u,v}\}\in\varLambda_0$ and $\eta=\{\eta_{t,u,v}\}\in\varLambda_0$. Let us assume that the sequence $\xi=\{\xi_{t,u,v}\}$ as:

$$\xi_{t,u,v}=\{\lambda_{t,u,v}\eta_{0,0,0}+\lambda_{t-1,u-1,v-1}\eta_{1,1,1}+\cdots+\lambda_{0,0,0}\eta_{t,u,v}\},\tag{1}$$

for all $t,u,v\in\mathbb{N}$.

Now let us prove that $\eta\in\varLambda$. Since $\lambda,\eta\in\varLambda$, then we have:

$$\sum_{t,u,v=0}^{\infty}|\xi_{t,u,v}|=\left|\sum_{t,u,v=0}^{\infty}\begin{pmatrix}\lambda_{t,u,v}\eta_{0,0,0}+\lambda_{t-1,u-1,v-1}\eta_{1,1,1}\\+\cdots+\lambda_{0,0,0}\eta_{t,u,v}\end{pmatrix}\right|$$
$$\leq\sum_{t,u,v=0}^{\infty}|\lambda_{t,u,v}||\eta_{0,0,0}|+\sum_{t,u,v=0}^{\infty}|\lambda_{t-1,u-1,v-1}||\eta_{1,1,1}|+\cdots+\sum_{t,u,v=0}^{\infty}|\lambda_{0,0,0}||\eta_{t,u,v}|$$
$$=\left(\sum_{t,u,v=0}^{\infty}|\lambda_{t,u,v}|\right)\left(\sum_{t,u,v=0}^{\infty}|\eta_{t,u,v}|\right)<\infty.$$

Let us consider $\{p^{\lambda}_{t,u,v}\}$ and $\{p^{\eta}_{t,u,v}\}$ be the $M^{stat}_{\lambda_{t,u,v}}$ and $M^{stat}_{\eta_{t,u,v}}$ transformation of $x=\{x_{g,h,i}\}$, respectively. $\{p^{\xi}_{t,u,v}\}$ is also, $M^{stat}_{\xi_{t,u,v}}$ transformation of $x=\{x_{g,h,i}\}$, where,

$$\{p^{\xi}_{t,u,v}\}=\lim_{t,u,v\to\infty}\tfrac{1}{tuv}|\{(t,u,v)|\in\mathbb{N}: \sum_{g,h,i=0}^{\infty}\xi_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})|\geq\varepsilon\}|.$$

Therefore, using [5]:

$$\lim_{t,u,v\to\infty}\tfrac{1}{tuv}\left|\left\{\begin{matrix}\{t,u,v\}\in\mathbb{N}:\\ \sum_{g,h,i=0}^{\infty}\xi_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\end{matrix}\right|\geq\varepsilon\right\}\right|=$$
$$\lim_{t,u,v\to\infty}\tfrac{1}{tuv}|\{t,u,v\in\mathbb{N}:|\sum_{t,u,v=0}^{\infty}\lambda_{t,u,v}\eta_{0,0,0}(x_{0,0,0}-\mathcal{L})|+|\sum_{t,u,v=0}^{\infty}\lambda_{t-1,u-1,v-1}\eta_{1,1,1}(x_{1,1,1}-\mathcal{L})|+\cdots+|\sum_{t,u,v=0}^{\infty}\lambda_{1,1,1}\eta_{t-1,u-1,v-1}(x_{t-1,u-1,v-1}-\mathcal{L})|+|\sum_{t,u,v=0}^{\infty}\lambda_{0,0,0}\eta_{t,u,v}(x_{t,u,v}-\mathcal{L})|\geq\varepsilon\}||\}\leq$$
$$max\left\{\begin{matrix}\lim_{t,u,v\to\infty}\tfrac{1}{tuv}|\{t,u,v\in\mathbb{N}:\\ \sum_{t,u,v=0}^{\infty}\lambda_{t,u,v}\eta_{0,0,0}(x_{0,0,0}-\mathcal{L})|,\\ |\sum_{t,u,v=0}^{\infty}\lambda_{t-1,u-1,v-1}\eta_{1,1,1}(x_{1,1,1}-\mathcal{L})|,\ldots,\\ |\sum_{t,u,v=0}^{\infty}\lambda_{1,1,1}\eta_{t-1,u-1,v-1}(x_{t-1,u-1,v-1}-\mathcal{L})|,\\ |\sum_{t,u,v=0}^{\infty}\lambda_{0,0,0}\eta_{t,u,v}(x_{t,u,v}-\mathcal{L})|\geq\varepsilon\}|\end{matrix}\right\}\to 0.$$

Since, $\lim_{t,u,v\to\infty}p^{\xi}_{t,u,v}\to 0$, as $t,u,v\to\infty$.

Thus, $\{x_{g,h,i}\}\xrightarrow{M^{stat}_{\lambda_{t,u,v}}}\mathcal{L}$.

Hence $M^{stat}_{\eta_{t,u,v}}\subset M^{stat}_{\xi_{t,u,v}}$.

Similarly, we can show that $M^{stat}_{\lambda_{t,u,v}}\subset M^{stat}_{\xi_{t,u,v}}$.

Theorem 3.2: Let $\lambda,\eta,\xi\in\varLambda_0$. Then, $M^{stat}_{\lambda_{t,u,v}}\subset M^{stat}_{\eta_{t,u,v}}$ iff $\sum_{t,u,v=0}^{\infty}|r_{t,u,v}|<\infty$ and $\sum_{t,u,v=0}^{\infty}r_{t,u,v}=1$.

Proof. Let $\{p^{\lambda}_{t,u,v}\}$, $\{p^{\eta}_{t,u,v}\}$ and $\{p^{\xi}_{t,u,v}\}$ be the $M^{stat}_{\lambda_{t,u,v}}$, $M^{stat}_{\eta_{t,u,v}}$ and $M^{stat}_{\xi_{t,u,v}}$ transform of the sequences $x=\{x_{g,h,i}\}$, $y=\{y_{g,h,i}\}$ and $z=\{z_{g,h,i}\}$, respectively.

Supposing $|x|<1$, $|y|<1$ and $|z|<1$, then

$$\sum_{t,u,v=0}^{\infty} p_{t,u,v}^{\eta} x^{t,u,v} y^{t,u,v} z^{t,u,v}$$

$$= \lim_{t,u,v\to\infty} \frac{1}{tuv} \left|\left\{\left| \sum_{t,u,v=0}^{\infty} \eta_{t,u,v}(x_{0,0,0} - \mathcal{L}) x^{t,u,v} y^{t,u,v} z^{t,u,v} \right| + \cdots + \left| \sum_{t,u,v=0}^{\infty} \eta_{0,0,0}(x_{t,u,v} - \mathcal{L}) x^{t,u,v} y^{t,u,v} z^{t,u,v} \right|\right\} \geq \varepsilon \right|$$

$$= \left(\sum_{t,u,v=0}^{\infty} \eta_{t,u,v} x^{t,u,v} y^{t,u,v} z^{t,u,v} \right)$$

$$\left(\lim_{t,u,v\to\infty} \frac{1}{tuv} \left|\left\{\left| \sum_{g,h,i,t,u,v=0}^{\infty} (x_{g,h,i} - \mathcal{L}) x^{t,u,v} y^{t,u,v} z^{t,u,v} \right|\right\} \geq \varepsilon \right| \right).$$

Similarly, supposing |x|<1, |*y*|<1 and |z|<1, thus

$$\sum_{t,u,v=0}^{\infty} p_{t,u,v}^{\lambda} x^{t,u,v} y^{t,u,v} z^{t,u,v}$$

$$= \lim_{t,u,v\to\infty} \frac{1}{tuv} |\{| \sum_{t,u,v=0}^{\infty} \lambda_{t,u,v}(x_{0,0,0} - \mathcal{L}) x^{t,u,v} y^{t,u,v} z^{t,u,v} | + \cdots + | \sum_{t,u,v=0}^{\infty} \lambda_{0,0,0}(x_{t,u,v} - \mathcal{L}) x^{t,u,v} y^{t,u,v} z^{t,u,v} |\} \geq \varepsilon |$$

$$= \left(\sum_{t,u,v=0}^{\infty} \lambda_{t,u,v} x^{t,u,v} y^{t,u,v} z^{t,u,v} \right)$$

$$\left(\lim_{t,u,v\to\infty} \frac{1}{tuv} \left|\left\{\left| \sum_{g,h,i,t,u,v=0}^{\infty} (x_{g,h,i} - \mathcal{L}) x^{t,u,v} y^{t,u,v} z^{t,u,v} \right|\right\} \geq \varepsilon \right| \right).$$

It is clear from definition (6) that *η*(x,y,z)=*λ*(x,y,z)*r*(x,y,z) for all |x|<1, |*y*|<1 and |z|<1, we have:

$$r(x,y,z)\lambda(x,y,z) \lim_{t,u,v\to\infty} \frac{1}{tuv} \left|\left\{\left| \sum_{g,h,i,t,u,v=0}^{\infty} (x_{g,h,i} - \mathcal{L}) x^{t,u,v} y^{t,u,v} z^{t,u,v} \right|\right\} \geq \varepsilon \right|$$

$$= \eta(x,y,z) \left|\left\{\left| \sum_{g,h,i,t,u,v=0}^{\infty} (x_{g,h,i} - \mathcal{L}) x^{t,u,v} y^{t,u,v} z^{t,u,v} \right|\right\} \geq \varepsilon \right|.$$

That is,

$$r(x,y,z) \sum_{t,u,v=0}^{\infty} p_{t,u,v}^{\lambda} x^{t,u,v} y^{t,u,v} z^{t,u,v} = \sum_{t,u,v=0}^{\infty} p_{t,u,v}^{\eta} x^{t,u,v} y^{t,u,v} z^{t,u,v}.$$

Thus,

$$p_{t,u,v}^{\eta} = r_{0,0,0} p_{t,u,v}^{\lambda} + r_{1,1,1} p_{t-1,u-1,v-1}^{\lambda} + \cdots + r_{t,u,v} p_{0,0,0}^{\lambda} = \sum_{g,h,i=0}^{\infty} a_{t,u,v,g,h,i} p_{g,h,i}^{\lambda}.$$

where, $a_{t,u,v,g,h,i} = \begin{cases} r_{g-t,h-u,i-v}; & if \;\; g \leq t, h \leq u, i \leq v; \\ 0; & if \;\; otherwise. \end{cases}$

Now, we need to show that $M_{\lambda_{t,u,v}}^{stat} \subset M_{\eta_{t,u,v}}^{stat}$ iff the matrix ($a_{t,u,v,g,h,i}$) is regular. So,

$$\sup_{t,u,v\geq 0} \sum_{g,h,i=0}^{\infty} |a_{t,u,v,g,h,i}| = \sup_{t,u,v\geq 0} \sum_{g,h,i=0}^{\infty} |r_{g-t,h-u,i-v}| < \infty.$$

That is $\sum_{t,u,v=0}^{\infty} |r_{t,u,v}| < \infty$.

Also,

$$\lim_{t,u,v\to\infty} \sum_{g,h,i=0}^{\infty} a_{t,u,v,g,h,i} = \lim_{t,u,v\to\infty} \sum_{g,h,i=0}^{\infty} r_{g-t,h-u,i-v} = 1.$$

That is $\sum_{t,u,v=0}^{\infty} r_{t,u,v} = 1$.

Theorem 3.3 Let *λ* and *η* be sequences of non-negative integers tending to ∞ from *Λ* such that *λ*≡*η*.
Then, $M_{\lambda_{t,u,v}}^{stat} = M_{\eta_{t,u,v}}^{stat}$.

Proof. Let $x = \{x_{g,h,i}\} \in M_{\lambda_{t,u,v}}^{stat}$ be a sequence such that:

$$\lim_{t,u,v\to\infty} \frac{1}{tuv} \left|\left\{ \begin{matrix} \{t,u,v\} \in \mathbb{N}: \\ \left| \sum_{g,h,i=0}^{t,u,v} \lambda_{g-t,h-u,i-v}(x_{g,h,i} - \mathcal{L}) \right| \geq \varepsilon \end{matrix} \right\}\right| = 0 \tag{2}$$

for any *ε*>0. Therefore,

$$\lim_{t,u,v\to\infty} \frac{1}{tuv} \left|\left\{ \begin{matrix} \{t,u,v\} \in \mathbb{N}: \\ \left| \sum_{g,h,i=0}^{t,u,v} \eta_{g-t,h-u,i-v}(x_{g,h,i} - \mathcal{L}) \right| \geq \varepsilon \end{matrix} \right\}\right| = \lim_{t,u,v\to\infty} \frac{1}{t,u,v} \left|\left\{ \begin{matrix} \{t,u,v\} \in \mathbb{N}: \\ \left| \sum_{g,h,i=0}^{t,u,v} \frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}} \lambda_{g-t,h-u,i-v}(x_{g,h,i} - \mathcal{L}) \right| \geq \varepsilon \end{matrix} \right\}\right|.$$

where, $\lim_{t,u,v\to\infty} \frac{\lambda_{t,u,v}}{\eta_{t,u,v}} = 1$. Since $\lambda \equiv \eta$. Then

$$\lim_{t,u,v\to\infty} \frac{1}{tuv} \left|\left\{ \begin{matrix} \{t,u,v\} \in \mathbb{N}: \\ \left| \sum_{g,h,i=0}^{t,u,v} \eta_{g-t,h-u,i-v}(x_{g,h,i} - \mathcal{L}) \right| \geq \varepsilon \end{matrix} \right\}\right|$$

$$= \lim_{t,u,v\to\infty} \frac{1}{tuv} \left|\left\{ \begin{matrix} \{t,u,v\} \in \mathbb{N}: \\ \left| \sum_{g,h,i=0}^{t,u,v} \frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}} \lambda_{g-t,h-u,i-v}(x_{g,h,i} - \mathcal{L}) \right| \geq \varepsilon \end{matrix} \right\}\right|$$

$$= \lim_{t,u,v\to\infty} \frac{1}{tuv} |\{\{t,u,v\} \in \mathbb{N}: \left| \sum_{g,h,i=0}^{t,u,v} \frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}} \lambda_{g-t,h-u,i-v}(x_{g,h,i} - \mathcal{L}) + \sum_{g,h,i=0}^{t,u,v} \frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}} \lambda_{g-t,h-u,i-v}(\mathcal{L})\right.$$

$$-\sum_{g,h,i=0}^{t,u,v}\frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}}\lambda_{g-t,h-u,i-v}(\mathcal{L})\right|\geq\varepsilon\Bigg\}\Bigg|\leq$$

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\begin{array}{c}\sum_{g,h,i=0}^{t,u,v}\frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}}\\ \lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\end{array}\right|\geq\varepsilon\end{array}\right\}\right|$$

$$+\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\begin{array}{c}\sum_{g,h,i=0}^{t,u,v}\frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}}\\ \lambda_{g-t,h-u,i-v}(\mathcal{L}-\mathcal{L})\end{array}\right|\geq\varepsilon\end{array}\right\}\right|$$

$$\leq max\left\{\begin{array}{c}\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\}|\\ \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}}\lambda_{g-t,h-u,i-v}(\mathcal{L}-\mathcal{L})\right|\geq\varepsilon\}|\end{array}\right\}$$

$$\leq max\left\{\begin{array}{c}\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\}|,0\end{array}\right\}$$

$$\leq max\left\{\begin{array}{c}\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\left|\frac{\eta_{t,u,v}}{\lambda_{t,u,v}}\lambda_{t,u,v}(x_{0,0,0}-\mathcal{L})\right|,\right.\right.\\ \dots,\left|\frac{\eta_{t-1,u-1,v-1}}{\lambda_{t-1,u-1,v-1}}\lambda_{t-1,u-1,v-1}(x_{1,1,1}-\mathcal{L})\right|,\\ \dots,\left|\frac{\eta_{g-t,h-u,i-v}}{\lambda_{g-t,h-u,i-v}}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|,\\ \dots,\left|\frac{\eta_{t,u,v-t,u,v}}{\lambda_{t,u,v-t,u,v}}\lambda_{t,u,v-t,u,v}(x_{t,u,v}-\mathcal{L})\right|\geq\varepsilon\}|\end{array}\right\}=0.$$

Thus,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\eta_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\end{array}\right\}\right|=0.$$

Therefore, $x_{g,h,i}\xrightarrow{M_{\eta_{t,u,v}}^{stat}}\mathcal{L}$. Hence $M_{\lambda_{t,u,v}}^{stat}\subset M_{\eta_{t,u,v}}^{stat}$.
Conversely, we can show that $M_{\lambda_{t,u,v}}^{stat}\supset M_{\eta_{t,u,v}}^{stat}$.
Since $M_{\lambda_{t,u,v}}^{stat}=M_{\eta_{t,u,v}}^{stat}$.

Theorem 3.4 If the bounded sequence $x=\{x_{g,h,i}\}$ is statistical convergence to $\mathcal{L}$ then, x is statistically $M_{\lambda_{t,u,v}}^{stat}$-summable to $\mathcal{L}$ for every regular matrix A.

Proof. Let $x=\{x_{g,h,i}\}$ be bounded and $M_{\lambda_{t,u,v}}^{stat}$-statistically convergent to $\mathcal{L}$.

To prove that the sequence $x=\{x_{g,h,i}\}$ is statistically $M_{\lambda_{t,u,v}}^{stat}$-summable to $\mathcal{L}$. Now,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\end{array}\right\}\right|=$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\ \sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})+\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\mathcal{L}-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\mathcal{L}|\geq\varepsilon\}|\leq$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\end{array}\right\}\right|+$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}(\mathcal{L}-\mathcal{L})\right|\geq\varepsilon\end{array}\right\}\right|\leq$$

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\end{array}\right\}\right|+$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\mathcal{L}(\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v})|\geq\varepsilon\}|$$

$$\leq max\left\{\begin{array}{c}\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\}|,\\ \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|\mathcal{L}(\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v})\right|\geq\varepsilon\}|\end{array}\right\}$$

$$\leq max\left\{\begin{array}{c}\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\}|,0\end{array}\right\}=0.$$

That is, $\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\end{array}\right\}\right|=0.$

Thus the sequence $x=\{x_{g,h,i}\}$ is statistically $M_{\lambda_{t,u,v}}^{stat}$ -summable to $\mathcal{L}$.

Theorem 3.5 If the bounded sequence $x=\{x_{g,h,i}\}$ is statistically $M_{\lambda_{t,u,v}}^{stat}$ -summable to $\mathcal{L}$ then, x is statistically convergent to $\mathcal{L}$.

Proof. Consider $x=\{x_{g,h,i}\}$ is statistically $M_{\lambda_{t,u,v}}^{stat}$-summable to $\mathcal{L}$, for all $\varepsilon>0$.

That is, $stat-\lim_{t,u,v\to\infty}\lambda_{g-t,h-u,i-v}x_{g,h,i}=\mathcal{L}$. Then,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\{t,u,v\}\in\mathbb{N}:x_{g,h,i}-\mathcal{L}\right|\geq\varepsilon\right\}\right|=0. \tag{3}$$

To prove that the sequence $x=\{x_{g,h,i}\}$ is statistically convergent to $\mathcal{L}$.

Now,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\{t,u,v\}\in\mathbb{N}:x_{g,h,i}-\mathcal{L}\right|\geq\varepsilon\right\}\right|=$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:x_{g,h,i}+\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}x_{g,h,i}-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}x_{g,h,i}-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\mathcal{L}+\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\mathcal{L}-\mathcal{L}|\geq\varepsilon\}|\leq$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|x_{g,h,i}-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}x_{g,h,i}\right|\geq\varepsilon\end{array}\right\}\right|+$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}x_{g,h,i}-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\mathcal{L}|\geq\varepsilon\}|+$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\mathcal{L}-\mathcal{L}\right|\geq\varepsilon\end{array}\right\}\right|$$
$$\leq\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|x_{g,h,i}(1-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v})\right|\geq\varepsilon\end{array}\right\}\right|$$
$$+\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\end{array}\right\}\right|$$
$$+\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{array}{c}\{t,u,v\}\in\mathbb{N}:\\ \left|\mathcal{L}(\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}-1)\right|\geq\varepsilon\end{array}\right\}\right|$$

$$\leq max\begin{Bmatrix} \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: \\ \left|x_{g,h,i}\left(1-\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\right)\right|\geq\varepsilon\}|, \\ \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon\}|, \\ \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: \\ \left|\mathcal{L}(\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}-1)\right|\geq\varepsilon\}| \end{Bmatrix}$$

$$\leq max\begin{Bmatrix} 0,\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon\}|,0 \end{Bmatrix}.$$

By our assumption that, $\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\begin{Bmatrix} \{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}\lambda_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon \end{Bmatrix}\right|=0.$

Therefore, $\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\{t,u,v\}\in\mathbb{N}:\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon\right\}\right|=0.$

Then, the sequence $x=\{x_{g,h,i}\}$ is statistically convergent to $\mathcal{L}$.

4. STATISTICAL $A_{(t,u,v)}$-SUMMABILITY FOR TRIPLE SEQUENCE

In this section, some inclusion relation between statistical convergence and statistical $\boldsymbol{A_{(t,u,v)}}$-summability method has been discussed in triple sequences.

Definition 4.1 Let $A=(a_{(t,u,v,g,h,i)})$ be a six dimensional infinite matrix and $x=\{x_{g,h,i}\}$ a triple sequence. Then the transformation sequence is $A(x)=((Ax)_{t,u,v})$, where $\sum_{t,u,v=0}^{\infty}a_{(t,u,v,g,h,i)}x_{g,h,i}$.

If $\lim_{t,u,v\to\infty}\left((Ax)_{t,u,v}\right)=s$, we say that the triple sequence $x=\{x_{g,h,i}\}$ is A- summable to s, written as $x_{g,h,i}\to s(A)$.

If $\lim_{t,u,v\to\infty}\left((Ax)_{t,u,v}\right)=s$, where $\lim_{t,u,v\to\infty}\left(x_{t,u,v}\right)=s$, we say that the six dimensional infinite matrix $A=(a_{(t,u,v,g,h,i)})$ is regular.

Definition 4.2 Let a triple sequence $x=\{x_{g,h,i}\}$ is said to be A-statistically convergent to $\mathcal{L}$ if for any $\varepsilon>0$, where $g\leq t$, $h\leq u$, $i\leq v$, $g,h,i\in\mathbb{N}$,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: \left|\sum_{t,u,v=0}^{\infty}a_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\}\Big|=0.$$

It is symbolized by $stat_{A_{t,u,v}}-\lim_{g,h,i\to\infty}|x_{g,h,i}|=|\mathcal{L}|$ or $|x_{g,h,i}|\xrightarrow{A_{t,u,v}^{stat}}|\mathcal{L}|$.

Definition 4.3 Let $A=(a_{(t,u,v,g,h,i)})$ be a regular matrix and $x=\{x_{g,h,i}\}$ be a sequence, we say that x is statistically A-summable to $\mathcal{L}$ if for every $\varepsilon>0$,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: \left|\sum_{t,u,v=0}^{\infty}(y_{t,u,v}-\mathcal{L})\right|\geq\varepsilon\}\Big|=0$$

where, $y_{(t,u,v)}=(Ax_{t,u,v})$.

Theorem 4.1 The six dimensional infinite matrix $A=(a_{(t,u,v,g,h,i)})$ is regular if and only if,

(i) $sup_{t,u,v}\sum_{t,u,v=0}^{\infty}\left|a_{(t,u,v,g,h,i)}\right|<\infty$;
(ii) $\lim_{t,u,v\to\infty}a_{(t,u,v,g,h,i)}=0;\ g,h,i=0,1,2,...,$
(iii) $\lim_{t,u,v\to\infty}\sum_{g,h,i=0}^{t,u,v}a_{(t,u,v,g,h,i)}=1;$
(iv) $\lim_{t,u,v\to\infty}\sum_{g=0}^{\infty}a_{(t,u,v,g,h,i)}=0;h,i=0,1,2,...,$
(v) $\lim_{t,u,v\to\infty}\sum_{h=0}^{\infty}a_{(t,u,v,g,h,i)}=0;g,i=0,1,2,...,$
(vi) $\lim_{t,u,v\to\infty}\sum_{i=0}^{\infty}a_{(t,u,v,g,h,i)}=0;g,h=0,1,2,....$

Theorem 4.2 If the bounded sequence $x=\{x_{g,h,i}\}$ is statistical convergence to $\mathcal{L}$ then, x is statistically $A_{t,u,v}^{stat}$-summable to $\mathcal{L}$ for every regular matrix A.

Proof. Let $x=\{x_{g,h,i}\}$ be bounded and $A_{t,u,v}^{stat}$-statistically convergent to $\mathcal{L}$.

To prove that the sequence $x=\{x_{g,h,i}\}$ is statistically $A_{t,u,v}^{stat}$-summable to $\mathcal{L}$. Now,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\begin{Bmatrix} \{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon \end{Bmatrix}\right|$$
$$=\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})+\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}\mathcal{L}-\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}\mathcal{L}|\geq\varepsilon\}|$$
$$\leq\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\begin{Bmatrix} \{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon \end{Bmatrix}\right|+\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\begin{Bmatrix} \{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}(\mathcal{L}-\mathcal{L})\right|\geq\varepsilon \end{Bmatrix}\right|$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\begin{Bmatrix} \{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon \end{Bmatrix}\right|+\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\{t,u,v\}\in\mathbb{N}:\mathcal{L}\left(\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}-\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}\right)\right|\geq\varepsilon\right\}\right|\leq$$
$$max\begin{Bmatrix} \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon\}|, \\ \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: \\ \left|\mathcal{L}(\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}-\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v})\right|\geq\varepsilon\}| \end{Bmatrix}$$
$$\leq max\begin{Bmatrix} \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: \\ \left|\sum_{g,h,i=0}^{t,u,v}a_{g-t,h-u,i-v}(x_{g,h,i}-\mathcal{L})\right|\geq\varepsilon\}|,0 \end{Bmatrix}=0.$$

That is, $\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\begin{Bmatrix} \{t,u,v\}\in\mathbb{N}: \\ \left|a_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon \end{Bmatrix}\right|=0.$

Thus the sequence $x=\{x_{g,h,i}\}$ is statistically $A_{t,u,v}^{stat}$-summable to $\mathcal{L}$.

Theorem 4.3 If the bounded sequence $x=\{x_{g,h,i}\}$ is statistically $A_{t,u,v}^{stat}$ -summable to $\mathcal{L}$ then, x is statistically convergent to $\mathcal{L}$.

Proof. Consider $x=\{x_{g,h,i}\}$ is statistically $M_{\lambda_{t,u,v}}^{stat}$-summable to $\mathcal{L}$, for all $\varepsilon>0$.

That is, $stat-\lim_{t,u,v\to\infty} a_{g-t,h-u,i-v}x_{g,h,i}=\mathcal{L}$. Then,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\{t,u,v\}\in\mathbb{N}: x_{g,h,i}-\mathcal{L}\right|\geq\varepsilon\right\}\right|=0. \tag{4}$$

To prove that the sequence $x=\{x_{g,h,i}\}$ is statistically convergent to $\mathcal{L}$. Now:

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\{t,u,v\}\in\mathbb{N}: x_{g,h,i}-\mathcal{L}\right|\geq\varepsilon\right\}\right|=$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}: x_{g,h,i}+$$
$$\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}x_{g,h,i}-\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}x_{g,h,i}-$$
$$\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}\mathcal{L}+\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}\mathcal{L}-\mathcal{L}|\geq\varepsilon\}|\leq$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{matrix}\{t,u,v\}\in\mathbb{N}:\\ x_{g,h,i}-\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}x_{g,h,i}\end{matrix}\right|\geq\varepsilon\right\}\right|+$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}x_{g,h,i}-$$
$$\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}\mathcal{L}|\geq\varepsilon\}|+$$
$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{matrix}\{t,u,v\}\in\mathbb{N}:\\ \sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}\mathcal{L}-\mathcal{L}\end{matrix}\right|\geq\varepsilon\right\}\right|$$
$$\leq max\left\{\begin{matrix}\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|x_{g,h,i}\left(1-\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}\right)\right|\geq\varepsilon\}|,\\ \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon\}|,\\ \lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|\mathcal{L}(\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}-1)\right|\geq\varepsilon\}|\end{matrix}\right\}$$
$$\leq max\left\{\begin{matrix}0,\lim_{t,u,v\to\infty}\frac{1}{tuv}|\{\{t,u,v\}\in\mathbb{N}:\\ \left|\sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon\}|,0\end{matrix}\right\}.$$

By our assumption that,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\begin{matrix}\{t,u,v\}\in\mathbb{N}:\\ \sum_{g,h,i=0}^{t,u,v} a_{g-t,h-u,i-v}\left(x_{g,h,i}-\mathcal{L}\right)\end{matrix}\right|\geq\varepsilon\right\}\right|=0.$$

Therefore,

$$\lim_{t,u,v\to\infty}\frac{1}{tuv}\left|\left\{\{t,u,v\}\in\mathbb{N}:\left(x_{g,h,i}-\mathcal{L}\right)\right|\geq\varepsilon\right\}\right|=0.$$

Then, the sequence $x=\{x_{g,h,i}\}$ is statistically convergent to $\mathcal{L}$.

5. CONCLUSIONS

Our aim in this article is to extend the known results in Archimedean fields to NAF. This paper aims to explore the concepts of statistical convergence sequence and statistical summability in non-Archimedean fields (NAF). The goal of this study is to provide characterizations for triple sequences using the $(M_{\lambda_{t,u,v}})$ method of summability and statistical $A_{(t,u,v)}$-summability method over NAF. Specifically, we proved the inclusion relations between statistical convergence and statistical $(M_{\lambda_{t,u,v}})$ summability for triple sequences over such fields. Also we proved some results on statistical $A_{(t,u,v)}$-summability method for triple sequences in NAF.

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